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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

Vol. LXVII. November 9, 1906. No. 1

W. H. Maw, Esq., President, in the Chair.

Ralph Falcon, M.A. Oxon., Camerton Hall, Workington, Cumberland; and
Arthur Grant Stillhamer, Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Rodney Boyce, Soudan Survey Department, c/o the Royal Colonial Institute, Northumberland Avenue, S.W. (proposed by E. P. Cotton);
Arthur Cleminson, Deputy Commissioner of Lands, Lagos, West Africa (proposed by E. P. Cotton);
George Innes, M.P.S., Chemist and Optician, Olive Bank, Edinburgh (proposed by Rev. John Spence);
Arthur Kent Lucke, Suez Canal Company's Service, Transit Department, Ismaïlia, Egypt (proposed by E. W. Maunder); and
George Street, M.A., Tutor, Merton House, Southwick, Sussex (proposed by Arthur R. Hinks).

One hundred and sixty-five presents were announ[illegible] having been received since the last meeting, including, amo[illegible]thers:—

Galileo, Opere, Edizio Nazionale, vols. 16, 17, [illegible]ented by the Italian Government; Catalog der Astronom[illegible] Gesell-schaft, Abtheilung ii. Stück 1, [illegible] bis −6° ([illegible]) pre-sented by Professor Becker [and [illegible]ther copy pr[illegible] the *Astronomische Gesellschaft*]; [illegible] Taylor, [illegible] of Applied Optics, presented by the [illegible]

Seventy-eight charts of the Astrographic Chart of the Heavens, presented by the Royal Observatory, Greenwich; and ninety-six charts, presented by the Paris Observatory.

Two enlarged transparencies and lantern slide from negatives of the total eclipse of August 1905, taken at Alcalá Chisvert, Spain, presented by Count de la Baume Pluvinel.

The Early Eclipses of the Sun and Moon. By E. Nevill.

With the view of furnishing Mr Cowell with the further information he desires (*Monthly Notices*, vol. lxvi. p. 474), I have extracted from my memoir on the Observed Errors of Hansen's *Tables de la lune* my investigation of the results yielded by the observations of the ancient Eclipses of the Moon contained in Ptolemy's *Almagest*.

The eclipses and errors of Hansen's Tables are as follow:—

	Date.	Place.	Phase.	Error of Hansen's Tables.	Weight.	Remarks.
1	−720 March 19	Babylon	Beginning	-23^m	2	Total
2	−719 March 8	,,	Middle	−63	½	Mag. = 0·25 on S
3	−719 Sept. 1	,,	Beginning	−43	2	Mag. = 0·55 on N
4	−620 April 21	,,	Beginning	−44	3	Mag. = 0·25 on S
5	−522 July 16	,,	Beginning	−93	1	Mag. = 0·50 on N
6	−501 Nov. 19	,,	Beginning	−28	1	Mag. = 0·25 on S
7	−490 April 25	,,	Beginning	−28	1	Mag. = 0·17 on S
8	−382 Dec. 22	Athens	Beginning	−51	½	Small. Place uncer
9	−381 June 18	Babylon	Beginning	−43	2	Partial
9a	,,	,,	End	−77	1	Duration $3^h\ 0^m$
10	−381 Dec. 12	,,	Beginning	−59	1	Total
11	−200 Sept. 22	,,	Beginning	−26	1	Partial
11a	,,	,,	End	−30	2	Duration $3^h\ 2^m$
12	−199 March 19	Alexandria	Beginning	−38	3	Total
13	−199 Sept. 11	,,	Beginning	−25	2	Total
13a	...	,,	Middle	−19	1	..
14	−173 April 30	,,	Beginning	−44	1	Mag. = 0·58 on N
14a	...	,,	End	−46	1	Duration $2^h\ 43^m$
15	−143 Jan. 27	Rhodes	Beginning	−83	½	Mag. = 0·25 on S
16	+125 May 5	Alexandria	Beginning	−26	2	Mag. = 0·17 on S
17	+133 May 6	,,	Middle	−30	3	Total
18	+134 Oct. 20	,,	Middle	−13	2	Mag. = 0·83 on N
19	+136 March 5	,,	Middle	−48	2	Mag. = 0·50 on N

The data have been taken, primarily, from Professor Newcomb's *Researches on the Motion of the Moon*, but in certain cases I have interpreted the record in a somewhat different manner, on the strength of additional information, and have assumed that the Eclipse No. 8 was observed at Athens, the actual place of observation being left unspecified. This Eclipse No. 8 and Eclipse No. 15 observed at Rhodes stand in a different category to the others, seemingly having been selected from outside Ptolemy's usual authorities for some special reason, possibly because no eclipses observed at either Babylon or Alexandria were observed under the desired conditions. Neither has been used in the discussion which follows.

Writing—

(Tabular—Observed) Longitude $= \Delta L + \sin\alpha \,.\, A_s + \cos\alpha \,.\, A_c + \sin\beta \,.\, B_s + \cos\beta \,.\, B_c + (T - 1800) \,.\, 30'' \,.\, \delta l^* + (T - 1800)^2\, 1'' \,.\, 0\, A$

where α denotes the Moon's mean anomaly and β denotes the Moon's mean argument of latitude. Then the preceding data lead to the expressions—

1	$\Delta L = +11''\cdot1$	$+\cdot88\,A$	$+\cdot47\,A_c$	$-\cdot01\,B_s$	$-\cdot99\,B_c$	$+4\cdot20\,\delta l^*$	$-10\cdot58\,A$
2	$= +28\cdot5$	$+\cdot21$	$+\cdot98$	$-\cdot15$	$-\cdot99$	$+4\cdot20$	$-10\cdot57$
3	$= +25\cdot3$	$+\cdot33$	$-\cdot94$	$+\cdot13$	$+\cdot99$	$+4\cdot20$	$-10\cdot57$
4	$= +19\cdot9$	$-\cdot31$	$+\cdot95$	$-\cdot18$	$+\cdot98$	$+4\cdot03$	$-9\cdot76$
5	$= +42\cdot7$	$+\cdot45$	$+\cdot89$	$+\cdot15$	$-\cdot99$	$+3\cdot87$	$-8\cdot99$
6	$= +12\cdot6$	$+\cdot06$	$+\cdot99$	$-\cdot14$	$+\cdot99$	$+3\cdot84$	$-8\cdot85$
7	$= +14\cdot7$	$+\cdot99$	$-\cdot07$	$-\cdot20$	$+\cdot98$	$+3\cdot82$	$-8\cdot75$
8	$= +28\cdot7$	$-\cdot78$	$-\cdot62$	$+\cdot20$	$-\cdot98$	$+3\cdot65$	$-7\cdot95$
9	$= +18\cdot9$	$+\cdot42$	$+\cdot91$	$-\cdot15$	$+\cdot99$	$+3\cdot64$	$-7\cdot93$
9a	$= +33\cdot8$	$+\cdot42$	$+\cdot91$	$-\cdot15$	$+\cdot99$	$+3\cdot64$	$-7\cdot93$
10	$= +35\cdot0$	$-\cdot01$	$-\cdot99$	$+\cdot07$	$-\cdot99$	$+3\cdot64$	$-7\cdot93$
11	$= +12\cdot4$	$-\cdot82$	$+\cdot57$	$-\cdot11$	$+\cdot99$	$+3\cdot34$	$-6\cdot66$
11a	$= +14\cdot3$	$-\cdot82$	$+\cdot57$	$-\cdot11$	$+\cdot99$	$+3\cdot34$	$-6\cdot66$
12	$= +20\cdot3$	$+\cdot97$	$-\cdot22$	$-\cdot02$	$-\cdot99$	$+3\cdot33$	$-6\cdot65$
13	$= +13\cdot5$	$-\cdot96$	$-\cdot29$	$+\cdot00$	$+\cdot99$	$+3\cdot33$	$-6\cdot65$
13a	$= +10\cdot2$	$-\cdot96$	$-\cdot29$	$+\cdot00$	$+\cdot99$	$+3\cdot33$	$-6\cdot65$
14	$= +25\cdot8$	$+\cdot33$	$-\cdot95$	$+\cdot12$	$+\cdot99$	$+3\cdot29$	$-6\cdot49$
14a	$= +27\cdot0$	$+\cdot33$	$-\cdot95$	$+\cdot12$	$+\cdot99$	$+3\cdot29$	$-6\cdot49$
15	$= +48\cdot9$	$+\cdot05$	$-\cdot99$	$-\cdot15$	$-\cdot99$	$+3\cdot24$	$-6\cdot27$
16	$= +13\cdot9$	$-\cdot95$	$-\cdot29$	$-\cdot17$	$+\cdot98$	$+2\cdot83$	$-4\cdot68$
17	$= +14\cdot1$	$-\cdot69$	$+\cdot72$	$+\cdot08$	$-\cdot99$	$+2\cdot81$	$-4\cdot63$
18	$= +6\cdot3$	$+\cdot86$	$+\cdot50$	$+\cdot08$	$+\cdot99$	$+2\cdot81$	$-4\cdot63$
19	$= +21\cdot7$	$+\cdot62$	$-\cdot78$	$+\cdot19$	$-\cdot98$	$+2\cdot80$	$-4\cdot62$

Let—

A denote the values *obtained from* Hansen's Tables, with his

tabular mean motions and secular accelerations replaced by Hansen's finally deduced values for these elements.

B denote the values obtained from Hansen's Tables after replacing his tabular values for the secular accelerations by those derived theoretically from the secular diminution of the eccentricity of the terrestrial orbit, removing his empirical Venus term of long period, and making the necessary corrections to the mean motions requisite to bring the amended Tables as far as possible into accord with the modern observations.

C denote the values obtained from Hansen's Tables after applying the corrections indicated by the discussion of the observations of the Moon made between 1650 and 1890, employing for the secular acceleration of the mean longitude the value $+7''\cdot 43$, of the longitude of the node the value $6''\cdot 07$, and of the longitude of the lunar perigee the value $-32''\cdot 53$.

Then the preceding expressions yield for the tabular errors of the Tables as modified in B and C the values

No.	Date.	Tables B.—Obs. ΔL.	Tables B.—Obs. ΔE.N.P.D.	Tables C.—Obs. ΔL.	Tables C.—Obs. ΔE.N.P.D.
		′	′	′	′
1	−720 March 19	−38·1	...	−17·8	...
2	−719 March 8	−19·2	+1·2	+ 3·9	+1·9
3	−719 Sept. 1	−28·9	+1·8	−16·2	+1·1
4	−620 April 21	−24·5	+6·0	− 4·2	+5·6
5	−522 July 16	+ 2·6	−2·4	+21·2	−1·8
6	−501 Nov. 19	−26·3	+4·2	− 7·6	+3·6
7	−490 April 25	−26·2	+5·4	−11·8	+4·8
8	−382 Dec. 22	− 9·5 ?	...	+ 1·5 ?	...
9	−381 June 18	−10·4	+8·3	+ 5·9	+7·8
10	−381 Dec. 12	− 3·9	...	+ 5·8	...
11	−200 Sept. 22	−16·2	+2·0	− 3·3	+1·6
12	−199 March 19	−10·0	...	+ 0·5	...
13	−199 Sept. 11	−18·0	...	− 7·7	...
14	−173 April 30	− 4·1	+2·6	− 4·0	+1·8
15	−140 Jan. 27	+19·4 ?	−1·2	+27·3 ?	−0·8
16	+125 April 5	− 6·0	+2·5	− 0·9	+2·2
17	+135 May 6	− 4·9	...	+ 4·2	...
18	+134 Oct. 20	−12·8	+1·4	− 4·2	+1·7
19	+136 March 5	+ 1·5	−5·0	+ 7·5	−4·7

The Eclipse No. 8, whose place of observation is not stated, but has been assumed to be Athens, yields values fairly accordant with those from the other eclipses. The Eclipse No. 15 observed at Rhodes is quite discordant, indicating either that Zech was correct in supposing the time to be an hour in error, or else that the phase observed was the middle of the eclipse: the first supposition would

render the error in longitude $B = -14'{\cdot}2$ $C = -6'{\cdot}3$, and the second the values $B = -12'{\cdot}4$ $C = -4'{\cdot}5$; results in fair accord with those from the other eclipses. Neither eclipse has been included in the investigation of the errors in longitude, however, owing to the inherent uncertainty.

From the preceding, there are derived the following values for the mean outstanding errors of the Tables as amended :—

Epoch.	Tables B.—Obs.	Tables C.—Obs.	Weight.
-720	$\Delta L = -28'{\cdot}7$	$= -15'{\cdot}2$	3
-620	$= -24{\cdot}5$	$= -4{\cdot}2$	2
-504	$= -16{\cdot}6$	$= +0{\cdot}6$	2
-381	$= -8{\cdot}8$	$= +5{\cdot}9$	4
-192	$= -11{\cdot}7$	$= -1{\cdot}2$	10
$+125$	$= -6{\cdot}3$	$= +0{\cdot}9$	2
$+134$	$= -5{\cdot}3$	$= +2{\cdot}6$	6
Weighted mean	$\Delta L = -12{\cdot}8$	$= -0{\cdot}9$	

It is obvious, therefore, that these eclipses cannot be reconciled with the theoretical value of the secular acceleration in mean longitude embodied in Tables B, or with any value less than that adopted in Tables C, or below $+7''{\cdot}40$.

Considering the correction $\Delta B''$ to the secular acceleration of the lunar node, the mean of the results from the observed phases 3, 9a, 11a, and 14 yield the expression—

			Tables B.—Obs.	Tables C.—Obs.
$+0{\cdot}70\,\Delta B''$	$-3{\cdot}62\,\delta l^*$	$+7{\cdot}91\,A$	$= -15{\cdot}22$	$= -0{\cdot}25$

whilst the mean of the results from the observed phases 4, 5, 6, 7, 8, 9, 11, 14a, and 16 gives

$$+1{\cdot}38\Delta B'' + 3{\cdot}59\delta l^* - 7{\cdot}78A = +14{\cdot}20 = +0{\cdot}03.$$

Hence

$$\Delta B'' - 0{\cdot}03\delta l^* + 0{\cdot}13A = -0''{\cdot}49 = +0''{\cdot}11.$$

The correction obtained is small.

The values of the corrections to the tabular E.N.P.D. regarding them all as of equal weight yield the values—

Epoch.		Obs.		Obs.	Weight
-565	Tables B. ΔE.N.P.D.	$= +3'{\cdot}50$	Tables C. ΔE.N.P.D	$= +$ [illegible]	[illegible]
-172		$= +0{\cdot}85$		$= +$ [illegible]	[illegible]
$+132$		$= -0{\cdot}27$		$= -$ [illegible]	[illegible]

Regarding these as indicating a p [illegible]
they correspond to the values—

Tables B.	ΔE.N.P.D.	$= +0'{\cdot}10$
Tables C.		$= -0{\cdot}02$

These values are too large to be real, corresponding to a change of some 20″ per century,—an impossible amount if assumed to extend up to the present time.

If they be regarded as due to the effect of corrections required by the adopted tabular value of the argument of latitude β, necessitated by an erroneous tabular value of the secular acceleration of the Moon's node, then the values indicated are—

		Obs.	Cal.		Obs.	Cal.
−565	Tables B. $\frac{1}{11}\,\Delta\beta=$	$-3'.84$	$-3'.84$	Tables C. $\frac{1}{11}\,\Delta\beta=$	$-3'.23$	$-3'.23$
−172		-1.45	-2.47	$=$	-1.05	-2.19
+132		-2.77	-1.70	$=$	-3.07	-1.51

corresponding to the corrections to the secular acceleration of the node of

$$\text{Tables B.}\quad \Delta B = -4''.25\,(T-1800)^2$$
$$\text{Tables C.}\quad \Delta B = -3.77\,(T-1800)^2$$

The discordance between the observed and calculated values is very great, so that such a correction can have no real weight. Nor can it be improved by any change in the tabular mean motion of the node.

Taking the values as they stand, they would appear to involve an inequality in the complete expression for the Moon's latitude with a coefficient of about a minute of arc and a period of from seven to eight hundred years. Such a term would undoubtedly reconcile these values of $\Delta\beta$, and render consistent the values found above for the correction to the secular acceleration of the node. But it could not be rendered consistent with the modern observations, for it would mean a change in the Moon's latitude of at least ten to twelve seconds of arc within the period 1820 and 1890; though in all probability the change would be at least two or three times as great, as it is very unlikely that the epoch of the term should be such as to render the amount of change a minimum.

Such a change of the Moon's latitude is quite inconsistent with the observations.

The Arabian Eclipses.

The Arabian eclipses quoted by Professor Newcomb in § 5 of his *Researches on the Motion of the Moon*, from Caussin's edition of "le Livre de la grande Table Hakémite," have to be taken into account to obtain further information on the points which have been discussed in the previous investigation of the Ptolemaic eclipses of the Moon.

Professor Newcomb points out that some of the observed altitudes of the Moon are impossible, and some of the observed altitudes of the clock stars are irreconcilable with each other. Mr Knobel, in *Monthly Notices*, vol. xxxix. pp. 338–340, suggests that these impossible values are due to errors in copying

the Arabian figures, which often differ so slightly that errors can easily be made. I have had no means of referring to the original authorities, but as the result of some inquiries made for me by Mr Marth, I think that the altitude of Arcturus on 925 April 11, which is given as 11°, should be read as 31°; that the altitude of Arcturus on 929 January 27, which is given as 18°, should be 33°; and that the altitude of the Moon on 983 March 1, which is given as 66°, should be 62°.

When the magnitude of the eclipses is given, it does not state whether they were estimated from the upper or lower limb, as is done by Ptolemy, so that when the eclipse is large it is uncertain whether the central line of eclipse passed north or south of the centre of the Sun. This renders the interpretation of the solar eclipses of 993 August 19 and 1004 January 24 somewhat uncertain; and though in each case the most probable path has been taken, some doubt must attach to the results.

The results of the investigation of these early eclipses observed by the Arabian astronomers are as follows:—

The Early Arabian Eclipses:—

Eclipses of the Sun observed at *Bagdad*:—

No.	Date.	Phase.	Tabular Errors.	Weight.	Magnitude.	Remarks.
			m			
1	829 Nov. 29	Beginning	−45·9	0		Rejected.
1a	,,	End	−18·0	1		
2	923 Nov. 10	End	−16·7	1		
3	928 Aug. 17	End	−10·0	2		

Eclipses of the Sun observed at *Cairo*:—

No.	Date.	Phase.	Tabular Errors.	Weight.	Magnitude.	Remarks.
4	977 Dec. 12	Beginning	− 4·5	1		
4a	,,	End	− 3·2	1		
5	978 June 8	Beginning	−19·7	2		
5a	,,	End	− 3·5	2		
6	979 May 28	Beginning	− 6·3	2		
7	985 July 20	Beginning	−24·3	½	0·25	
7a	,,	End	−14·7	½		
8	993 Aug. 19	Beginning	− 0·3	2	0·74	
8a	,,	End	−19·9	1		
9	1004 Jan. 24	Beginning	−11·1	1	0·92	

Eclipses of the Moon observed at *Bagdad*:—

No.	Date.	Phase.	Tabular Errors.	Weight.	Magnitude.	Remarks.
1	854 Aug. 11	Beginning	− 7·1	1		
2	856 June 21	Beginning	− 3·7	2		
3	923 June 1	End	− 6·7	2		
4	925 April 11	Beginning	− 4·5	1		
4a	,,	End	+ 2·2	3		

Eclipses of the Moon observed at *Bagdad—continued.*

No.	Date.	Phase.	Tabular Errors. m	Weight.	Magnitude.	Rem
5	927 Sept. 13	Beginning	+13·2	0	Doubtful. Time possibly calcu	
6	929 Jan. 27	Beginning	–17·8	1	Altitude corrected for error in c	
7	933 Nov. 4	Beginning	– 1·8	2		

Eclipses of the Moon observed at *Cairo* :—

No.	Date.	Phase.	Tabular Errors.	Weight.	Magnitude.	Rem
8	979 May 14	End	–11·3	½	Doubtful. Perhaps calculated	
9	979 Nov. 6	Beginning	– 3·4	1		
9a	,,	End	–13·9	1		
10	980 May 2	End	– 4·8	1		
11	981 April 21	Beginning	– 2·8	1	Magnitude =0·25.	
11a	,,	End	+ 1·6	1		
12	981 Oct. 15	Beginning	–14·8	1	Magnitude =0·42.	
13	983 March 1	Beginning	– 4·9	1	Altitude corrected for error in c	
13a	,,	End	–17·7	2		
14	986 Dec. 18	Beginning	–19·2	2	Magnitude =0·83.	
15	1002 March 1	Beginning	– 2·5	2	Total.	

From these observations are obtained the following expressions :—

Eclipses of the Sun :—

1	$\Delta L = +\ 7·6$	$+·98\,A_s$	$+·16\,A_c$	$-·06\,B_s$	$+·99\,B_c$	$+9·70\,\delta l$ *	–
2	$= +\ 6·7$	$-·90$	$+·42$	$-·10$	$+·99$	$+8·76$	–
3	$= +\ 4·5$	$-·99$	$+·14$	$+·05$	$+·99$	$+8·72$	–
4	$= +\ 1·7$	$-·19$	$-·98$	$-·11$	$+·99$	$+8·42$	–
4a	$= +\ 1·1$	$-·19$	$-·98$	$-·11$	$+·99$	$+8·42$	–
5	$= +\ 5·4$	$-·26$	$+·96$	$+·01$	$-·99$	$+8·41$	–
5a	$= -\ 1·9$	$-·26$	$+·96$	$+·01$	$-·99$	$+8·41$	–
6	$= +\ 3·8$	$-·90$	$+·42$	$-·13$	$-·99$	$+8·40$	–
7	$= +10·6$	$+·99$	$-·14$	$-·09$	$+·99$	$+8·15$	–
7a	$= +\ 9·1$	$+·99$	$-·14$	$-·09$	$+·99$	$+8·15$	–
8	$= +\ 0·1$	$+·44$	$-·90$	$-·04$	$-·99$	$+8·06$	–
8a	$= +10·2$	$+·44$	$-·90$	$-·04$	$-·99$	$+8·06$	–
9	$= +\ 4·4$	$-·94$	$-·33$	$-·05$	$-·99$	$+7·96$	–

Eclipses of the Moon:—

1	$\Delta L = +\ 3·6$	$-·66$	$-·76$	$-·01$	$-·99$	$+9·45$	–
2	$= +\ 1·9$	$+·99$	$+·00$	$+·20$	$-·98$	$+9·43$	–
3	$= +\ 3·4$	$-·39$	$+·92$	$+·14$	$+·99$	$+8·77$	–
4	$= -\ 2·3$	$+·93$	$-·38$	$-·09$	$+·99$	$+8·75$	–
4a	$= +\ 1·1$	$+·93$	$-·38$	$-·09$	$+·99$	$+8·75$	–

Eclipses of the Moon—*continued*

= + 9·1	− ·90	− ·44	− ·10	+ ·99	+8·71	− 1.26
= + 0·9	− ·57	+ ·82	+ ·02	+ ·99	+8·64	− 1·25
= + 1·7	− ·96	− ·27	+ ·11	− ·99	+8·20	− 1·12
= + 7·0	− ·96	− ·27	+ ·11	− ·99	+8·20	− 1·12
= + 2·4	+ ·80	+ ·60	+ ·06	+ ·99	+8·20	− 1·11
= + 1·4	+ ·07	+ ·99	+ ·02	− ·99	+8·19	− 1·11
= − 0·8	+ ·07	+ ·99	+ ·02	− ·99	+8·19	− 1·11
= + 7·7	+ ·37	− ·92	− ·16	− ·99	+8·18	− 1·11
= + 2·5	− ·85	− ·53	− ·09	+ ·99	+8·17	− 1·10
= + 8·9	− ·85	− ·53	− ·09	+ ·99	+8·17	− 1·10
= + 9·7	+ ·17	+ ·98	− ·10	+ ·99	+8·15	− 1·10
= + 1·3	− ·07	− ·99	+ ·06	− ·99	+7·98	− 1·06

From these there are derived the following values :—

			Tables B.—Obs.		Tables C.—Obs.		Weight.	
			ΔL.	ΔE.N.P.D.	ΔL.	ΔE.N.P.D.	ΔL.	ΔE.N.P.D.
Eclipses of the Sun :—								
			′	′	′	′		
1	829 Nov.	29	+2·8	...	+5·1	...	1	...
2	923 Nov.	10	+3·8	...	+6·0	...	1	...
3	928 Aug.	17	+1·7	...	+3·9	...	2	...
4	977 Dec.	12	−1·4	+2·4	+ ·8	+2·3	2	1
5	978 June	8	− ·8	+4·5	+1·3	+4·6	4	1
6	979 May	28	+1·1	...	+3·3	...	2	...
7	985 July	20	+7·4	+2·2	+9·4	+2·1	1	2
8	993 Aug.	19	+1·0	−4·1*	+3·0	−4·0*	3	1
9	1004 Jan.	24	+2·0	+2·9*	+3·9	+2·8*	1	1
Mean (weighted)			+1·01	+1·68	+3·21	+1·65		
Eclipses of the Moon :—								
1	854 Aug.	11	−0·9	...	+1·3	...	1	...
2	856 June	21	−2·4	...	−0·2	...	2	...
3	923 June	1	+0·5	...	+2·8	...	2	..
4	925 April	11	−2·7	+3·1	−0·5	+3·0	4	1
6	929 Jan.	27	+6·2	...	+8·4	...	1	...
7	933 Nov.	4	−1·9	...	+0·3	..	1	...
9	979 Nov.	6	+1·8	+2·7	+3·9	+2·8	2	1
10	980 May	2	+0·0	...	+2·0	...	1	...
11	981 April	21	−2·1	−0·6	+0·0	−0·7	2	2
12	981 Oct.	15	+5·2	+1·2	+7·2	+1·3	1	1
13	983 March	1	+4·3	+6·2	+6·3	+6·1	3	1
14	986 Dec.	18	+7·3	−1·2	+9·3	−1·3	2	1
15	1002 March	1	−1·2	...	+0·8	..	2	
Mean (weighted)			+1·15	+1·34	+2·73	+1·50		

The mean results being the same for both solar and lunar eclipses, they may be united into one series and yield the values—

Epoch.	Tables B.—Obs.	Tables C.—Obs.	Weight.	Tables B.—Obs.	Tables C.—Obs.	Weight
849	$\Delta L = -0'{\cdot}75$	$= +1'{\cdot}50$	4	ΔE.N.P.D. =		
927	$= +0{\cdot}11$	$= +2{\cdot}25$	10	$= +3{\cdot}10$	$= +3{\cdot}00$	1
981	$= +1{\cdot}79$	$= +3{\cdot}88$	20	$= +2{\cdot}11$	$= +2{\cdot}07$	10
999	$= +1{\cdot}22$	$= +2{\cdot}44$	6	$= -1{\cdot}65$	$= -1{\cdot}65$	2
Mean (weighted)	$= +1{\cdot}23$	$= +3{\cdot}21$	36	$= +1{\cdot}45$	$= +1{\cdot}41$	13

These Arabian eclipses yield results in accord with the theoretical values of the secular acceleration, and admit of being properly represented by no value of the secular acceleration in mean longitude greater than

$$+6''{\cdot}20$$

whereas it has been shown that the Ptolemaic eclipses of the Moon can be represented by no value less than

$$+7''{\cdot}40$$

Hence both series cannot be properly represented by any one value of the secular acceleration, as the mean value

$$+6''{\cdot}80$$

fails to represent either.

Considering the tabular errors in E.N.P.D., there are seven eclipses by which the error can be determined through the observed duration of the eclipse. These are—

Solar Eclipses.	Tables B.—Obs.		Lunar Eclipses.	Tables B.—Obs.	
	E.N.P.D.	$\frac{1}{11}\Delta\beta$		E.N.P.D.	$\frac{1}{11}\Delta\beta$
977 Dec. 12	$+2'{\cdot}4$	$-2'{\cdot}4$	925 April 11	$+3'{\cdot}1$	$-3'{\cdot}1$
978 July 8	$+4{\cdot}5$	$+4{\cdot}5$	979 Nov. 6	$+2{\cdot}7$	$+2{\cdot}7$
985 July 20	$+2{\cdot}2$	$-2{\cdot}2$	981 April 21	$+0{\cdot}5$	$-0{\cdot}5$
			983 March 1	$+6{\cdot}2$	$-6{\cdot}2$
Mean	$+3{\cdot}03$	$-0{\cdot}03$		$+3{\cdot}12$	$-1{\cdot}77$

The errors regarded as errors in E.N.P.D. are the same from both solar and lunar eclipses, but regarded as due to errors in the adopted longitude of the node, they are discordant. If united, they yield the mean values—

$$\Delta\text{E.N.P.D.} = +3'{\cdot}09 \qquad \tfrac{1}{11}\Delta\beta = -1'{\cdot}03$$

Hence, like the Ptolemaic eclipses of the Moon, they indicate a uniform correction to the tabular E.N.P.D. rather than the need of a correction to the tabular longitude of the node.

The six eclipses where the recorded magnitude is available for determining the tabular error in E.N.P.D. are—

Solar Eclipses.		Tables B.—Obs.		Lunar Eclipses.		Tables B.—Obs.	
		ΔE.N.P.D.	$\frac{1}{11}\Delta\beta$			ΔE.N.P.D.	$\frac{1}{11}\Delta\beta$
985 July	20	$+2'.2$	-2.2	981 April	21	-1.8	$+1.8$
993 Aug.	19	-4.1	-4.1	981 Oct.	15	$+1.2$	$+1.2$
1004 Jan.	24	$+2.9$	$+2.9$	986 Dec.	18	-1.2	$+1.2$
Mean		$+0.33$	-1.13			-0.60	$+1.40$

If these be united they yield the mean values—

$$\Delta\text{E.N.P.D.} = -0'.13 \qquad \tfrac{1}{11}\Delta\beta = +0'.13$$

But the separate results are quite discordant, and disagree with the results obtained from the observed durations. A systematic difference between the results obtained from the magnitude and duration of an eclipse is quite possible in the case of these early observations with unassisted vision, especially in the case of a lunar eclipse, but the observations do not indicate any difference of sensible extent. Thus, in the case of the solar eclipse of 985 July 20, the observed duration and magnitude yield the same value for the error in E.N.P.D. In the case of the lunar eclipse of -173 April 30 the observed duration is $2^{m}.4$ greater than that corresponding to the observed magnitude, whilst in the case of the lunar eclipse of 981 April 21 the observed duration is $4^{m}.5$ less than that indicated by the magnitude.

If the two series be combined they yield the values

	Table B.—Obs.	Table B.—Obs.	Weight.
Solar Eclipses	$\Delta\text{E.N.P.D} = +1'.68$	$\frac{1}{11}\Delta\beta = -0'.58$	6
Lunar Eclipses	$= +1.53$	$= -0.41$	7
Mean	$\Delta\text{E.N.P.D.} = +1.60$	$\frac{1}{11}\Delta\beta = -0.49$	13

From the preceding investigations the mean values for the errors in E.N.P.D. for the different epochs may be written as

-565	$\Delta\text{E.N.P.D.} = +3'.50$	$\frac{1}{11}\Delta\beta = -3'.84$
-172	$= +0.85$	$= -1.45$
$+132$	$= -0.27$	
$+980$	$= +1.60$	
$+1800$	$= .00$	

It is obvious that no correction to th[illegible] N.P.D. nor to the mean motion and secular accel[illegible] node will serve to represent these values, thoug[illegible] e to bring them into closer ac[illegible] [illegible]od with a coefficient of ab[illegible] hundred years.

Disregarding the apparently periodical variations, the observed values are best represented by the expression

$$\tfrac{1}{11}\,\Delta\beta = -0''\cdot50\,(T-1800) - 0''\cdot33\,(T-1800)^2$$

but the weight of such a correction must be very small—to be, in fact, a mere possibility.

By assuming that the observed tabular error is due to a gradual change in both E.N.P.D. and argument of latitude β, a closer agreement may be obtained. Separating the observations into those made at the two nodes, the results are

Ascending Node.			*Descending Node.*		
−620	ΔE.N.P.D. = $-0'\cdot60$	weight 2	−540	ΔE.N.P.D. = $+5'\cdot14$	weight 5
− 2	$= -3\cdot10$	2	− 31	$= +1\cdot76$	5
+987	$= +1\cdot44$	5	+978	$= +1\cdot70$	8

These may be regarded as indicating

	Obs.		Obs.	Cal.
−580	ΔE.N.P.D. = $+2'\cdot27$		$\tfrac{1}{11}\,\Delta\beta = -2'\cdot87$	$-2'\cdot41$
− 17	$= -0\cdot67$		$= -2\cdot43$	$-1\cdot46$
+983	$= +1\cdot57$		$= -0\cdot13$	$-0\cdot20$

corresponding to the correction

$$\tfrac{1}{11}\,\Delta\beta = +1''\cdot00\,(T-1800) - 0''\cdot30\,(T-1800)^2$$

but it is obvious that the results for the middle epoch are discordant.

It is to be remembered that the observations in the three groups were made under somewhat different conditions. The first group consists of observations made by Babylonian astronomers without the aid of any instrumental means; the second group by Grecian astronomers with some instrumental aid; and the third group by Arabian astronomers by the aid of pinule graduated astrolabes. It is possible, therefore, that the estimates of magnitudes may be systematically different; and it is noteworthy that if it be supposed that the Grecian astronomers under-estimated the magnitude of the five eclipses observed between −200 and +136 by a digit as taken from the southern limbs, it would change the observed errors of the middle group to

$$-\;2\ \Delta E.N.P.D. = -1'\cdot26 \qquad -31\ \Delta E.N.P.D. = +3'\cdot28$$

corresponding to the derived correction

$$-17\ \Delta E.N.P.D. = +1'\cdot01 \qquad \tfrac{1}{11}\,\beta\Delta = +2'\cdot27$$

thus reducing the discrepancy, but still leaving it far too large for any confidence to be placed in the resulting value for the correction to the node.

How far do the results obtained in the preceding investigations of the observations of these early eclipses agree with Mr Cowell's conclusion that the Moon's argument of latitude shows a secular

acceleration about four and a half seconds greater than that theoretically indicated by the secular decrease in the eccentricity of the terrestrial orbit?

It will be seen that the contact observations of the Ptolemaic eclipses do not indicate the existence of such a correction, and the similar observations of the Arabian eclipses do not extend over a sufficiently long period to enable any conclusion to be formed.

The observed magnitudes of these Ptolemaic eclipses, however, may be regarded as indicating the existence of a secular term of this nature, though of smaller dimensions; but the deduced correction is entitled to little weight, as the separate results are far from accordant unless a considerable term of long period is assumed to exist, and, as they stand, seem more consistent with a secular change in the E.N.P.D. of the Moon.

The magnitude and duration of the early eclipses of the Sun and Moon observed by the Arabian astronomers, likewise, indicate the existence of a correction to the tabular E.N.P.D. of the Moon, which might be ascribed to a secular term in the expression for the Moon's argument of latitude of similar character to that suggested by Mr Cowell; but the separate results show discordances greater than can be ascribed to outstanding errors of observation, which renders it difficult to say whether what is indicated is a change in the adopted secular variation in the argument of latitude, or an analogous change in the E.N.P.D.'s themselves.

When the two series of eclipse observations are united, unfortunately it becomes still less easy to reach a definite conclusion as to which cause the observed deviations from the tabular E.N.P.D. are due to. The observed discordances are so considerable, whatever interpretation be given, that little weight can be assigned to the conclusion which is drawn. Still, though the weight may be small, the observations can be held to support the existence of a secular term in the Moon's argument of latitude greater than can be deduced by theory from the observed decrease in the eccentricity of the terrestrial orbit. Yet they are also consistent, though in a lesser degree, with a secular increase in the observed E.N.P.D., and in this connection it is to be remembered that Tycho Brahe's observations indicate an E.N.P.D. greater by 40″ than that assigned by Hansen's Tables, and that the eclipses of the Sun during the period 1620–1670 indicate an E.N.P.D. from 10″ to 15″ greater than that assigned by the tables.

The only conclusion that can be legitimately drawn from these early eclipse observations appears to be that they are certainly not inconsistent with Mr Cowell's conclusion that the M[illegible] nent of latitude requires an increased secular acceleratio[illegible]

In my memoir it was decided that the uncertai[illegible] o the correction to the tabular E.N.P.D. indicated [illegible] e observations was too great for them to be taken [illegible] determining the correction to the tabular value of the lunar node.

Natal Observatory:
1906 *August* 17.

On the Early Eclipses. By E. Nevill.

The following notes will serve to elucidate some points in Mr Cowell's paper on the *Ancient Eclipses* in *Monthly Notices*, vol. lxvi. p. 473.

As it was not proposed to use these records of early eclipses of the Sun for deriving corrections to the lunar tables, but only to ascertain how far they were represented by the amended tables, it was judged sufficient to employ approximative forms of calculation; and the data for this purpose were obtained during my visit to England in 1890, from Oppolzer's *Canon der Finsternisse* (Wien, 1887), by applying to his data the corrections necessary to reduce them to the theoretical values of the secular acceleration in mean longitude, longitude of perigee, and longitude of node.

The elements employed by Oppolzer are stated to correspond to the following:—

Mean longitude	$\zeta =$ Hansen's Tables +	$\{\; 0''.00 - 26''.34T - 3''.574T^2 - 0.004964T$
Longitude of Perigee	$A =$ Hansen's Tables +	$\{\; 0.00 - 26.34T - 16.073T^2 - 0.016841T$
Longitude of Node	$B =$ Hansen's Tables +	$\{\; -5.50 + 53.06T - 2.956T^2 - 0.004331T$

where T denotes centuries reckoned from the epoch 1800.

To reduce these to the theoretical values of the secular accelerations, it is necessary to apply the corrections

$$\begin{aligned} &\text{Mean longitude} &&= -\ 2''.496T^2 \\ &\text{Longitude of Perigee} &&= +13.128T^2 \\ &\text{Longitude of Node} &&= +\ 2.672T^2 \end{aligned}$$

At the same time, in order to bring the amended tables into accord with modern observations, it is necessary to apply the further corrections

$$\begin{aligned} &\text{Mean longitude} &&= -30''.00T \\ &\text{Longitude of Perigee} &&= +10.80T \\ &\text{Longitude of Node} &&= +\ 5.00T \end{aligned}$$

Hence the total corrections to be applied to the elements of the Moon's orbit employed by Oppolzer in order to reduce them to the theoretical values are:

$$\begin{aligned} \Delta\zeta &= +0''.00 - 3''.66T - 2''.496T^2 \\ \Delta A &= +0.00 + 37.14T + 13.128T^2 \\ \Delta B &= +0.00 - 48.06T + 2.672T^2 \end{aligned}$$

Oppolzer's formulæ for calculating the latitude ϕ and longitude λ of the position of the curve of central totality for the beginning, middle, and end of an eclipse are

Beginning	$\sin\phi$	$= -\cos\delta'\cos(N'+W)$	$\lambda = -\mu + \frac{15^\circ}{n}\cos W - \tan(N'+W)\,\mathrm{cosec}$
Middle	$\sin(\phi-\delta') =$	$\sin W\,\mathrm{cosec}\,N'$	$\lambda = -\mu - \frac{15^\circ}{n}\cot N'\sin W$
End	$\sin\phi$	$= +\cos\delta'\cos(N'-W)$	$\lambda = -\mu - \frac{15^\circ}{n}\cos W - \tan(N'-W)\,\mathrm{cosec}$

where

$$\sin W = \frac{\sin(b-b')}{\sin(\pi-\pi')} \cdot \sin(N'+h)$$

$$\tan h = \tan \epsilon \cdot \cos L'$$

$$\mu = 15^\circ.H - \frac{15^\circ}{n} \cdot \sin W \cdot \cot(N'+h) - \text{Equation of Time.}$$

Here

H denotes the epoch of conjunction measured from Greenwich Noon.
b, b' denote the latitude of the Moon and Sun.
π, π' denote the horizontal parallaxes of the Moon and Sun.
L', δ' denote the true longitude and declination of the Sun.
ϵ denotes the obliquity of the Ecliptic.

The other quantities are taken as they stand from Oppolzer's *Canon*.

It is obvious that it is only W and μ that will suffer any material change by the introduction of the correction $\Delta\zeta$, ΔA, and ΔB.

Then α denoting the Moon's mean anomaly and β the Moon's argument of latitude,

$$\Delta \sin W = \Delta B \cdot \cos(b-b') \operatorname{cosec}(\pi-\pi') \sin(N'+h)$$

where approximately

$$\Delta B = -\cos\beta\{(0{\cdot}092 - 0{\cdot}006 \cos\alpha)\Delta B + 0{\cdot}007\Delta\zeta\}$$

and

$$\Delta\mu = 15^\circ\{\Delta H - \Delta B \cdot n^{-1} \cdot \cos(N'+h)\}$$

where approximately

$$\Delta\mu = -(28^\circ{\cdot}65 - 1^\circ{\cdot}38 \cos\alpha)\Delta\zeta + 2^\circ{\cdot}64 \cos\alpha \cdot \Delta A + 0^\circ{\cdot}25\Delta B$$

The changed path of the curve of central totality can be deduced from the comparison of the new positions of the beginning, middle, and the end of eclipse with those given by Oppolzer.

For the required purpose this approximation was sufficiently accurate.

It was in the same manner that the approximate paths of the different eclipses were calculated on Mr Cowell's original data. In the case of the eclipse of -1116 June 18 an accidental error in copying out the final correction as plus instead of minus led to an inaccurate result, and this eclipse cannot have been that observed at Babylon.

I cannot concur with Mr Cowell's method of bringing -1062 July 31 within the month of *Sivan*. The intercalary months never inserted at the beginning of the year, and the first of could not have been later than the early part of April, so month of *Sivan* must have ended in the be[illegible] latest. Nor can I concur with Mr M[illegible] though July 31st could not have fallen [illegible] the known Babylonian calendar, yet t[illegible] assumed that, as the epoch is prior to [illegible] calendar may be supposed to be quite [illegible]

be used after that era. If such arbitrary assumptions have to be made, the records of these eclipses are absolutely valueless.

On this question I would observe, merely, that it must not be forgotten that for many centuries prior to the epoch of this eclipse the Babylonians had possessed a well-regulated definite calendar, sufficiently certain and predicate to enable its being incorporated in deeds, leases, and commercial agreements. For centuries the beginning of the year was fixed by the occurrence of the equinox, and it is most unlikely that they were dependent for the fixing of this event on the observation of actual New Moon, or on the heliacal or other fancy rising or setting of a star, when, in the passage of the shadow from the Sun over a fixed line on the Temple floor or courtyard, the line midway between the longest and shortest shadow during the year, they had a means of fixing the date of the equinox to the very day. Then, when the equinox fell after the first half of the fixed month, the year was made full by the insertion of an intercalary month at the end of the year. The reckoning of time in this manner by the dimensions of the shadows occurs in the early history of all races, and is universal in the East even now. The heliacal rising and setting of stars and the visibility of the new moon would have their religious and astrological signification, but so clumsy and indirect a method of fixing the equinox would not have been used when so much easier a method was available. Recently, every year has brought further evidence, in the shape of astronomical records and calculations, that the Assyrians, Babylonians, and even the early Chaldeans possessed a much better knowledge of astronomy and much better means of making astronomical observations than had been assigned to them, and that they could measure epochs and intervals of time with some certainty, probably by the equivalent to a dial on the floor of the Temple or its courtyard, as well as by contrivances of the nature of clepsammia or clepsydra. This, however, is beyond the present subject.

Fuller discussion of the eclipse seen at Babylon "on the 26th *Sivan*" is useless until the full details of the record are before us, and it is possible to critically examine the reasons for assigning it to some date in the eleventh century before our era, for at present there exist no data for estimating its value as the record of an eclipse at any place or at any epoch. If the record can be established as proving the occurrence of a total eclipse visible at Babylon in the eleventh century before our era, it is undoubtedly of the very highest importance as fixing both astronomical and chronological elements, but we must have the record with all ancillary evidence before its true bearings can be properly discussed.

Mr Cowell points out that the failure of the observations of the Sun during the period 1750–1900 to show any signs of the assumed secular acceleration in its mean longitude may be due to the existence of an unknown term of very long period. That is indubitable; and on that basis it must be admitted that these

observations of the Sun cannot be said to be inconsistent with the extended hypothesis.

The evidence that has been advanced serves to show that the facts brought forward by Mr Cowell in support of his views can be explained without having recourse to a new secular acceleration in the mean motion of the Earth, so that these facts do not suffice to establish the existence of such a secular acceleration. But if this is all, they certainly do not suffice to disprove the existence of such a term. If that is to be done, it must be by further direct evidence, to which I have no access, though it may exist. During past years in my researches I have repeatedly found evidence of an unexplained apparent secular acceleration in the motion of the Moon's argument of latitude, but I have doubted its reality, as no explanation of its origin seemed available. The explanation advanced by Mr Cowell, that it might be due to a secular acceleration in the mean motion of the Earth, did not occur to me, or I should have taken the necessary steps to obtain the data to investigate the matter. This, I presume, will be done by Mr Cowell. I do not see any theoretical explanation of such a secular acceleration which does not seem to involve great difficulties. Mr Cowell's tentative explanation (*Monthly Notices*, vol. lxvi. p. 352) I think he will find untenable on review.

Natal Observatory:
1906 *August* 23.

The Mediæval Eclipses of Celoria. By P. H. Cowell.

Professor Celoria has collected a number of references to the total eclipses of 1239 June 3 and 1241 October 6. He considers that the former eclipse was certainly total at Piacenza and at Lesina, and that the latter was total at Stadt and at Ellwangen. He draws the limits of totality according to Hansen's Tables (without Newcomb's corrections), and concludes that these tables are in serious error at the epoch of the two eclipses.

Professor Celoria's diagrams show that the conditions that he imposes are very stringent ones. The zone of totality for the 1239 eclipse is to be displaced not less than enough to secure totality at Piacenza, and not so much as to destroy totality at Lesina, and these limits are very narrow. They become still narrow- [illegible] it be remembered that Hansen's semi-diameter is consider[illegible]
for eclipse purposes. A similar remark applies to [illegible]
1241. Professor Celoria's assumptions, therefore, [illegible]
equations of condition, one for each eclipse, in whic[illegible]
uncertainty is very slight. The combination of [illegible]
therefore give an excellent [illegible]
required by the mean elongatio[illegible]

The form of the equation[illegible]
found. Let k denote, as in [illegible]

ratio of the Moon's motion in latitude to the difference of motion in longitude as seen by the observer, and let $\delta(V - V')$ and δU be the corrections required by the difference of longitude and by the Moon's latitude. Then the equation of condition must be

$$\delta U - k\delta(V - V') = a$$

where a is a constant, that depends upon the tables to be corrected.

That this is the true equation of condition is most easily seen; for if δU, $\delta(V - V')$ be one solution, $\delta U + k\delta t$, $\delta(V - V') + \delta t$ must be another solution, corresponding to a small change in the assumed time at which the eclipse occurred.

For any place on the Earth's axis, including the centre of the Earth and the two poles, the parallactic displacements are constants, and the value of k will be $\pm 0{\cdot}10$, the upper sign corresponding to the ascending node. For moderate latitudes k is approximately

$$\pm\ 0{\cdot}14 + 0{\cdot}14 \cos h$$

where h is the sidereal time.

Of course this formula is only a rough one, but it serves the purpose, before undertaking laborious calculations, of predicting that the equations of condition will involve

$$\delta U - 0{\cdot}12\ \delta(V - V') \text{ for the 1239 eclipse}$$
$$\text{and } \delta U + 0{\cdot}26\ \delta(V - V') \text{ for the 1241 eclipse.}$$

Writing
$$\delta U = \pm 0{\cdot}09\ \delta F$$
$$\delta\ (V - V') = \delta D$$

the unknown corrections to the argument of latitude and mean elongation enter in the form

$$0{\cdot}09\ \delta F - 0{\cdot}12\ \delta D$$
and
$$-0{\cdot}09\ \delta F + 0{\cdot}26\ \delta D$$

We shall not therefore have any trouble arising from the determinant of the coefficients being small.

Professor Celoria has obtained the equations

$$9{\cdot}8635\ \delta v - 5{\cdot}4914\ \delta\Omega = \ \ 35'{\cdot}40$$
$$-57{\cdot}7624\ \delta v - 1{\cdot}4098\ \delta\Omega = -42{\cdot}60$$

These equations cannot be reconciled with the form of mine, which has been justified by such elementary reasoning.

I obtain the following expressions for the difference of apparent latitude at the instant of apparent conjunction in longitude, δF, δD being corrections to the values of F, D that correspond to the formulæ in *Monthly Notices*, vol. lxvi., p. 525.

1239, for Piacenza	$0{\cdot}13\delta D - 0{\cdot}09\delta F + 89''$
1239, for Lesina	$0{\cdot}11\delta D - 0{\cdot}09\delta F - 47''$
1241, for Stadt	$-0{\cdot}25\delta D + 0{\cdot}09\delta F + 29''$
1241, for Ellwangen	$-0{\cdot}27\delta D + 0{\cdot}09\delta F - 57''$

Hence, in order to produce totality at all four places, we must have

$$0{\cdot}12\delta D - 0{\cdot}09\delta F = -21''$$

and

$$-0{\cdot}26\delta D + 0{\cdot}09\delta F = +14''$$

with errors of not more than three or four seconds on the right-hand sides.

The solution of these equations is $\delta D = 50''$, $\delta F = +300''$.

The value of δD is a possible one, but the value of δF is utterly impossible. It cannot possibly be reconciled with modern observations. Matters are even worse if the comparison be made with the present tables. The latter require an even larger correction δF.

I interpret allusions to darkness and to the appearance of stars to mean that the place of observation was not necessarily within the zone of totality, but within a somewhat wider zone, within which not more than about twenty seconds of the Sun's diameter was visible.

Now this explanation not only serves to dispose of an inconveniently large correction to the formulæ used in calculation, but it is absolutely required by the records. For at Altenzelle in 1241 we have the record " . . . . et stellæ apparuerunt," and at Vienna we have two records to the same effect. Professor Celoria confines the places to which he assigns black rings on his maps to within a zone not wider than the zone of totality. As the distance between Ellwangen and Vienna is about as great, when projected perpendicularly to the central line, as the greatest width that can be ascribed to the zone within which stars appeared, the central line probably was about equidistant from the two places. This fits my formulæ well, but I cannot use the principle to discriminate between my formulæ and the present tables.

In 1239 the record at Milan has to be rejected, as Professor Celoria suggests, on the ground that the writer lived 160 years later. We are then left with the records that stars were seen at Piacenza, and that the eclipse was certainly total at Lesina.

To sum up, therefore, my formula cannot be accepted unless it be admitted that at Piacenza in 1239 stars might have been seen with about 20″ of the Sun's diameter visible; on the other hand, if the theoretical secular acceleration be attributed to the argument of latitude, 30″ of the Sun's diameter would have been visible. I am not prepared to say that 20″ is possible and 30″ impossible, and therefore no proof can be based upon these eclipses. I feel bound, however, to examine all evidence that I come across, and I can only say that I am perfectly satisfied with the wa my formulæ stand the test in the present instances.

The outline of the calculations is as follows:-

Changes in a Julian century $\times 10^{-7}$.

1230

True Longitude of Mo

" " Su

True Elongation .

	1239 Piacenza.	1239 Lesina.	1241 Stadt.	1241 Ellwangen.
Parallax in Longitude .	55′·3	55′·2	45′·0	49′·5
Apparent Elongation .	130·8	130·9	141·6	137·1
Geocentric Latitude of Moon	+18·42		−18·24	
Parallax in Latitude	− 1·2	− 4·2	− 17·7	− 19·6
Apparent Latitude of Moon . . .	+ 17·2	+ 14·2	− 35·9	− 37·8
Hence k . . .	+ 0·13	+ 0·11	− 0·25	− 0·27

	1239	1241
T in centuries from 1800 Jan. 0·06 MT	−5·6054754	−5·5820399
G.M.T. in degrees . . .	+3°·9654	−2°·6451

	°	′	″	°	′	″
F . . .	5	23	36·4	169	28	27·5
D . . .	2	0	1·6	357	3	26·9
g . . .	344	26	51·5	7	50	14·0
g' . . .	168	39	16·1	292	18	35·7
−Ω . . .	284	51	39·0	330	11	20·6

	1239	1241
Inequalities of Moon's Longitude :—		
α. 21 largest terms, coefficients over 20″	− 5096″·2	+ 3299″·2
β. 30 next largest solar terms, coefficients over 1″·35 . . .	− 5·9	+ 53·5
γ. 58 next largest solar terms, coefficients over 0″·15 . . .	+ 4·0	− 0·4
δ. Figure of Earth terms . . .	+ 7·1	+ 3·2
Inequalities of Moon's Latitude :—		
α. 14 largest terms, coefficients over 10″	+ 1271·3	+ 3118·0
β. 42 next largest solar terms, coefficients over 0″·55 . . .	− 21·1	− 63·7
γ. 62 next largest solar terms, coefficients over 0″·06 . . .	− 0·4	− 3·9
δ. Figure of Earth terms . . .	− 7·5	+ 1·2
Moon's sine parallax. Constant, 3422″·7 :—		
α. 5 largest terms, coefficients over 10″	+ 3671·5	+ 3678·7
β. 9 next largest solar terms, coefficients over 0″·55 . . .	− 0·9	+ 4·2
γ. 17 next largest solar terms, coefficients over 0″·06 . . .	+ 1·0	− 0·2
Inequalities of Sun's Longitude :—		
Three terms of equation of centre	+ 22′ 33″·1	− 1° 49′ 7″·7

	Piacenza.	Lesina.	Stadt.	Ellwangen.
Geocentric Latitude	44°51′·0	42°58′·6	53°24′·6	48°45′·6
Radius Vector ρ .	·99830	·99841	·99779	·99807
Longitude, East of Greenwich . .	9°41′·55	16°28′·22	9°28′·43	10°9′·92
Parallax in Longitude :—				
α. Terms of first order	− 740″·3	− 1055″·0	+ 850″·1	+ 726″·7
β. " second "	− 11·9	− 16·8	+ 7·1	+ 6·9
γ. " third "	− 0·2	− 0·2	+ 0·2	+ 0·2

Parallax in Latitude :—

α. Terms of first order	− 1329·8	− 1230·1	− 3122·2	− 3006·7
β. Smaller terms	− 1·4	+ 0·2	− 0·7	+ 0·3

The formulæ used for the arguments are those given in *M. N.*, lxvi. p. 525.

The inequalities of the Moon's Longitude, Latitude, and Parallax are taken from Brown, *M. N.*, lxv. p. 276.

The inequalities of the Sun's Longitude and the obliquity of the ecliptic are taken from Newcomb, Tables of the Sun.

The figure of Earth's terms used are :—

r Moon's Longitude $+ 6''\cdot 6 \sin \Omega - 0''\cdot 7 \cos \Omega + 1''\cdot 0 \sin \Omega \cos$

r Moon's Latitude $- 8''\cdot 1 \sin \mathrm{L} + 1''\cdot 5 \cos \mathrm{L} - 1''\cdot 0 \sin g \cos \mathrm{L} - 0''\cdot 3 \sin (\mathrm{F} - \Omega)$

Nutation is not applied, since it is the same for both Sun and Moon, and not sufficiently important in its effect on the parallax.

The planetary inequalities have not been calculated for either Sun or Moon.

The formulæ used for parallax are

$$q = \rho\ (\sin p - p')$$

$$\text{Parallax in longitude} - bq - abq^2 + (\tfrac{1}{3}b^3 - a^2b)q^3 - \tfrac{1}{2}bq\mathrm{U}^2$$

$$\text{,,} \quad \text{latitude} - cq + aq(\mathrm{U} - cq) - \tfrac{1}{3}\mathrm{U}^3$$

$$\text{where } a = \cos\lambda \cos h \cos \mathrm{V} + \cos\lambda \sin h \cos\epsilon \sin \mathrm{V} + \sin\lambda \sin\epsilon \sin \mathrm{V}$$

$$b = -\cos\lambda \cos h \sin \mathrm{V} + \cos\lambda \sin h \cos\epsilon \cos \mathrm{V} + \sin\lambda \sin\epsilon \cos \mathrm{V}$$

$$c = -\cos\lambda \sin h \sin\epsilon + \sin\lambda \cos\epsilon$$

Less exact calculations would have been sufficient if I could have foreseen that the only result is that my formulæ expose less of the Sun's disc at the places named as places where stars appeared than the theoretical formulæ.

The present paper is most easily read in connection with Professor Celoria's diagrams. These, however, can be sufficiently reproduced from the following extracts from Professor Celoria's papers. The latitudes are geographical.

1239 June 3. Limits of Totality.

North		South	
43° 6′·84 N	0° 29′·79 E	40° 49′·52 N	1° 12′·58 E
44 18·97	8 23·34	41 58·58	8 33·51
44 43·80	17 1·14	42 24·64	16 34·23

1241 October 6. Limits of Totality.

North		South	
58° 35′·96 N	6° 42′·01 E	55° 56′·48 N	4° 59′·05 E
51 57·27	12 48·14	49 41·56	10 49·69
46 37·08	17 24·99	44 34·64	15

and

Place.	Latitude.	Lo
Milan	45° 27′ 59″	9
Altenzelle	51 3′·55	13
Vienna	48 12·55	16

The positions of the other places mentioned in this paper are given above.

Professor Celoria's extracts from mediæval records represent a vast amount of valuable labour. It is advisable, however, to warn readers that his figures for the 1239 eclipse must not be taken from his original paper, but from his own revised figures in his second paper.

The Irregular Movement of the Earth's Axis of Rotation: a Contribution towards the Analysis of its Causes. By Professor J. Larmor, F.R.S., and Major E. H. Hills, C.M.G.*

Much material, defining with increasing accuracy the irregular wanderings of the Earth's axis of rotation, has now been accumulating for a long series of years. The attempt to decompose the movement into regular harmonic components excited interest some six or eight years ago. Since that time the more systematic data obtained and analysed by the International Organisation have given greater precision to the path of the Pole; and, while a definite astronomical discussion must rest with the experts, curiosity as to the general physical causes of the phenomenon is legitimate. It has long been recognised that displacement of material on the Earth's surface due to meteorological changes (melting of polar ice, long-period barometric fluctuations, etc.) must be a prominent agent, and may indeed be taken to be the main one; while Newcomb has pointed out that the free Eulerian oscillatory period must be very different, for a nearly spherically balanced Earth, if it is elastically deformable under the centrifugal force, from what it would be if it were rigid, thus accounting for the unexpected value of the Chandler period.

It is shown below that, without making any hypothesis except the natural one, that this free precessional period is fixed in duration and determines the average duration of the revolutions of the Pole of rotation, it is easy by a graphical process to deduce from the path of the pole a map of the varying torque which must be acting in order to produce that path, and thence to infer as to the character of the displacements of terrestrial material that must be taking place in order to originate that torque on the Earth as a whole. It has seemed worth while to carry this out in a preliminary way. It has also been thought worth while to set down various dynamical considerations which may prove useful in a systematic analysis of the observational results.

Let ω_1, ω_2, Ω be the component angular velocities of the Earth referred to axes moving with itself, the latter being around the axis of figure. Thus $\omega_1/\Omega, \omega_2/\Omega$ are the angular co-ordinates of the pole of rotation measured on the Earth's surface, and represent

* Read in part at the British Association, August 1906.

directly the change of latitude; they are different from the absolute co-ordinates of the pole on the celestial sphere, in the order of the ratio $C/(C-A)$, as appears from the Poinsot geometrical representation of the free precessional motion, C and A being the *effective* polar and equatorial moments of inertia.* The dynamical equations of the Earth's free precession are, with sufficient accuracy, in terms of the angular momentum (h_1, h_2, H),

$$\frac{dh_1}{dt} - h_2\Omega + H\omega_2 = 0$$

$$\frac{dh_2}{dt} - H\omega_1 + h_1\Omega = 0,$$

where, D, E, F being products of inertia,

$$h_1 = A\omega_1 - F\omega_2 - E\Omega, \quad h_2 = B\omega_2 - D\Omega - F\omega_1, \quad H = C\Omega - D\omega_2 - E\omega_1$$

Thus

$$A\dot{\omega}_1 - (C-A)\Omega\omega_2 = -L'$$
$$A\dot{\omega}_2 + (C-A)\Omega\omega_1 = -M',$$

where Ω may be taken constant, because its variation would multiply in these equations two factors, each small of the first order; and where L', M' include, in addition to the kinetic forcives due to the location of mobile material attached to the earth, the terms

$$\dot{A}\omega_1 - \dot{F}\omega_2 - \dot{E}\Omega, \quad \dot{B}\omega_2 - \dot{D}\Omega - \dot{F}\omega_1$$

Here the terms on the left would be rates of change of angular momentum if the configuration remained constant. The terms on the right include the reversed rates of change of the same, taking the angular velocity constant but the configuration altering. To determine these latter terms, as arising from the reaction of mobile terrestrial material, we can consider them directly as representing the centrifugal forces of this loose material, together with the reversed gradient of the angular momentum, due to the Earth's rotation, of new negative material in the original position and of new positive material in the altered position.

The difference between this procedure and the usual one adopted by previous writers † is that the investigation of the slight

* *Cf.* Routh, *Dynamics*, ii. §§ 180–2, 533. On account of the largeness of this factor $C/(C-A)$ the axis of rotation is practically fixed *in space*, except in so far as the extraneous attractions of the Sun and Moon operate in causing forced precession, so that the effect [illegible] tion is simply a change of latitude. *Cf.* Maxwell ["On a Dynar [illegible] with some Suggestions as to the Earth's Motion," *Trans.* [illegible] 7. *Scientific Papers* i. (pp. 259–61)], who appears to have [illegible] apply the Euleriar theory precisely to the Earth, and to e [illegible] ich observations in search of a 306-day period in latitud [illegible] rofessor Newcomb, *Monthly Notices*, lii., 1892, pp. 336–41 [illegible] the latitude, of this period, instituted at Pulkowa by [illegible] gton, in 1862–7, had already yielded negative results. [illegible] which pointed out and estimated roughly the incre [illegible] uced by elastic yielding, the influer [illegible] ments was also considered.

† For an accou [illegible] 24, 533–5.

changes of position of the Earth's principal axes of inertia arising from displacement of material is evaded, by considering an unchanging Earth, with effective moments of inertia (A, A, C), which is subject to force arising from the kinetic reaction exerted on it by this additional and independent material, moving over it and at the same time maintained by it in diurnal rotation.

The effect of centrifugal force, in flattening elastically the terrestrial spheroid, simply modifies its effective or dynamical moments of inertia according to the principle arrived at in a previous discussion,* viz. that the moments of inertia which would exist in the absence of diurnal rotation, but on the assumption that the Earth's form when centrifugal force is thus removed is determined by a linear law of elasticity, are to be employed in dynamical investigations which take account of the elastic yielding of the Earth. There would be consequently an increase in the free precessional period (actually as observed it is from 306 to about 428 days) in the manner first pointed out by Newcomb.

Refer now the problem thus formulated to axes of ω_1 and ω_2 rotating with angular velocity $\Omega(C-A)/A$, that of the undisturbed

* "On the Earth's Free Eulerian Precession," *Proc. Camb. Phil. Soc.*, May 25, 1896, p. 186.

The argument there employed, briefly stated in more analytic form, is as follows. Let A′, B′, C′ be the principal moments of inertia of the Earth when unstrained by centrifugal force; and let I be the change of moment of inertia round its own axis, due to the equatorial protuberance raised by that force. This axis is in the direction of the resultant angular velocity $(\omega_1, \omega_2, \omega_3)$; and it is implied that it is very near to the principal axis of greatest moment C′, so that $\omega_3(=\Omega)$ is practically constant and great compared with ω_1 and ω_2. It is involved also in this restriction that I is a constant up to the first power of ω_1/Ω or ω_2/Ω. Referred to the principal axes, the total component angular momenta are

$$h_1 = A'\omega_1 + I\omega_1, \quad h_2 = B'\omega_2 + I\omega_2, \quad h_3 = C'\omega_3 + I\omega_3$$

The equations of motion referred to the rotating axes are of the well-known vector type

$$\dot{h}_1 - h_2\omega_3 + h_3\omega_2 = L$$

When A and B are equal, the third of them is

$$\frac{d}{dt}(C\omega_3) = N$$

where C is the effective moment of inertia C′ + I: when N is null ω_3 is thus constant, say Ω, up to the first order. The other two equations are

$$\frac{d}{dt}\cdot(A'+I)\omega_1 + (C'-B')\Omega\omega = L_2$$

$$\frac{d}{dt}\cdot(B'+I)\omega_2 - (C'-A')\Omega\omega_1 = M,$$

which in the case of approximate symmetry involve a free period $2\pi(A'+I)/(C'-A')\Omega$, and similarly in the general case, thus depending only on A′, B′, C′ when I is small.

The present procedure absorbs the effect of this regular change of form due to strain into modified moments of inertia, while it sets out the effect of erratic displacement of (additional) material on the rotating Earth as a kinetic forcive.

free Eulerian precession, viz., about 428 days. The equations of movement assume the form

$$A\dot{\omega}_1 = -L', \qquad A\dot{\omega}_2 = -M',$$

the same as if the axes were fixed and there were no diurnal rotation; that is, the polar axis moves in the earth, relative to these rotating axes of co-ordinates, along the direction of the reversed resultant of the torques L′ and M′.

It seems useful, therefore, to plot the course of the Pole relative to co-ordinate axes rotating with the mean Chandler period, marking, at intervals along the curve, both the time, and the longitude of one of the revolving axes of co-ordinates at that time; for the velocity along the curve at any instant will then give the direction and magnitude of that part of the rate of change of (L′, M′), the transverse component of the centrifugal torque of the loose material and the time-gradient of the angular momentum arising from transport of this material, when the velocity of rotation is imagined unaltered. Such change can therefore be partially located, as *infra*; if it is mainly due to displacements of surface-material, of thermal or meteorological type, it should show seasonal recurrences, and may prove to be in part due to slight change in oceanic or barometric levels.

Aggregate rough estimates of mere order of magnitude are easiest made directly, without use of these rotating axes. Thus a surface depression of 1 foot over a square mile, extending down in gradually diminishing amount to 30 miles, would involve an effective displacement of a layer 1 foot thick through 15 miles downward. In latitude 45°, where the effect in this respect would be greatest, this displacement would change the resultant transverse component of angular momentum of the earth by $4000\,\Omega\frac{2\frac{3}{4}\cdot 15}{5280}\cos^2 45^\circ$, the whole angular momentum of the earth being in the same units $\Omega.\ 5\frac{1}{2}.\ \frac{4}{3}\pi(4000)^3.\ \frac{2}{5}(4000)^2$; this takes the density of surface material to be $2\frac{3}{4}$ and that of the whole Earth $5\frac{1}{2}$. The polar axis would thereby be displaced through an angle equal in absolute measure to the ratio of these quantities; in seconds of arc it would be about 3.10^{-13}. Thus local displacements by earthquakes can have no sensible direct effect on motion of the Pole. But more important is the centrifugal effect rep[illegible] sentable by displacement of the axis of inertia of the E[illegible] round which the free precession of the polar axis is taking [illegible] This angular displacement is $h/(C-A)$, when h is the prod[illegible] inertia thus introduced: this is of $C/(C-A)$ times (abou[illegible] times) the order of the direct effect on the Pole of rotation, [illegible] in the present way of viewing the matt[illegible] [illegible]g or precession is practically everything.* [illegible] b[illegible] by Milne (Bakerian Lecture, *Roy. Soc.* [illegible]

* Sir G. H. Darwin has recently express[illegible] the effect of earthquakes may be to bring th[illegible] axis of figure, and thus damp the polar move[illegible]

small range of time (two years) then investigated by him, sharp curvatures in the polar movement appeared to be on the whole concomitant with earthquakes; the latter may be promoted perhaps by the changes of superficial or internal loading along meridians, that are the main cause of the irregular motion of the Pole and are greatest when the curvature of its path is sharpest. The procedure indicated in this note would locate to some extent this displacement or change of loading, and thus test that theory.*

The effect of transfer of water from the Poles towards mean latitudes, arising from melting of Arctic ice, may be estimated either by considering an added layer, of thickness positive or negative according to the locality, and of null aggregate amount spread over the whole ocean, or by estimating directly as above the change of angular momentum involved in the displacement of each portion of the material. A displacement of the Pole of rotation in the Earth in a given direction, when referred to the rotating axes as above, would imply alteration of intrinsic angular momentum of surface load in the neighbourhood of the meridian circle containing that direction, which would be a defect in the northern quadrant in front of that direction or an increase in the northern quadrant behind it, and *vice versa* for the southern quadrants. Thus water rapidly moved from the Poles, where it has little angular momentum, so as to cover to a depth of 1 foot a region 4000 miles square, in middle latitudes, would displace the Pole of rotation in the Earth by something of the order of 2 seconds of arc; for it would involve a new transverse angular momentum $\Omega \cos^2 45^\circ \; 4000^5/5280$ in the same units as above. It is readily seen that the principal axis of inertia, about which the free precession would continue, would be displaced in the opposite direction through an angle of the same order.

In reducing the International Observations of change of latitude at the selected observatories extending round the Earth, it has been found necessary to include a change common to all longitudes, which at first sight could only arise from change of form of the spheroid which represents the terrestrial sea-level. This might be in part due to gravitational influence of the displaced material; yet the removal by melting of 10 feet of ice over a polar area 500 miles in diameter would produce a change of attraction which could not, in middle latitudes, raise the Pole by more than

* The case here considered is 15 cubic miles of material displaced vertically 1 foot. Professor Milne informs us that the result of an actual earthquake might be 10,000,000 cubic miles displaced vertically or horizontally through 10 feet. This would multiply the figure in the text by 7.10^6, thus giving 2.10^{-6} seconds of arc. After this sudden shift of the axis of rotation in the Earth, the free precession would continue, but it would be around a new principal axis of inertia displaced from the original one by a quantity of the same small order of magnitude multiplied by $C/(C-A)$, that is by 300, giving a result of the order 10^{-3} seconds.

Sir G. H. Darwin has estimated (*Phil. Trans.*, 1876) that $\frac{1}{108}$ of the area of Africa rising or falling *in situ* through 10 feet would produce 0·2 seconds of change, the rising or sinking being presumably taken as not merely superficial.

1/1000 of the order of magnitude required. As the effect seems to be real, some indirect, perhaps seasonal instrumental, cause must apparently be sought for.

In trying, as here, to separate out the meteorological displacements of the Pole from the true free precession which would in their absence be a regular circular motion, by referring the whole to rotating axes, the essential point is to assign as correctly as possible the period of this precession, for that determines the velocity of rotation to be given to the axes of co-ordinates. Elastic yielding of the Earth will prolong the period beyond the 306 days that would belong to a rigid solid. Hough has shown * that an average modulus of rigidity of a solid Earth even so great as that of steel would involve the prolongation of the period to the above value, 428 days, which represents the periodicity of the observed path of the Pole. Now whatever be the cause, elastic or fluid displacement, or both, that thus alters the effective dynamical moments of inertia of the Earth, it may be presumed that it alters them to a constant extent over a fairly long period of time. Thus the true free, or Eulerian, precession would maintain a rotation of the Pole fairly constant, while the meteorological disturbance superposed on it would in the long-run have no rotational quality one way round or the other. In applying the method of analysis of the complex motion that is here proposed, the angular velocity of true precession may therefore be obtained by taking the mean of the observed times of revolution of the Pole, as Chandler originally pointed out.

The Eulerian principal axes fixed in the Earth, or still better the axes rotating as specified above, are appropriate to the analysis of the effects of moments or torques, which arise from change of loading and are therefore themselves revolving on the whole with the Earth. On the other hand, torques which only slowly change in direction in space, such as those arising from the attraction of the Sun and Moon on the protuberant parts of the oblate terrestrial spheroid, are most amenable to dynamical analysis when referred to fixed axes. They produce mainly the ordinary forced astronomical precessions and nutations, on which the varying elastic yielding of the Earth has no sensible kinetic influence; while the intensity of the torque of attraction depends on the instantaneous geometrical value of $C-A$, as determined by distribution alone. The distribution of mass thus governs the solar and lunar precessions.† These forced precessions depend on internal fluidity,

* *Phil. Trans.*, 1895. It is (as above) the near approach to spheri[illegible] makes so slight a yielding to the changes of centrifugal and o[illegible] effective in this manner.

† There is one case, however, in which, as the equations of above show, a small extraneous forcive would cause a wanderin[illegible] in the Earth itself, much greater than its change of direction in s[illegible] the astronomical precession thereby caused, that, namely, of a f[illegible] a period in longitude relative to the Earth's rotation, of the free period of 428 days. Such a term in the forcive would [illegible]

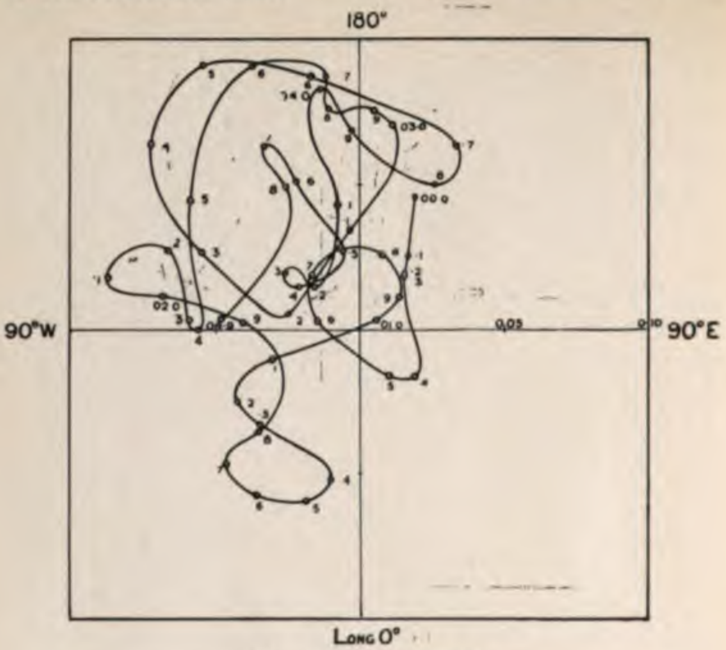
etc., only in so far as it modifies the effective inertia-moment C or A; while, on the other hand, the free precession depends on the effective or kinetic value of the small difference $(C - A)/C$.

The tides would be about the same at antipodal points if contours of the land were symmetrical, and the two opposite tidal protuberances would be additive in their effect on the free precessional moment. At first sight, it might appear that the estimate given above for a local overflow of a foot of water from the Polar regions would involve that, even apart from the irregularity of form of the oceans, the features of the various tidal components travelling round the Earth must be reproduced to some extent in an exact diagram of the torque, owing to the tidal flow demanding alteration of the angular momentum of the water relative to the Earth's rotation, and to its centrifugal force. But in so far as it is the Earth that turns round under the nearly stationary tide, the direct dynamical effects of the tidal movements are very slight and belong to the astronomical class, and are in fact merged in the lunar and solar nutations. But if the Earth's surface were divided by meridional barriers, so that the tidal flow would be in the main north and south instead of around the Earth, we might expect a tidal aberration in the latitude of amount not entirely insensible.

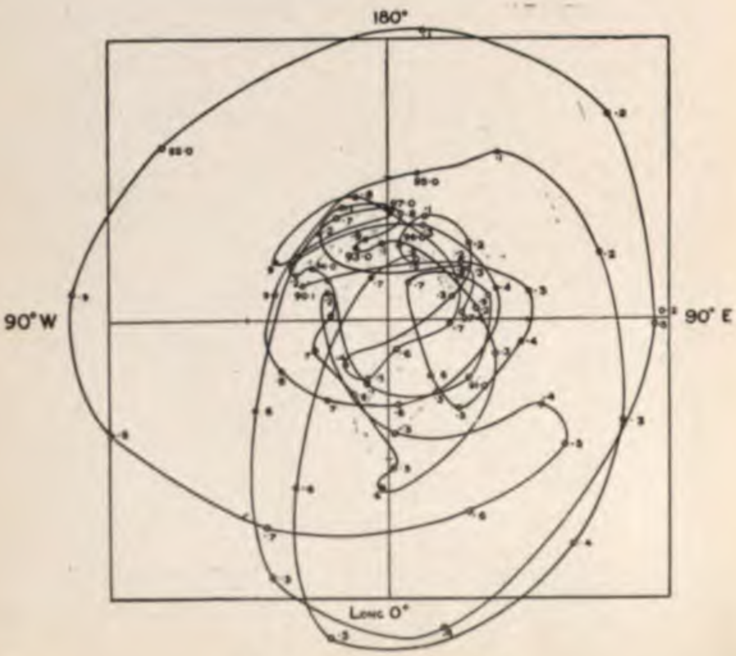
The magnitude of this tidal torque would then compare even with that of the precessional couple of the Moon's attraction on the protuberant parts of the terrestrial spheroid. The latter is $\frac{2}{3} Mr^{-3} (C-A) \sin 2\delta$, so that its amplitude is about $\frac{3}{2}(0''\cdot 1\, g/a)\, 10^{-2}$ C while the earth's angular momentum is $C\omega$. If this torque were to rotate with the Earth for six hours it would produce an angular displacement of the Pole of amount $0''\cdot 001\; g.6h/\omega a$, where $\omega^2 a/g = \frac{1}{289}$, that is, of amount $0''\cdot 001 \times 289.\frac{1}{2}\pi$ or $0''\cdot 5$; in contrast with the actual lunar fortnightly nutation of amplitude one or two seconds. It has just been seen that a partial tidal overflow from the poles covering $(4000)^2$ square miles to the depth of a foot, and carried on with the Earth's rotation, could in an extreme case account for a displacement of the Pole as much as $2''$: and the antipodal high waters reinforce each other. It is true that if the Earth were covered symmetrically with water, the Sun and Moon travelling in the equator would produce no effect. But the obliquity of their paths and the irregular distribution of the oceans must lead indirectly to nutations of the pole of the short periods of the various tidal components, which are not insensible in the present connection. Thus, for example, a forced nutation of this kind might introduce discrepancies into observations around a parallel of latitude, of character in part systematic owing to the progressive semi-diurnal change of phase, which would be eliminated in smoothing out the observations for each observing station of the International chain of longitude.

Possibilities in this direction are perhaps worth bearing in mind.

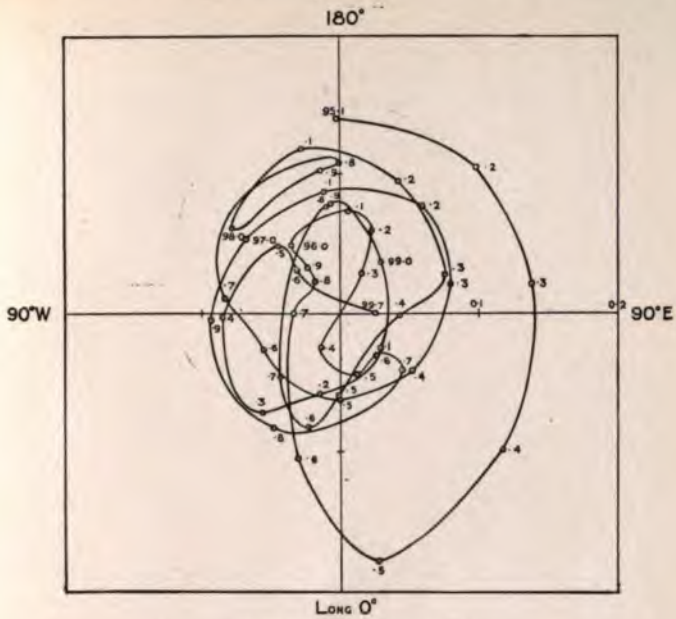
oscillation in latitude of its own period, which would be superposed on the free oscillation of a nearly equal period, thus producing an alternation in its amplitude.



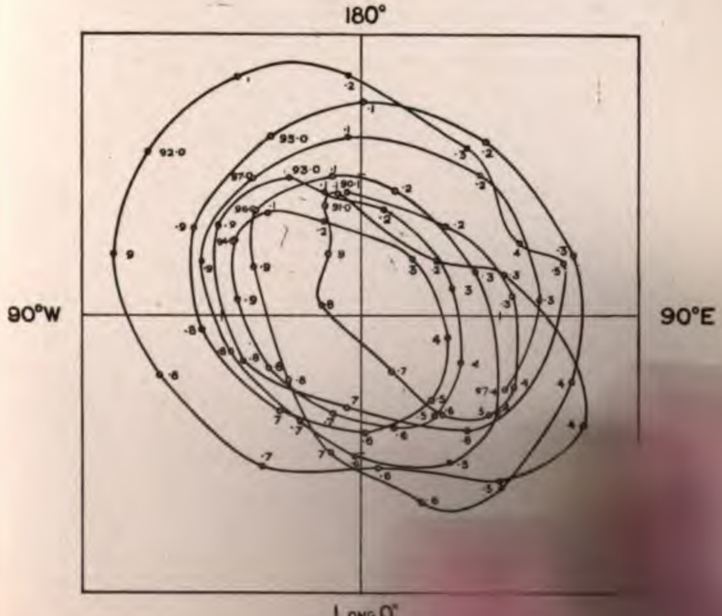
1. ALBRECHT, 1900-1905.



2. ALBRECHT, 1890-1897.



3. CHANDLER, 1895-1899.



4. CHANDLER, 1890-18

For example, in addition to the difference of phase at different stations mentioned above, if the same stars were observed at all the stations, a solar tidal nutation might partially simulate a change of latitude with a yearly period, common to all the stations, such as appears in the reduced observations. The deviation of the vertical by the attraction of the tidal water is in all cases very small compared with this deviation of the Pole (*cf.* Thomson and Tait's *Natural Philosophy*, ed. 2), and is in fact here negligible.

The influence of a nutational torque of fairly short period could be laid off by aid of the diagram referred to rotating axes; but even in this case it can be effected rather more easily in the usual astronomical manner, on account of the relatively slow change of its direction in space.

But whether these tidal influences are sensible or not, assuming 428 days to be the period of free precession, we can transform the curve of wandering of the Pole as above to axes of co-ordinates rotating with this period: and the hodograph of this new curve, when referred back again to axes connected with the Earth, will represent the distribution in direction and time of the torque arising from displacement of terrestrial material, which is continually modifying the motion of the Pole. From the point of view of geophysics, this curve would appear to be worth setting out, and might be expected to show seasonal recurrences.

The annexed diagram (1) gives, from the path of the Pole since 1900, as officially published by Professor Albrecht in the *Ast. Nachr.*,* the torque which must have been in action to cause that motion. It will be observed that the expected annual periodicity does not appear, but that the direction of the axis of the torque points preponderantly towards the side of the Pacific Ocean, an extraneous feature which we shall show how to eliminate later (p. 31).

Then follow, but based on more imperfect data, the torque-diagram (2) derived from the Albrecht diagram of Polar wandering for the period 1890–97. The other two are derived from the Chandler diagrams for the periods 1895–99 and 1890–97. It will be noticed that for the first one and a half years of the overlapping period 1895–97 the torque-diagram is much the same in both, but not for the later part. The appearance of all these diagrams is very different from that of the first, *e.g.* the one-sided bias in the torque does not appear.

The unit (0·1) marked on the axes of the diagrams represents the torque that would shift the Pole at the rate of 0″·05 in one-ten[illegible] year; the dates are marked along the curves in decimals of [illegible]

The analysis of Chandler made the motion of the Pole [illegible] his circular precession of 428 days period, with an addition [illegible]

* The most convenient way to use these diagrams is to pla[illegible] terrestrial globe, with the origin at the [illegible] lbrecht path-diagrams from which these force-dia[illegible] found in *Bericht über den Stand* . . . [illegible] vol. xix., respectively.

motion about the centre, and of yearly period, superposed on it. The component torques that would originate such an addition to a free precession of 428 days, being proportional to $\dot\omega_1 - n\omega_2$ and $\dot\omega_2 + n\omega_1$, would have an annual elliptic periodicity, in rough agreement with this diagram. For, referring to the axis of the ellipse, this theoretical motion would be

$$\omega_1 = \mathrm{A}\cos(nt+\alpha) + a\cos pt$$
$$\omega_2 = \mathrm{A}\sin(nt+\alpha) + b\sin pt,$$

so that

$$\dot\omega_1 - n\omega_2 = -(ap+bn)\sin pt$$
$$\dot\omega_2 + n\omega_1 = (an+bp)\cos pt,$$

in which the amplitude A of the free precession is not involved.

We have now to inquire into the kind of information that these diagrams can convey. On our plan of analysis, on the basis of a definite elastic Earth on which additional matter can be displaced, the forcive necessary to supply new angular momentum to material that has come into a position of greater diurnal velocity has to be supplied by this earth, while it has also to sustain the centrifugal force of this material. The other ways in which the mobile material reacts on the motion of the Earth on which it is superposed are negligible in comparison with these two.*

For a mass m in co-latitude θ the centrifugal torque is $m\Omega^2r^2\sin\theta\cos\theta$ around an axis at right angles to the meridian of m. The aggregate torque corresponding to its angular momentum in its present position is $m\Omega r^2\sin\theta\cos\theta$, as regards the equatorial component which is in the meridian of m. The former operates as a whole as a forcive, but only the time-gradient of the latter thus acts, which is $2m\Omega rv\cos 2\theta$, where v is the velocity of m along the meridian. The former preponderates, in the ratio $\frac{1}{2}\Omega r$ to v, therefore usually very much so, as $\frac{1}{2}\Omega r$ is of the order of 300 miles per hour. On the other hand, the centrifugal force of a new steady local load merely makes the steady precession occur about a new axis of inertia, thus is not progressive or cumulative. This is readily verified by reversing the graphical procedure; a steady torque, represented by the end of a radius vector, becomes represented by an arc of a circle when referred to axes rotating with the free precessional velocity, and this corresponds to a velocity of circular precession of the Pole while it lasts.

This double mode of action of transported material, through centrifugal force and through change of momentum of diurnal motion, renders interpretation of the torque-diagram to some extent indefinite.

But, neglecting the effect of change of intrinsic angular momentum, which may be as much as one-tenth of the whole, the torque will be due to the centrifugal force of the distribution

* This is readily seen by the procedure of the note, p. 24, if we introduce the exact formula $h_1 = \mathrm{A}'\omega_1 - \mathrm{E}\omega_3 - \mathrm{F}\omega_2 + \mathrm{I}\omega_1$, where for an additional mass m at xyz, $\mathrm{F}=mxy$, $\mathrm{E}=mxz$; we are in fact merely neglecting $\Omega\omega_1$ and $\Omega\omega_2$ compared with Ω^2.

of the mobile load at each instant, and will thus indicate the general features of that distribution; while the rate of change of the torque, *i.e.* the velocity in this torque-diagram, will give an indication of the movement of the load. The radius vector O P of the torque-diagram at any instant will imply a proportional accumulation of materials in middle latitudes on the meridian at right angles to O P, so that antipodal accumulations reinforce, but adjacent ones, north and south of the equator, counteract each other. The marked tendency of the torque-diagram for the period 1900–05 towards the side of the Pacific Ocean might thus be due to simultaneous accumulation of load not on the side of the Pacific, but in the neighbourhood of the perpendicular meridian. There is, however, a possible alternative to be kept in view, as follows.

It has been suggested to us by Professor Turner that the position of the origin to which the curve of wandering of the Pole of rotation is referred, is subject to considerable uncertainty. The observations and their reduction do not, however, seem to be at fault specially in this direction; unless the unexplained constant (Kimura) term may be taken to indicate a radius of uncertainty due to seasonal instrumental changes. This term, which recent discussion has confined to a smaller amplitude and to an annual period, was referred roughly to a displacement (mainly N and S) of the Earth's centre of gravity: we have verified above, however, that no likely meridional transfer due to seasonal change of temperature could produce an effect so great.

There does not, in fact, seem to be any ground, apart from mere uncertainties, for taking the origin to which the wanderings of the Pole of rotation are referred to be other than a fixed point on the Earth. But, on the other hand, this fixed point may not be the Pole of inertia of the solid Earth. We can, however, make it so, by separating from the solid Earth a thin superficial sheet, and counting this with the mobile material. The centrifugal forcive due to this sheet will then constitute a torque invariable with respect to the Earth: and we have merely to subtract this from the torque-diagram referred to the Earth in order to obtain the torque due to the loose material alone. This subtraction of a constant vector term amounts simply to a change of origin. Thus on the final torque-diagram the origin is uncertain, and would naturally be placed in as central a position as possible.

The process here carried out graphically may be compared with the procedure by successive steps as employed by Newcomb, in which free precession occurs for an infinitesimal time δt round an axis of inertia O supposed fixed in the Earth, then O is moved on to O_1 as the result of the change of the mobile load during that time, then free precession takes place round O_1 for a time δt_1, then O_1 is moved on to O_2, etc. Our method has virtually amounted to the elimination of the free precessional motion, w [illegible] tant angular velocity, thus leaving the causes which d [illegible] f inertia on the Earth's surface open to inspectio[illegible]

shift of the axis of rotation in space being neglected for the nearly spherical Earth, as *infra*.

How far the results may throw light on their causes depends largely on a comparison of the diagrams with the displacements of matter on the Earth's surface that are known to meteorology and oceanography. It may be that not much certain information may yet be derivable; but, considering the long time that observations of the wandering of the Pole have been accumulating, it can hardly be said that it is too soon to prepare for their preliminary discussion from the geophysical point of view.

The mode of reduction on which this paper is founded gives a force diagram which exhibits the torque sustained by the rest of the Earth owing to the displacement in and over it of the movable masses treated as independent bodies. It remains valid, however rapid the free precession may be. In the case of the actual earth the latter is slow, being $C/(C-A)$ sidereal days, in which the difference of moments of inertia has its effective or dynamical value, thus lengthening the free period from 306 to 428 solar days. In this case various features assume simple forms, as appears in the Poinsot representation by a rolling ellipsoid. When this ellipsoid is very nearly spherical, the axis of rotation, drawn from its fixed centre to the point of contact with the plane on which it rolls, is at a small inclination to the invariable direction of resultant momentum (which is normal to that plane) compared with its inclination to the axis of inertia; thus the axis of rotation is practically fixed in direction in space—except as regards the superposed luni-solar precession. When the ellipsoid of inertia is nearly of revolution, as in the case of the Earth, the pole of inertia thus revolves in space with the uniform free precessional velocity, around the fixed direction of the pole of rotation, while at the same time it is undergoing such shifts as the redistribution of material geometrically requires. Our procedure in the above has been to eliminate the uniform precession, and the residue is a graphical representation of the irregular shifts, or rather of the torques which produce them.

The discussion of the question whether in past geological history the pole of rotation has wandered extensively in the Earth seems also capable of being based on simple graphic representation. We shall assume, as before, that the dynamics of the Earth's rotation are based at each instant on a simple kinetic energy T given by

$$2T = A\omega_1^2 + B\omega_2^2 + C\omega_3^2,$$

where A, B, C are effective moments of inertia; it follows that the angular momentum (L, M, N) is given by the formula

$$(L, M, N) = (A\omega_1, B\omega_2, C\omega_3)$$

During the history of the Earth (L, M, N), resultant G, must have remained constant, while T will probably have diminished through

frictional agency. Abstraction is here made of the solar and lunar forced precessions, which compensate the torque of extraneous attractions without affecting the position of the axis of rotation in the Earth. The free motion is represented, after Poinsot, by the angular motion of a momental ellipsoid, say

$$Ax^2 + By^2 + Cz^2 = K$$

The direction of the axis of instantaneous rotation intersects its surface at the point (xyz) such that

$$\frac{x}{\omega_1} = \frac{y}{\omega_2} = \frac{z}{\omega_3}, \text{ therefore} = \left(\frac{K}{2T}\right)^{\frac{1}{2}}$$

The distance p of the tangent plane at this point (xyz) from the centre is given by

$$\frac{1}{p^2} = \frac{A^2}{K^2}x^2 + \frac{B^2}{K^2}y^2 + \frac{C^2}{K^2}z^2$$
$$= \frac{G^2}{2KT}$$

Thus if $K^{-1} = 2T$, we have $p = G^{-1} = \text{constant}$; this tangent plane is then fixed as regards distance from the centre, as well as regards its direction, which is perpendicular to the invariable momentum (L, M, N). Thus it is entirely fixed.

Thus the free precessional motion of the slowly changing Earth is represented by the varying ellipsoid

$$Ax^2 + By^2 + Cz^2 = (2T)^{-1}$$

rolling with centre fixed, so as to keep in contact with this fixed plane whose distance from the centre is G^{-1}.

If the kinetic energy keeps constant, this is simply Poinsot's representation. The axis of rotation will circulate in the body around the axis of greatest moment of inertia, and in space around the axis of resultant angular momentum. The amplitude of this free precession will be kept small by internal friction, so that the axis of rotation will always be near the principal axis, and can never wander further from its original position than the latter does. It will require a good deal of change of distribution of mass to move this principal axis very far: to move it into the equator the radius of the Earth must shrink to the order of 40 miles along the equator near the new Poles, and expand to about an equal extent near the original Poles.

If the Earth is shrinking uniformly, the moments of inertia vary as l^2, where l represents linear dimensions; thus the angular velocities vary as l^{-2} and the kinetic energy varies as l^{-2}. Thus the dimensions of this rolling ellipsoid remain unaltered. The distance of the plane on which it rolls also keeps fixed. Hence uniform shrinkage without frictional loss of energy would not affect the amplitude of the free precession, or cause the Poles to migrate.

Diminution of the energy of rotation through internal friction, without change of (A, B, C), would increase the dimensions of the rolling ellipsoid (in the proportion of $T^{-\frac{1}{2}}$), and so would make strongly for stability of the axis of rotation, as above remarked. Such increase can only proceed to a limited extent, determined by the ellipsoid just failing to intersect, and so touching the fixed plane at the extremity of its axis.

A discussion of the geological problem of displacement of the polar axis in the Earth must take account of considerations such as these.

The Systematic Motions of the Stars.

By A. S Eddington, B.A., M.Sc.

1. *Introduction*

Of late years astronomers who have investigated the proper motions of stars have been mainly interested in determining the direction of the solar motion. It is usual to assume that, if we consider a sufficient number of stars, their true motions will be at random; on this assumption the direction of the sun's motion has been calculated.

Professor J. C. Kapteyn* has examined the Bradley proper motions to find out whether this assumption is approximately true. He concludes that it is incorrect. Relative to the sun, he finds two "favoured" directions of motion instead of one. Kapteyn suggested that there are two systems or "drifts" of stars; these two drifts are in motion relative to one another. If the whole universe forms one system (or one chaos) we can speak of its motion relative to the sun; but it is more natural, though perhaps misleading, to speak of the sun's motion relative to it. But if there are two systems, we may as well drop the idea of the solar motion altogether, and speak of the motions of the two drifts relative to the sun.

In this paper I have attempted to subject Kapteyn's theory to a quantitative test by examining the Greenwich-Groombridge proper motions. I was led to undertake this by the following considerations:—

(1) The importance and revolutionary character of Kapteyn's discovery render independent confirmation very desirable.

(2) The Greenwich-Groombridge proper motions, recently determined by Dyson and Thackeray, afford new material for this purpose. I had the advantage of access to Dyson and Thackeray's calculations used in their determination of the so-called solar apex.

(3) The Bradley stars are all bright stars. The Groombridge catalogue includes a large proportion of stars between the seventh and ninth magnitudes; it is desirable to find out whether these fall into Kapteyn's two drifts.

* *Brit. Assoc. Report*, 1905.

(4) The Groombridge stars number about 4500 and are all within 52° of the north pole. The 2500 Bradley stars cover most of the sky. For some purposes there is an advantage in comparing the phenomena in different parts of the sky, but for quantitative tests of the theory I have found density of distribution essential; this the Groombridge catalogue affords.

I have also examined in the same manner the Greenwich-Carrington proper motions.

When more determinations of line-of-sight velocities are published, perhaps a complete demonstration of the two-drift theory will be possible. What is here attempted is to show that there exist anomalies of a remarkable and systematic character in the proper motions (or what are believed to be the proper motions) of the stars; and that these anomalies are, so far as our evidence goes, entirely explained if we suppose the visible universe to consist in the main of two streams of stars crossing through one another.

The test of a hypothesis is its power of predicting facts. It is easy to pick out half-a-dozen stars which, our theory predicts, will have an average velocity towards the sun, and another half-dozen from the same part of the sky which will have a velocity away from the sun.

Until some such simple test is applied, it would be wrong to regard the theory as established. On the other hand, it must not be forgotten that, should the two-drift theory fail when the crucial test is applied, some other explanation will have to be found for the remarkable phenomena to which Kapteyn has called attention.

2. *Mathematical Theory.*

I define a "drift of stars" to be a system of stars whose velocities relative to some system of axes are quite haphazard. The velocity of the drift is the velocity of the aforesaid system of axes. The "peculiar" velocity of a star is its velocity relative to that system. For example, the ordinary hypothesis on which discussions of the solar motion are based regards the universe as forming one such drift. It is sometimes further assumed that this drift is at absolute rest; but, of course, referred to the sun, it is in relative motion.

I shall accept, as the best mathematical equivalent of "haphazard," a distribution of velocities according to Maxwell's law. Of course there is no analogy between the causes which determine the velocities of the molecules of a gas and those of the stars. In the former case it is the collisions which make the molecules obey this law. But Jeans* has shown that of all distributions *having the same energy*, that according to Maxwell's law is the most probable; and as the number of bodies increases, the probability of a Maxwellian distribution approaches certainty.

* *The Dynamical Theory of Gases*, § 56. We take as our *basis of probability* that all values of a velocity component are equally probable.

For our purposes it will be sufficient to accept the Maxwellian distribution as a *standard*; and it will be the object of the comparison of observation and theory to discover any marked deviations from this standard distribution of stellar velocities.

Consider a small region of the sky which can be considered approximately plane, and suppose the stars in it belong to one drift. We shall consider their *linear* velocities, but ignore motion in the line of sight.

Let the drift velocity be V along Ox, and let the number of stars having component peculiar velocities between (u, v) and $(u+du, v+dv)$ be in accordance with Maxwell's law

$$\mathrm{A}e^{-h^2(u^2+v^2)}du\,dv$$

I shall call h the drift-constant; it is connected with the mean peculiar speed Ω of the stars of the drift by the relation $\Omega = \sqrt{\frac{4}{\pi}} \cdot \frac{1}{h}$.

If r be the resultant velocity, θ its inclination to Ox, we shall have

$$u^2+v^2 = r^2 + \mathrm{V}^2 - 2\mathrm{V}r\cos\theta$$
$$du\,dv = r\,dr\,d\theta$$

Hence the number of stars having proper motions in directions inclined to Ox between θ and $\theta+d\theta$

$$= \mathrm{A}d\theta\int_0^\infty e^{-h^2(r^2+\mathrm{V}^2-2\mathrm{V}r\cos\theta)}r\,dr$$

or putting $z = r - \mathrm{V}\cos\theta$,

$$= \mathrm{A}d\theta e^{-h^2\mathrm{V}^2\sin^2\theta}\int_{-\mathrm{V}\cos\theta}^\infty e^{-h^2z^2}(z+\mathrm{V}\cos\theta)dz$$

now put $hz = x$ and $h\mathrm{V}\cos\theta = \tau$ the number becomes

$$= \frac{\mathrm{A}}{h^2}e^{-h^2\mathrm{V}^2}d\theta\left\{e^{\tau^2}\int_{-\tau}^\infty e^{-x^2}(x+\tau)dx\right\}$$

$$= \mathrm{B}d\theta\left\{\tfrac{1}{2}+\tau e^{\tau^2}\int_{-\tau}^\infty e^{-x^2}dx\right\}$$

where B is independent of θ.

The above function of τ is very important in what follows. Table I. gives the values of $\log f(\tau)$, where

$$f(\tau) = \frac{2}{\sqrt{\pi}}\left\{\tfrac{1}{2}+\tau e^{\tau^2}\int_{-\tau}^\infty e^{-x^2}dx\right\}$$

TABLE I.

τ.	log f(τ).	τ.	log f (τ).	τ.	log f(τ).	τ.	log f(τ).
−1·3	$\bar{2}$·997	−·4	$\bar{1}$·471	·5	0·188	1·3	1·152
−1·2	$\bar{1}$·041	−·3	·536	·6	·288	1·4	·300
−1·1	·087	−·2	·605	·7	·395	1·5	·455
−1·0	·135	−·1	·676	·8	·506	1·6	·618
−·9	·185	0	·751	·9	·623	1·7	·787
−·8	·238	·1	·830	1·0	·746	1·8	·964
−·7	·292	·2	·913	1·1	·875	1·9	2·148
−·6	·349	·3	0·000	1·2	1·010	2·0	·339
−·5	·409	·4	·091				

Consider the curve obtained by drawing the radius vector in any direction proportional to the number of stars having proper motions in that direction.

The polar equation of the curve will of course be $r \propto f\,(h\mathrm{V} \cos\theta)$,

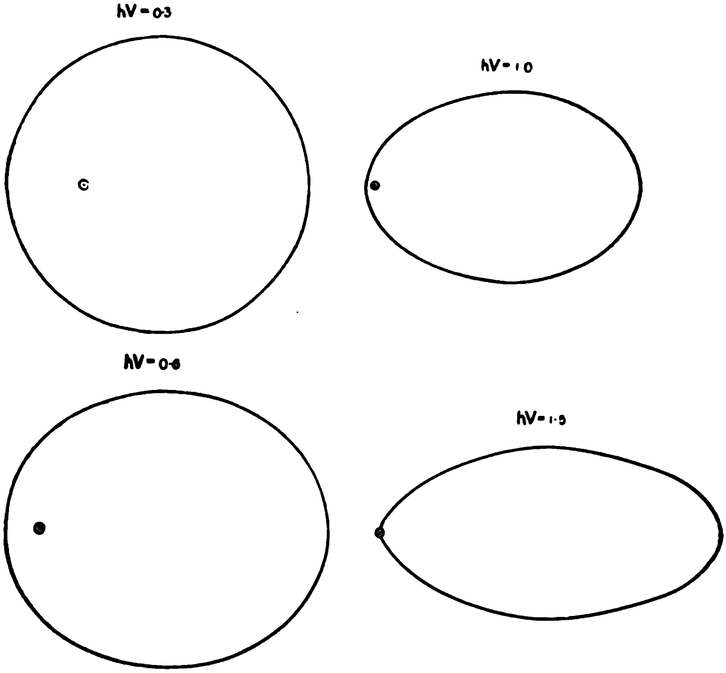


FIG. 1.

or more generally, if the drift is in the direction θ_0 instead of being along the initial line,

$$r \propto f(h\mathrm{V} \cos \overline{\theta - \theta_0})$$

The shape of the curve depends on the parameter hV; in other words, it depends on the ratio of the drift velocity to the mean peculiar velocity. It does not in any way depend on the distances of the stars.

Fig. 1 shows the curves drawn for four different values of hV. It will be noticed how rapidly the shape of the curve changes for small variations in hV.

3. *Treatment of the Observed Proper Motions.*

The region covered by the Groombridge catalogue comprises that part of the sky within 52° of the north pole. I divided it into seven regions as follows :—

Region A	N.P.D. 0°–20°	
,, B	N.P.D. 20°–52°	R.A. 22^h– 2^h
,, C	,,	,, 2^h– 6^h
,, D	,,	,, 6^h–10^h
,, E	,,	,, 10^h–14^h
,, F	,,	,, 14^h–18^h
,, G	,,	,, 18^h–22^h

In making this division it was necessary to steer between two evils. If too large a region is chosen, the proper motions will not follow the same law at all parts of it, and they cannot fairly be treated as all in one plane. But the regions must be large enough to contain a sufficient number of stars; it is difficult to deal mathematically with a region containing less than 400 stars.

In discussing one of these regions, a table was first made showing how many stars are moving in each direction. The angle θ which indicates the direction was measured in a counter-clockwise direction (as seen in the sky) from the direction of increasing R.A. at the centre of the region.* Thus $\theta = 90°$ would mean towards the equator and parallel to the central meridian.

Fig. 2 shows curves drawn to represent these tables. As in the theoretical curves of fig. 1, the radius vector in any direction is proportional to the number of stars whose proper motions are in that direction.

Let us consider, for example, the curve B (corresponding to region B), and compare it with the theoretical single-drift curves of fig. 1. Perhaps the most remarkable feature is the extraordinary minimum between *oa* and *ob*. The number of stars having directions of motion within this right angle is found on reference to the table to be only 63 out of 862. But the curve differs

* In the case of region A, the direction $\theta = 0°$ is along the meridian 6^h.

completely from the theoretical curves in that the maximum is not by any means opposite to this, but is along *oc*; in fact, there is a sort of secondary minimum along *od*, where we should have expected the maximum to be.

It is no exaggeration to say that the hypothesis that the stars

Fig. 2.

form a single drift does not represent with tl ima- tion the observed distribution of proper region B at any rate. No matter what orie the theoretical curve is taken, at the vei between theory and observation will amo quadrant. Regions C, F, and G in like hypothesis of one drift, whilst there are se

cases of D and E. In region A we have two maxima of the rad
vector nearly opposite one another, so that the disagreement w
theory is in some respects more remarkable still. Kapteyn in
investigation found this same disagreement, and pointed out
explanation—that there are really two drifts of stars.

I have accordingly analysed each region separately to see if
can be accounted for on the assumption that each curve is co
pounded of two simple drift curves. For instance, in fig. 3, X
Y are theoretical curves for single drifts; X represents a drift c

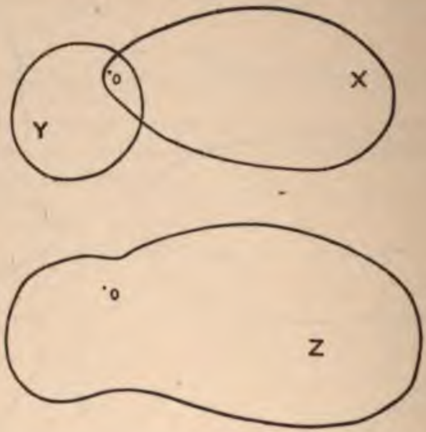


FIG. 3.

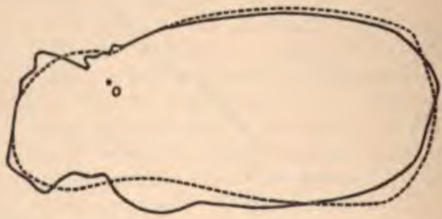


FIG. 4.

taining 418 stars, and for which $hV = 1{\cdot}55$; Y a drift contain
430 stars, and for which $hV = {\cdot}45$. The directions of motion of
two drifts are inclined at an angle of 120°. The compound cu
Z is drawn with its radius vector in any direction equal to
sum of the radii vectores of X and Y in that direction. It th
fore corresponds to a mixture of the stars of the two drifts ab
mentioned. The curve Z is the best approximation to fig. 2 B wh
I am able to obtain by combining two simple curves. Fig
shows a comparison of the two. There is a small difference in
two curves between the radii $\theta = 50°$ and 80°, amounting to

excess of 24 stars altogether (*i.e.* the observed curve has 114 stars, where the theoretical curve has 90 stars). Elsewhere the agreement is practically perfect. For this and the other regions, the agreement is better seen by referring to the tables which follow, and especially to figs. 5 and 6.

In Tables II. to VII., corresponding to various values of θ, the fifth column shows the actual number of stars having proper motions in that direction *—the radius vector of the diagrams. The fourth column shows the theoretical number given by my analysis, and the second and third columns show how the two drifts contribute to this theoretical number. The comparison of the observed and theoretical numbers is shown in figs. 5 and 6 for four regions. The number of stars and the corresponding direction of motion θ are plotted as ordinate and abscissa instead of (as in the previous diagrams) as polar co-ordinates.

It must not be supposed that I claim that two drifts represent completely the phenomena of the distribution of proper motions, and that the discordances are necessarily either errors of observation or the purely accidental departures which the theory of probability allows. Local irregularities must be expected; perhaps a third smaller drift exists throughout the sky, or one of the two principal drifts may prove to be compound. What is claimed for the two-drift hypothesis is that it is a good first approximation, whereas the one-drift hypothesis is no approximation at all.

Each region has been analysed without reference to the others. The six disposable constants of each theoretical curve have been determined by trial and error, so as to make it coincide as nearly as possible with the observed curve. Of course, if the same two drifts persist from region to region, there must be relations between the constants of the drifts in the different regions. The fulfilment of these relations is a good test of the theory.

* More precisely—one-third the number of stars having proper motions in directions between $\theta - 15°$ and $\theta + 15°$. The other three columns have been "smoothed" in the same way.

TABLE II.—REGION A.

Direction θ.	Calculated. Drift I.	Drift II.	Total.	Observed.	Difference observed–calculated.
5°	42	5	47	49	+2
15°	35	5	40	39	−1
25°	26	5	31	27	−4
35°	16	5	21	20	−1
45°	9	6	15	18	+3
55°	5	7	12	15	+3
65°	2	8	10	14	+4
75°	1	8	9	12	+3
85°	1	9	10	12	+2
95°	1	10	11	11	0
105°	0	11	11	10	−1
115°	0	12	12	10	−2
125°	0	12	12	9	−3
135°	0	13	13	13	0
145°	0	13	13	11	−2
155°	0	13	13	14	+1
165°	0	13	13	12	−1
175°	0	13	13	14	+1
185°	0	12	12	12	0
195°	0	12	12	13	+1
205°	0	11	11	11	0
215°	0	10	10	10	0
225°	0	9	9	7	−2
235°	0	8	8	7	−1
245°	0	8	8	9	+1
255°	0	7	7	9	+2
265°	1	6	7	7	0
275°	1	5	6	7	+1
285°	1	5	6	9	+3
295°	2	5	7	11	+4
305°	5	5	10	12	+2
315°	9	5	14	15	+1
325°	16	4	20	21	+1
335°	26	4	30	29	−1
345°	35	4	39	39	0
355°	42	5	47	47	0
Totals .	276	293	569	585	

TABLE III.—REGION B.

Direction θ.	Calculated.			Observed.	Difference observed—calculated.
	Drift I.	Drift II.	Total.		
5°	59	6	65	67	+2
15°	59	7	66	65	−1
25°	51	8	59	54	−5
35°	38	9	47	47	0
45°	25	10	35	37	+2
55°	15	12	27	31	+4
65°	8	14	22	30	+8
75°	5	16	21	28	+7
85°	2	18	20	25	+5
95°	1	20	21	19	−2
105°	1	22	23	20	−3
115°	1	23	24	20	−4
125°	1	24	25	26	+1
135°	1	24	25	25	0
145°	0	23	23	26	+3
155°	0	22	22	22	0
165°	0	20	20	20	0
175°	0	18	18	18	0
185°	0	16	16	18	+2
195°	0	14	14	15	+1
205°	0	12	12	16	+4
215°	0	10	10	10	0
225°	0	9	9	5	−4
235°	0	8	8	7	−1
245°	1	7	8	6	−2
255°	1	6	7	6	−1
265°	1	6	7	7	0
275°	1	5	6	6	0
285°	1	5	6	8	+2
295°	2	5	7	8	+1
305°	5	5	10	10	0
315°	8	5	13	13	0
325°	15	5	20	18	[illegible]
335°	25	5	30	27	[illegible]
345°	38	5	43	42	[illegible]
355°	51	6	57	[illegible]	[illegible]
Totals .	416	430	846	[illegible]	[illegible]

TABLE IV.—REGION C.

Direction θ.	Calculated. Drift I.	Drift II.	Total.	Observed.	Difference observed—calculated.
5°	5	6	11	11	0
15°	9	7	16	15	−1
25°	16	7	23	24	+1
35°	24	8	32	34	+2
45°	32	9	41	42	+1
55°	35	9	44	43	−1
65°	32	10	42	38	−4
75°	24	11	35	35	0
85°	16	12	28	27	−1
95°	9	13	22	23	+1
105°	5	13	18	16	−2
115°	2	14	16	14	−2
125°	1	14	15	12	−3
135°	1	14	15	13	−2
145°	1	13	14	14	0
155°	0	13	13	14	+1
165°	0	12	12	12	0
175°	0	11	11	11	0
185°	0	10	10	10	0
195°	0	9	9	10	+1
205°	0	9	9	9	0
215°	0	8	8	8	0
225°	0	7	7	7	0
235°	0	7	7	6	−1
245°	0	6	6	6	0
255°	0	6	6	4	−2
265°	0	5	5	4	−1
275°	0	5	5	4	−1
285°	0	5	5	5	0
295°	0	5	5	5	0
305°	0	5	5	6	+1
315°	0	5	5	7	+2
325°	1	5	6	7	+1
335°	1	5	6	6	0
345°	1	5	6	6	0
355°	2	6	8	8	0
Totals .	217	309	526	516	

TABLE V.—REGION D.

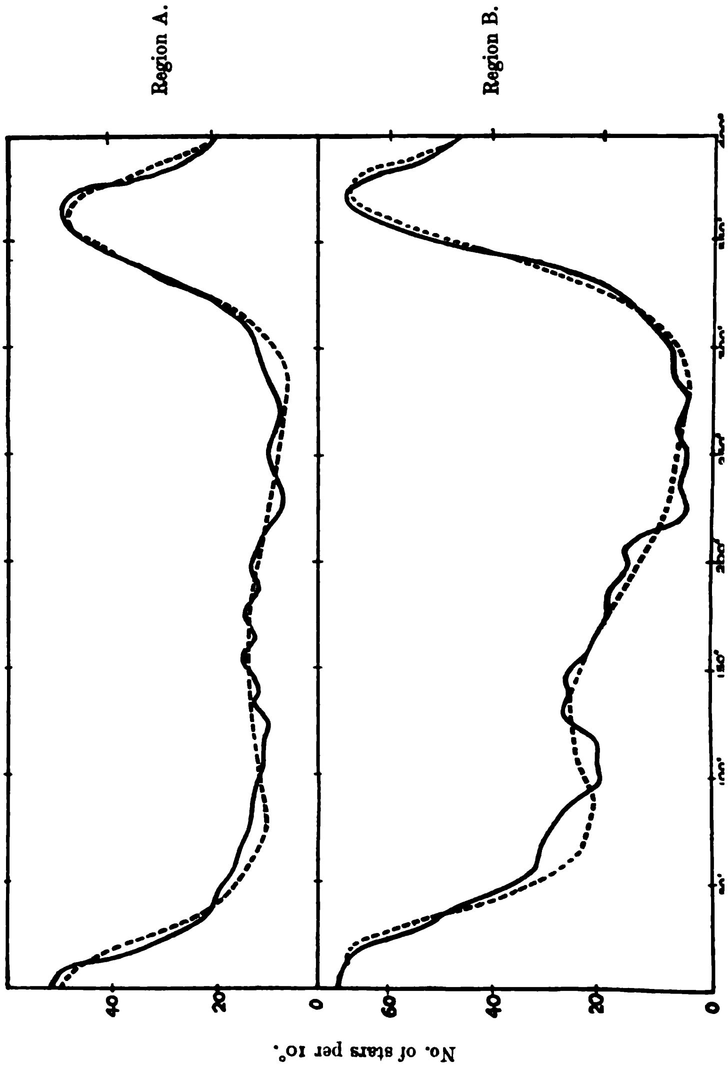
Direction θ.	Calculated. Drift I.	Calculated. Drift II.	Calculated. Total.	Observed.	Difference observed—calculated.
5°	0	5	5	4	−1
15°	0	5	5	5	0
25°	1	5	6	5	−1
35°	1	5	6	7	−1
45°	1	5	6	8	+2
55°	2	5	7	9	+2
65°	4	5	9	12	+3
75°	6	6	12	12	0
85°	10	6	16	16	0
95°	16	6	22	20	−2
105°	22	6	28	30	+2
115°	27	7	34	35	+1
125°	29	7	36	37	+1
135°	27	7	34	30	−4
145°	22	7	29	27	−2
155°	16	8	24	22	−2
165°	10	8	18	19	+1
175°	6	8	14	16	+2
185°	4	8	12	15	+3
195°	2	8	10	15	+5
205°	1	8	9	10	+1
215°	1	8	9	8	−1
225°	1	7	8	6	−2
235°	0	7	7	7	0
245°	0	7	7	8	+1
255°	0	7	7	8	+1
265°	0	6	6	6	0
275°	0	6	6	5	−1
285°	0	6	6	5	−1
295°	0	6	6	5	−1
305°	0	5	5	5	0
315°	0	5	5	5	0
325°	0	5	5	5	0
335°	0	5	5	5	0
345°	0	5	5	6	+1
355°	0	5	5	5	0
Totals .	209	225	434	443	

TABLE VI.—REGION F.

Direction θ.	Calculated. Drift I.	Drift II.	Total.	Observed.	Difference observed—calculated.
5°	0	6	6	4	−2
15°	0	7	7	5	−2
25°	0	8	8	6	−2
35°	0	10	10	9	−1
45°	0	11	11	10	−1
55°	0	12	12	14	+2
65°	0	12	12	14	+2
75°	0	13	13	14	+1
85°	0	13	13	13	0
95°	0	12	12	12	0
105°	1	12	13	10	−3
115°	1	11	12	11	−1
125°	1	10	11	10	−1
135°	1	8	9	10	+1
145°	2	7	9	7	−2
155°	3	6	9	9	0
165°	5	6	11	9	−2
175°	7	5	12	14	+2
185°	11	4	15	14	−1
195°	15	4	19	16	−3
205°	19	3	22	21	−1
215°	23	3	26	27	+1
225°	24	3	27	29	+2
235°	23	3	26	26	0
245°	19	3	22	19	−3
255°	15	3	18	17	−1
265°	11	3	14	12	−2
275°	7	3	10	11	+1
285°	5	3	8	11	+3
295°	3	3	6	8	+2
305°	2	3	5	7	+2
315°	1	3	4	6	+2
325°	1	4	5	6	+1
335°	1	4	5	5	0
345°	1	5	6	5	−1
355°	0	6	6	4	−2
Totals .	202	232	434	425	

TABLE VII.—REGION G.

Direction θ.	Calculated.			Observed.	Difference observed—calculated.
	Drift I.	Drift II.	Total.		
5°	26	9	35	39	+4
15°	22	11	33	39	+6
25°	17	12	29	34	+5
35	13	14	27	28	+1
45°	11	16	27	27	0
55°	8	18	26	28	+2
65°	7	21	28	30	+2
75°	5	24	29	29	0
85°	5	26	31	31	0
95°	4	29	33	34	+1
105°	3	30	33	33	0
115°	3	31	34	32	−2
125°	3	31	34	34	0
135°	3	30	33	35	+2
145°	3	29	32	33	+1
155°	3	26	29	27	−2
165°	3	24	27	28	+1
175°	3	21	24	27	+3
185°	3	18	21	24	+3
195°	4	16	20	22	+2
205°	5	14	19	20	+1
215°	5	12	17	20	+3
225°	7	11	18	17	−1
235°	8	9	17	18	+1
245°	11	8	19	19	0
255°	13	8	21	20	−1
265°	17	7	24	25	+1
275°	22	7	29	29	0
285°	26	6	32	32	−0
295°	32	6	38	34	−4
305°	36	6	42	38	−4
315°	40	6	46	46	0
325°	41	7	48	48	0
335°	40	7	47	46	−1
345°	36	8	44	40	−4
355°	32	8	40	37	−5
Totals .	520	566	1086	1103	


Region A.
Region B.
40
20
0
60
40
20
0
No. of stars per 10°.

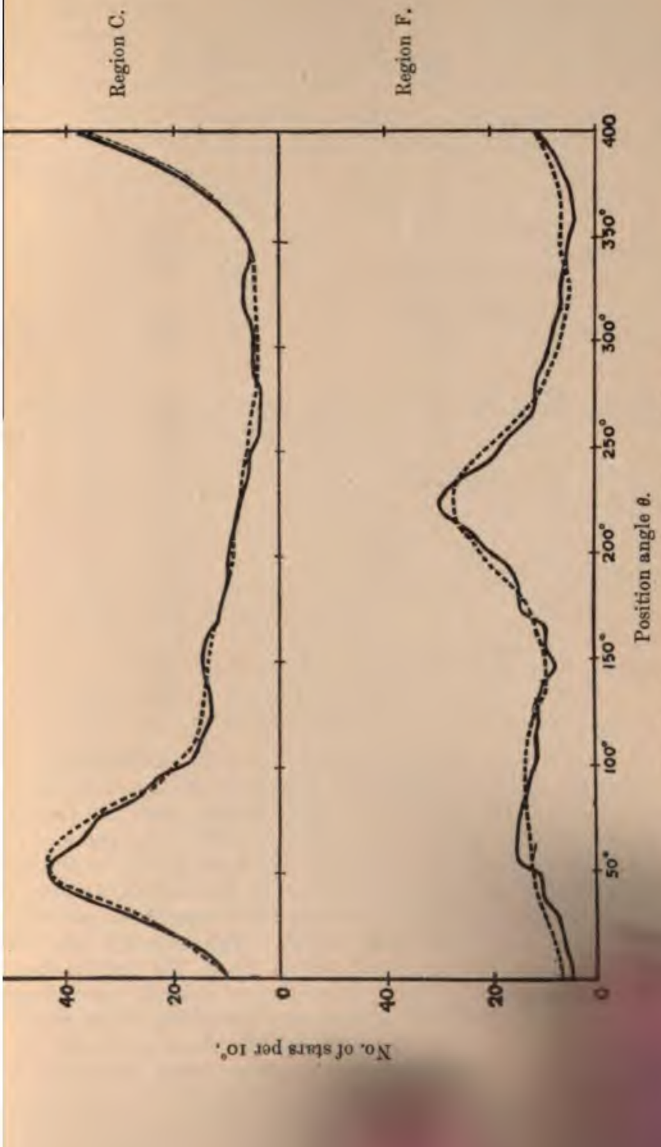


Fig. 6.—Comparison of observation and theory.

4. *Results of the Analysis.*

The following are the constants of the drifts in the several regions. They are the values on which the columns headed "calculated" in the preceding tables are based.

TABLE VIII.

		Drift I.	Drift II.
Region A	Number of stars	277	293
	hV	1·65	·30
	θ_0 (direction of drift) . . .	0°	155°
Region B	Number of stars	418	430
	hV	1·55	·45
	θ_0	10°	130°
Region C	Number of stars	217	309
	hV	1·65	·30
	θ_0	55°	125°
Region D	Number of stars	209	225
	hV	1·40	·15
	θ_0	125°	185°
Region F	Number of stars	202	232
	hV	1·20	·45
	θ_0	225°	80°
Region G	Number of stars	520	566
	hV	·75	·45
	θ_0	325°	120°

Region E contained too few stars to analyse satisfactorily.

Using a globe, I entered the directions of the drifts as great circles on it. The great circles were drawn from the centres of regions (which in the case of regions B C D G were assumed to be at N.P.D. 40°*) in the directions indicated by θ_0 in the above results. Supplementing the drawing by calculation, I found that for drift I. the great circles (produced backwards) all passed close to the point R.A. 18^h, Decl. +18°. The accuracy was rather surprising: five circles passed within 2° of the point; that for region G was about 5° out. The stars of drift I. have clearly a common velocity relative to the sun away from this point. I shall call the point Q_1. In spite of the close agreement of the great circles, they intersect at so acute an angle, all coming from

* This was verified by actual calculation of the centre of mean position of the stars in several regions. For F the centre was taken to be N.P.D. 36°; it was discovered that some stars between 0° and 20° N.P.D. had accidentally been included in the region. As the number of stars in region F would have been too few to analyse if they were excluded, the results for this augmented region have been retained.

a limited region of the sky, that Q_1 is not very accurately determined, especially as regards declination.

Theoretically, hV should be proportional to the sine of the angular distance of the area from Q_1, for V is the component velocity of the drift *across the line of sight.* The following table shows a comparison between observation and theory. The decrease in hV as we approach Q_1 is indicated, but the agreement is only rough. In particular, region G shows much too small a value.

Region.	Dist. from Q_1.	Sine of Distance.	hV.
A	72°	·95	1·65
B	76°	·97	1·55
C	107°	·96	1·65
D	107°	·96	1·40
F	43°	·68	1·20
G	40°	·64	·75

Examining the results for drift II. in the same way, we must bear in mind that the same degree of accuracy is not to be expected. The drift II. curves are not very elongated, and we cannot tell with precision which way they are pointing. If we take the velocity of the sun relative to drift I. to be 18 kilometres per sec., an error of 0·1 in hV corresponds to an error of 1 kilometre per sec. An error of 5° in the drift I. apex, or an error of 15° in the drift II. apex, corresponds to an error of about $1\frac{1}{2}$ kilometre per sec.

I find the best position for the drift II. apex Q_2 to be

$$\text{R.A. } 7^h\ 30^m,\ \text{Decl. } +58°$$

the great circles passing fairly near it, except for region G.

The values of hV are in good agreement with theory, as the following table shows :—

Region.	Dist. from Q_2.	Sine of Distance.	hV.
A	32°	·53	·30
B	59°	·86	·45
C	31°	·52	·30
D	9°	·16	·15
F	60°	·87	·45
G	72°	·95	·45

Partly owing to the very large number of stars in region G, and partly owing to its results not agreeing as well as those of the other regions, I made a special investigation, subdividing it into four divisions. Three of the divisions showed the two drifts, and, as far as I could tell, were in good agreement with theory. But the other division (called $G\gamma$), viz., N.P.D. 40°–52°, R.A. 18^h–20^h, containing 336 stars, was evidently quite exceptional. The diagram for it is shown in fig. 7. It will at once be seen that there is no indication here that we have to deal with a combination of two

drifts. There seems to be a third drift in the direction $\theta=$ which is apparently as important as either of the two fundam drifts. Whether the explanation is that we have here a local drift, or whether there is some local error in the p motions, it will at any rate be admitted that $G\gamma$ is an abn region, and no good can be done by retaining it in our investig of the nature of the two main drifts which prevail througho sky.

I have accordingly analysed the gnomon-shaped region

Fig. 7.

pounded of the three remaining divisions. Its centre is at $20^h\ 30^m$, N.P.D. 36°.

	Drift I.	Drift II.
Number of stars .	328	432
hV	1·0	·45
θ_0	330°	100°

The direction of drift II. is now in good agreement wi other regions, and so are the values of hV. The direction of I. is rather discordant.

5. *The Carrington Proper Motions.*

After the greater part of the preceding work had been pleted, another series of proper motions became availab discussion. These are the result of a comparison betwee rington's Catalogue (mean epoch 1855) and the Greenwich year Catalogue for 1900, not yet published. This series in about 1100 stars down to magnitude 9·5, all included within the pole. Of course the Groombridge Catalogue overlaps this 90 of the brighter stars are common to the two catalogues.

Thackeray has made a comparison of the proper motio these 90 stars derived from the two catalogues. From this found that the mean accidental difference between Grooml

and Carrington proper motions is 1·1″ per century in either co-ordinate. If the accidental error of either catalogue be assumed to be the same, the mean accidental error of a proper motion will be 0·8″ per century in either co-ordinate.

The result of the analysis of the Carrington proper motions was as follows:—

	Drift I.	Drift II.
Number of stars .	530	620
hV	1·30	·30
θ_0	5°	140

A comparison with the results for the corresponding region A of Groombridge shows fairly good agreement. The only noteworthy difference is the value 1·30 of hV for drift I. For region A it is 1·65; and although that value is probably rather high, we should expect a value of at least 1·50.

One naturally looks for the explanation of a small value of hV in large accidental error (see § 7). At first sight the above-mentioned comparison of Groombridge and Carrington seemed to forbid this explanation. However, on reflection it was realised that the comparison only referred to bright stars, viz., those which Groombridge observed. Now, the Carrington catalogue contains 364 stars between magnitude 9·0 and 9·5, whilst Groombridge does not include many stars below magnitude 8·5. The Greenwich places of these fainter stars are subject to considerable accidental error, and we can well suppose that the proper motions are not nearly so good as those of the brighter stars, for which the comparison was made.

Moreover, at about magnitude 8·5 magnitude equation begins to be of importance. In any other region magnitude equation would appear as a systematic error, but in a region symmetrical about the pole it is equivalent to an accidental error.

Analysing the Carrington stars brighter than 8·9 magnitude I found

	Drift I.	Drift II.
Number of stars .	383	386
θ_0	5°	140°
hV	1·40	·35

The increase in hV was very decided.*

Table IX. shows the analysis. The last column number of stars between magnitude 9·0–9·5; it is int note that the two drifts are conspicuous, so that the law extend beyond the ninth magnitude. ima are nor the minima deep—indicating sm and large accidental error.

* hV is still too small, but we are ent excluded faint stars a higher value of hV if we remove another half magnitude, to

TABLE IX.—CARRINGTON (mag. 0 – 8·9).

Direction θ.	Calculated.			Observed.	Difference observed—calculated.	Mag. obse
	Drift I.	Drift II.	Total.			
5°	51	6	57	57	0	1
15°	48	7	55	54	−1	1
25°	39	7	46	48	+2	1
35°	28	8	36	44	+8	1
45°	18	9	27	33	+6	1
55°	11	10	21	28	+7	1
65°	7	11	18	18	0	1
75°	4	13	17	17	0	1
85°	2	14	16	12	−4	1
95°	2	15	17	13	−4	1
105°	1	16	17	17	0	
115°	1	17	18	18	0	
125°	1	18	19	21	+2	1
135°	1	19	20	19	−1	
145°	1	19	20	21	+1	
155°	1	18	19	19	0	
165°	1	17	18	17	−1	1
175°	0	16	16	14	−2	
185°	0	15	15	12	−3	
195°	0	14	14	12	−2	
205°	1	13	14	13	−1	
215°	1	11	12	12	0	
225°	1	10	11	9	−2	
235°	1	9	10	9	−1	
245°	1	8	9	8	−1	
255°	1	7	8	9	+1	
265°	1	7	8	8	0	
275°	2	6	8	9	+1	
285°	2	6	8	8	0	
295°	4	6	10	11	+1	
305°	7	6	13	16	+3	
315°	11	5	16	20	+4	
325°	18	5	23	23	0	
335°	28	6	34	32	−2	1
345°	39	6	45	44	−1	1
355°	48	6	54	55	+1	1
Totals .	383	386	769	780	.	36

6. *Characteristics of the Drifts.*

It will have been noticed that the velocity of the first drift relative to the sun is very much greater than that of the second, the values being about $\frac{1\cdot7}{h}$ and $\frac{\cdot5}{h}$ respectively. For this reason the first drift is very much more prominent than the second in the diagrams. But the result of the analysis is to show that they contain nearly the same number of stars, drift II. containing a slightly greater proportion. Another very unexpected result is that the proportion of drift II. to drift I. stars is very nearly constant in the different regions, as can be seen by inspection of Table VIII. The distribution of the stars is exceedingly irregular, and we should hardly have expected the two drifts to possess the same irregularities. We know, for instance, that stars are much more numerous near the Galactic plane than elsewhere: we might expect this to be a peculiarity due to one drift alone. Now, the Galactic plane passes through region B and part of G, and we see that actually the special abundance of stars there is due to both drifts equally.

We now further inquire whether this constancy of proportion extends to all distances. Do the two drifts form a homogeneous mixture? We have seen that the proportion is the same in all parts of the sky (that we have investigated), but may not the average distance of the drift II. stars be different from that of the drift I.? For instance, may not drift II. be a cluster of stars containing the sun, and moving relatively to drift I., which represents the main background of more distant stars?

Mean Distances of the Stars.—We have deduced the various constants connected with the motions of the two drifts of stars without making use of the *magnitudes* of the proper motions. We may now use these in order to learn something of the distances of the stars.

Ceteris paribus, the magnitude of a proper motion will be inversely proportional to the distance of the star. Accordingly, if we use arithmetic means of the proper motions, the results deduced will refer to harmonic means of the distances. There is no special disadvantage in this under certain circumstances; but it is necessary to bear in mind that in using the harmonic mean a great deal of the advantage of taking the average of a lar number disappears. The stars which happen to be near us h very great weight in determining the resulting mean; but are few in number, and are therefore essentially irregularl tributed. The large number of more distant stars on whi should prefer to depend have little effect or

I have adopted the usual practice of ve motions; the limit I adopted was to r ? four times the mean (excluding them meet the difficulty; the nominal av really determined mainly by about 2

this in mind in considering the results about to be given. is plenty of room for accidental variations in discussing from 20 stars.

With the notation of § 2,

mean value of $h(r - V\cos\theta)$

$$= \bar{x} = \frac{\int_{-\tau}^{\infty} e^{-x^2}x^2dx + \tau\int_{-\tau}^{\infty} e^{-x^2}xdx}{\int_{-\tau}^{\infty} e^{-x^2}xdx + \tau\int_{-\tau}^{\infty} e^{-x^2}dx}$$

which after some reduction gives

$$\bar{x} = \frac{\frac{1}{2}e^{\tau^2}\int_{-\tau}^{\infty} e^{-x^2}dx}{\frac{1}{2} + \tau e^{\tau^2}\int_{-\tau}^{\infty} e^{-x^2}dx}.$$

$$\therefore\ h\bar{r} = \left\{\frac{\frac{1}{\sqrt{\pi}}e^{\tau^2}\int_{-\tau}^{\infty} e^{-x^2}dx}{f(\tau)} + \tau\right\}$$

The following table gives the values of $h\bar{r}$ corresponding to values of τ. From these I drew a graph which gave intern values with sufficient accuracy.

τ	$h\bar{r}$
−1·0	·565
−0·5	·701
0	·886
+0·2	·977
+0·5	1·134
+0·7	1·252
+1·0	1·449
+1·3	1·669
+1·6	1·908

Now if, out of n stars moving in a certain direction, n_1 to drift I., and for them $\tau = \tau_1$, and n_2 belong to drift II., a them $\tau = \tau_2$, if d_1 d_2 are the mean distances of the stars bel to the two drifts, and if $\bar{p}$ is the mean proper motion in the n stars,

$$\text{then}\quad \bar{p} = \frac{n_1}{n}\frac{\bar{r}_1}{d_1} + \frac{n_2}{n}\frac{\bar{r}_2}{d_2}\quad .\quad .$$

Considering stars in a different direction we can get another simultaneous equation, and solve the two for $\frac{1}{d_1}$ and $\frac{1}{d_2}$.

Example.—In region B, consider the proper motions in directions

(α) between 0° and 20°
(β) between 130° and 160°

(α) The analysis showed that we have here 124 drift I. stars and 13 drift II. stars.

Also for drift I., $hV = 1{\cdot}55$ and the direction of drift I. is 10°, which is also the mean direction of these proper motions.

$$\therefore \quad \tau_1 = 1{\cdot}55 \cos 0^\circ = 1{\cdot}55$$

Similarly,

$$\tau_2 = {\cdot}45 \cos 120^\circ = -{\cdot}22$$

From the graph I found,

corresponding to

$$\tau_1 = 1{\cdot}55, \quad hr_1 = 1{\cdot}87$$
$$\tau_2 = -{\cdot}22, \quad hr_2 = 0{\cdot}81$$

Also the mean proper motion came to be $5''{\cdot}44$, rejecting seven stars whose proper motions exceeded $4 \times 5''{\cdot}44$.

Substituting these values in equation (1),

$$5{\cdot}44'' = \frac{124}{137} \times 1{\cdot}87 \times \frac{1}{hd_1} + \frac{13}{137} \times {\cdot}81 \times \frac{1}{hd_2} \quad . \quad . \quad (2)$$

(β) These are all drift II. stars,

$$\tau_2 = {\cdot}45 \cos 15^\circ = {\cdot}43$$

$$\text{hence} \qquad hr_2 = 1{\cdot}09$$

The mean P.M. was found to be $3''{\cdot}07$, rejecting six stars whose P.M.'s exceeded $4 \times 3''{\cdot}07$,

$$\text{hence} \qquad \frac{1{\cdot}09}{hd_2} = 3''{\cdot}07 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Solving (2) and (3) we find

$$\frac{1}{hd_1} = 3''{\cdot}09 \quad \frac{1}{hd_2} = 2''{\cdot}82 \qquad \text{(unit of time} = 1 \text{ century)}$$

The results of a similar examination of the other regions are

Region.	$\frac{1}{hd_1}$	$\frac{1}{hd_2}$
A	$3''{\cdot}77$	$3''{\cdot}68$
B	$3''{\cdot}09$	$2''{\cdot}82$
C	$2''{\cdot}83$	$3''{\cdot}47$
G	$3''{\cdot}34$	$3''{\cdot}48$

It will be seen that the maximum difference between the numbers in the last two columns is only 25 per cent. of the whole. This perhaps is the only conclusion which can be considered established by the investigation. The figures seem to indicate that where the drift I. stars are nearer to us than the average, there also the drift II. stars are nearer, just as we found that where the stars of one drift were densely distributed, there also the stars of the other drift were dense. However, the main conclusion is that there is no appreciable difference in the mean distances of drift I. and drift II. stars.

Magnitude and Spectral Type.—We cannot, as a rule, pick out an individual star and decide (from its motion) to which drift it belongs. But, for example, in region G we find, on reference to Table VII., that of stars moving in directions

$\theta = 290^\circ - 360^\circ$ 257 belong to drift I. and 48 to drift II.
for $\theta = 90^\circ - 170^\circ$ 25 ,, ,, 230 ,,

We may thus roughly separate out stars typical of the two drifts and examine their characteristics.

The following table shows how stars of different magnitudes are divided between the drifts :—

		Number of Stars of Magnitudes								
		0–3·9	4·0–4·9	5·0–5·9	6·0–6·9	7·0–7·4	7·5–7·9	8·0–8·4	8·4–8·9	Total.
Region A	Drift I. .	3	5	28	48	36	21	22	11	174
	Drift II. .	1	4	20	48	30	18	26	13	160
Region B	Drift I. .	7	10	35	60	47	36	35	19	249
	Drift II. .	1	13	33	55	51	40	50	23	266
Region G	Drift I. .	6	14	23	63	53	51	47	21	278
	Drift II. .	1	6	28	66	44	55	56	25	281
A, B, and G combined	Drift I.	16	29	86	171	136	108	104	51	701
	Drift II.	3	23	81	169	125	113	132	61	707
Carrington	Drift I.	1	0	4	8	14	8	42	77	154
	Drift II.	0	0	6	11	8	16	34	92	167

There is a slightly greater proportion of very faint stars in drift II. in each case. This may be due to the fact that accidental error tends to falsely increase the number of stars apparently in drift II. in this investigation, because, for example, in region B, $\theta = 350^\circ - 30^\circ$ has been taken to typify drift I., and $\theta = 190^\circ - 270^\circ$ for drift II. If a star's motion is purely accidental, there is a much greater chance of its falling within the latter than within the former angular interval.

Most of the very bright stars seem to belong to drift I., but they are few in number altogether. With this possible exception, we may conclude that the drifts do not differ at all as regards the magnitudes of the stars they contain.

As regards *type of spectrum*, the evidence is inconclusive. The proportion of type II. stars seems distinctly higher in drift II. than in drift I. This is true in every region I have examined. On the other hand, the distribution of spectral types seems to be determined mainly by something in no way connected with the drift motions.

The following is the result of the examination of the four most suitable regions :—

Region.	Drift I.			Drift II.			Ratio Column 7. / Column 4.
	No. of Stars Type I.	No. of Stars Type II.	Ratio Type II. / Type I.	Type I.	Type II.	Ratio Type II. / Type I.	
B	95	35	·37	61	45	·74	2·0
G	58	39	·67	41	39	·95	1·4
C	61	23	·38	16	16	1·00	2·6
A	61	66	1·08	36	70	1·94	1·8

The numbers in the last column represent

$$\frac{\text{Ratio of Type II. to Type I. for Drift II.}}{\text{Ratio of Type II. to Type I. for Drift I.}}$$

and show that in each case the proportion of type II. stars is higher in drift II.

We must beware of attaching too much significance to this result, and remember that it applies only to a small part of the sky.

7. *Effect of Errors in the Proper Motions.*

Accidental Error.—The method I have adopted is open to the objection that the stars with very small proper motions, whose directions are presumably not so reliably determined, have the same weight as those with large proper motions. We can, however, take account of small accidental error in the following manner.

If the number of stars having an actual velocity component u be $Ne^{-h^2u^2}$ and the probability of an accidental error x in the observed component proper motion is

$$\frac{g}{\sqrt{\pi}}e^{-g^2x^2}$$

(x being a linear velocity, we confine our attention for the moment to stars at a definite distance).

Then the number of stars having an observed velocity u is

$$N\int_{-\infty}^{\infty} e^{-h^2(u-x)^2}\frac{g}{\sqrt{\pi}}e^{-g^2x^2}dx$$

which reduces to

$$\mathrm{N}\frac{g}{\sqrt{g^2+h^2}}e^{-\frac{g^2h^2}{g^2+h^2}u^2}$$

Thus the effect of accidental error is simply to change the true drift constant h into an apparent drift constant h_1 given by

$$h_1^2=\frac{g^2h^2}{g^2+h^2}$$

The mean *accidental* error of a Groombridge proper motion appears to be about 0·8″ per century in either co-ordinate (see p. 53). To convert this into linear velocity we must use the values of $\frac{1}{hd}$ found in § 6: the mean value of $\frac{1}{hd}$ is a little over 3″ per century, hence the mean accidental error in the component linear velocity of a star is

$$<\frac{\cdot 8''}{3''}\cdot\frac{1}{h}=\text{about }\frac{1}{4h}$$

But if the probability of an accidental error x be $\frac{g}{\sqrt{\pi}}e^{-g^2x^2}$ the mean error is $\frac{1}{\sqrt{\pi}\cdot g}$

$$\therefore\qquad\frac{1}{4h}=\frac{1}{\sqrt{\pi}\cdot g}$$

$$\text{whence}\qquad\frac{1}{h_1}=1\cdot 09\,\frac{1}{h}\text{ roughly}$$

Thus the effect of accidental error is that if the true value of h is used, the quantities hV given in the preceding sections should be increased by about 10 per cent.

Since g depends on the distance of the stars, this correction is only approximate, and would not hold if the accidental error were large.

Systematic Error.—It is possible that in any region all the proper motions may be liable to a systematic error. For various reasons it can be confidently asserted that in no case does the systematic error much exceed 1″ per century. For instance, let each proper motion be represented by a point whose Cartesian co-ordinates are its components; if there is no systematic error these points will be densest near the origin, but if there is considerable systematic error this will not be so. I have examined the more important regions in this way and found no indication of large systematic error.

Table X. shows the number of proper motions which are greater than 5″ per century in each region in the various directions. This table can hardly be appreciably affected by any admissible value of the systematic error. Nevertheless, the presence of two

TABLE X.—PROPER MOTIONS EXCEEDING 5″.

. θ	REGIONS A	B	C	D	E	F	G	Carrington.
5°	29	38	3	2	4	1	7	33
15°	23	31	4	1	2	1	8	30
25°	16	22	8	1	2	1	7	23
35°	10	15	13	2	2	3	4	18
45°	7	10	17	3	2	4	3	14
55°	5	6	18	5	3	5	3	10
65°	3	6	15	6	3	5	5	8
75°	2	6	13	7	2	6	7	7
85°	2	5	7	8	2	5	10	5
95°	2	4	5	10	3	5	11	4
105°	2	4	3	15	4	4	10	6
115°	3	4	4	18	4	4	10	5
125°	3	5	3	18	7	3	11	6
135°	5	6	4	15	8	3	13	7
145°	4	7	3	13	11	2	11	9
155°	5	6	5	9	12	2	7	8
165°	6	4	4	7	16	2	4	6
175°	6	3	3	5	21	2	3	5
185°	4	3	2	5	22	4	4	4
195°	5	3	2	3	19	6	4	5
205°	5	3	1	2	12	10	3	5
215°	5	2	0	1	8	13	3	6
225°	2	2	1	0	6	15	2	4
235°	2	1	1	1	4	12	2	3
245°	3	1	1	2	3	9	4	1
255°	3	0	1	1	2	6	7	1
265°	2	1	1	1	2	3	10	1
275°	1	1	0	0	1	2	10	2
285°	2	1	0	0	0	3	12	2
295°	3	1	0	1	0	3	13	4
305°	4	1	0	0	0	3	15	5
315°	7	2	0	0	1	2	18	8
325°	9	4	0	1	1	1	17	[illegible]
335°	14	10	0	1	2	1	16	[illegible]
345°	20	22	0	2	4	1	[illegible]	[illegible]
355°	26	34	1	2	4	[illegible]	[illegible]	[illegible]
rection drift II.	170°	135°	150°	350°?	?	80°	[illegible]	[illegible]
,, I.	2°	7°	55°	[illegible]	180°	225°	[illegible]	[illegible]

drifts is exhibited very clearly; in fact, more so than when all the proper motions are included. We may therefore feel sure that the phenomena which we have attributed to two drifts are real, and not in any way the effect of systematic error.

A calculation of the positions of the two apices from these large proper motions gives for

Q_1 R.A. 18^h, Decl. $+10°$
Q_2 R.A. 7^h, Decl. $+58°$

in quite good enough agreement with those previously found.

A very important possible systematic error—that due to an incorrect constant of precession—must now be briefly considered.

8. *The Constant of Precession.*

The determination of the proper motion of a star depends on a comparison of observations of its position referred to certain axes at different epochs. In this comparison two unknowns are concerned, viz. the motion of the axes of reference and the motion of the star. The same holds true when we consider a large number of stars; there are two unknowns—the motion of the axes and the systematic motions of the stars. For this reason, in former investigations, determinations of the solar motion and of the constant of precession were necessarily made together. The question arises: Can we attack the fundamental assumptions on which the determination of the solar motion was made without attacking the value of the precession-constant found at the same time?

This is a serious question from the point of view of the present paper, because the constructive and quantitative work in it depends on the correctness of the Struve-Peters precessions. As Dyson and Thackeray pointed out, the area covered by the Groombridge catalogue is not very sensitive to a change in the value of the precession.

If Newcomb's precessions were adopted, the correction to the proper motions would, in the extreme cases (regions C and D), only amount to $0''.8$ per century. This would make very little difference in my tables. But may not a much more widely different value of the precession be admissible?

There is an *a posteriori* justification for using the usual value of the precession-constant. We have arrived at the conclusion that the two drifts form a practically homogeneous mixture. If, and only if, this is the case, Airy's method will give the precession. As far as averages are concerned, a homogeneous mixture of two drifts is equivalent to a single drift. Thus the so-called solar motion is an important mathematical vector, but its physical meaning must be modified. We must not be surprised at the discordances which occur in the various attempts to estimate it; it only exists at all because of this roughly uniform arrangement of the drifts, and the precision with which it can be defined depends on this uniformity.

We conclude that the adopted value of the precession is not widely wrong; at the same time, if our theory is correct, we must not trust too far the present methods of calculating it. They are only approximately (and one might almost say accidentally) correct.

9. *Summary and Results.*

The Greenwich-Groombridge and Greenwich-Carrington proper motions strongly support Kapteyn's hypothesis of two star-drifts.

The additional results arrived at in this paper (which, of course, depend on the truth of this hypothesis) are :—

1. The numbers of the stars belonging to the two drifts are nearly equal.

2. One drift is moving relatively to the sun, with a speed between three and four times that of the other.

3. The proportion of stars belonging to each drift is about the same in every part of the sky and at every distance here dealt with.

4. The magnitudes of the stars belonging to the two drifts are about the same.

5. The spectral types of the stars seem to differ to some extent in the two drifts. But the evidence is not conclusive.

6. The results apply to all stars down to magnitude 9·5.

Royal Observatory, Greenwich:
1906 *August.*

On the Effects of Radiation on the Motion of Comets (Second Note). By H. C. Plummer, M.A.

1. Professor Poynting has recently pointed out to me the existence of an additional term in the forces which are due to the action of solar radiation on a small black body. The term has no effect when the motion of the particle is circular, but it becomes necessary to amend the theory which was given in §§ 5–8 of my former paper (*Monthly Notices*, vol. lxv. p. 232). This is due to the fact that when the body is moving with a radial velocity $\frac{dr}{dt}$ the light pressure is altered in the ratio $1 : 1 - \frac{1}{U} \cdot \frac{dr}{dt}$ where U is the velocity of light. The effect of the radial component of the motion was before omitted.

2. The radial accelerations of the particle are

(i.) that due to gravitation,

$$R_g = -\frac{\mu}{r^2}$$

(ii.) that caused by a "static" pressure due to radiation,

$$R_s = +\frac{\mu'}{r^2}$$

(iii.) that caused by a "kinetic" pressure due to radiation (this is the term now introduced),

$$R_k = -\frac{1}{U}\frac{dr}{dt}\cdot\frac{\mu'}{r^2}$$

There is also a tangential retardation Tv/r^2, which may be resolved into

(iv.) a radial component

$$R_t = -\frac{T}{r^2}\cdot\frac{dr}{dt}$$

(v.) a transversal component

$$V_t = -\frac{T}{r}\cdot\frac{d\theta}{dt}$$

Now if S is the solar constant at the Earth's distance b from the Sun, and a is the radius of the spherical particle of density ρ

$$T = Sb^2/4U^2a\rho \quad . \quad . \quad . \quad . \quad (1)$$

$$\mu' = 3Sb^2/4Ua\rho = 3UT \quad . \quad . \quad . \quad (2)$$

Hence the complete radial component is

$$\Sigma R = -\frac{\mu-\mu'}{r^2} - \frac{4T}{r^2}\cdot\frac{dr}{dt} \quad . \quad . \quad . \quad (3)$$

3. One equation of motion remains as before

$$\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = rV_t = -T\frac{d\theta}{dt}$$

or on integration

$$r^2\frac{d\theta}{dt} \equiv h = \frac{T}{c}(1-c\theta) \quad . \quad . \quad . \quad (4)$$

where c is a small constant of the same order as T. The other equation of motion is

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \Sigma R$$

which becomes on eliminating t and putting $u = 1/r$

$$h^2u^2\left(\frac{d^2u}{d\theta^2} + u\right) + hu^2\frac{dh}{d\theta}\cdot\frac{du}{d\theta} + \Sigma R = 0$$

By (3) and (4) this is reduced to

$$\frac{d^2u}{d\theta^2} + u + \frac{3c}{1-c\theta}\frac{du}{d\theta} = \frac{(\mu-\mu')c^2}{(1-c\theta)^2T^2} \quad . \quad . \quad (5)$$

which differs from the corresponding equation of the former paper by the addition of the third term on the left.

4. Neglecting second-order terms we replace (5) by

$$\frac{d^2u}{d\theta^2}+3c\frac{du}{d\theta}+u=(\mu-\mu')c^2\mathrm{T}^{-2}(1+2c\theta)$$

the integral of which, to the same order of approximation, is

$$u=(\mu-\mu')c^2\mathrm{T}^{-2}\{1+2c\theta+e\cos(\theta-\gamma)-\tfrac{3}{2}ce\theta\cos(\theta-\gamma)\} \quad (6)$$

Let the osculating orbit at the point θ be

$$u=l^{-1}\{1+e'\cos(\theta-\gamma')\}$$

Since r, $\frac{dr}{dt}$ and $\frac{d\theta}{dt}$ are the same for the two curves, u, $\frac{du}{d\theta}$ and h are also the same. Therefore by (4)

$$l=\frac{h^2}{\mu-\mu'}=\frac{\mathrm{T}^2}{c^2}\,\frac{(1-c\theta)^2}{\mu-\mu'} \quad . \quad . \quad . \quad (7)$$

where θ is to be considered constant at the point. Hence we have

$$\begin{aligned}\frac{u\mathrm{T}^2}{(\mu-\mu')c^2}&=1+2c\theta+e\cos(\theta-\gamma)-\tfrac{3}{2}ce\theta\cos(\theta-\gamma)\\&=(1+2c\theta)\{1+e'\cos(\theta-\gamma')\}\end{aligned}$$

and also $\frac{du}{d\theta}$ the same for both expressions, provided that in the second we regard $c\theta$ as constant. Hence, still neglecting terms beyond the first order,

$$\begin{aligned}&e'\cos(\theta-\gamma')-e\cos(\theta-\gamma)=-\tfrac{7}{2}ce\theta\cos(\theta-\gamma)\\&e'\sin(\theta-\gamma')-e\sin(\theta-\gamma)=-\tfrac{7}{2}ce\theta\sin(\theta-\gamma)+\tfrac{3}{2}ce\cos(\theta-\gamma)-2c\end{aligned}$$

From these can be derived

$$\begin{aligned}\delta e&=-\tfrac{7}{2}ce\theta+\tfrac{3}{4}ce\sin 2(\theta-\gamma)-2c\sin(\theta-\gamma) \quad . \quad (8)\\ e\,.\,\delta\gamma&=-\tfrac{3}{2}ce\cos^2(\theta-\gamma)+2c\cos(\theta-\gamma) \quad . \quad . \quad (9)\end{aligned}$$

where the increments refer to the osculating ellipse as compared with the ellipse obtained by suppressing the accents on e and γ.

5. The secular changes in the elements can now be deduced from (7), (8), and (9) by considering the effect of a complete revolution. The result of thus removing the periodic variations is clearly

$$\begin{aligned}\Delta l/l&=-4\pi c \quad . \quad . \quad . \quad . \quad . \quad (10)\\ \Delta e/e&=-7\pi c \quad . \quad . \quad . \quad . \quad . \quad (11)\\ \Delta\gamma&=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)\end{aligned}$$

Thus the result of the additional term in the action of solar radiation is by (11) to increase the secular change in the eccentricity in the ratio of 7 : 4.

Further,

$$\frac{\Delta a}{a}=\frac{\Delta l}{l}+\frac{2e\Delta e}{1-e^2}=-\frac{4+10e^2}{1-e^2}\cdot\pi c \quad . \quad . \quad (13)$$

and

$$\frac{\Delta n}{n} = -\tfrac{3}{2} \cdot \frac{\Delta a}{a} = \frac{2 + 5e^2}{1 - e^2} \cdot 3\pi c \quad . \quad . \quad . \quad (14)$$

The effect of the new term is thus to increase the change in the mean motion in the ratio

$$2 + 5e^2 : 2 + 2e^2$$

so that, with the same notation and constants as before, equation (1) of the previous paper becomes

$$\rho a = 4 \cdot 6 \times 10^{-15} \frac{(2 + 5e^2)(1 - e)^{\frac{1}{2}}}{(1 + e)^{\frac{3}{2}}} \cdot \frac{b^2}{q^2} \cdot \frac{1}{\Delta n} \quad . \quad (15)$$

6. For Encke's Comet, $e = 0 \cdot 85$, we now obtain

$$\rho a = 0 \cdot 0093 \text{ gm./cm.}^2 ; \ \mu'/\mu = 0 \cdot 008$$

These figures do not differ very materially from those previously found. They are of the same order, and equally fail to substantiate the suggestion that an acceleration in the mean motion of Encke's Comet can be explained in this particular way. This conclusion, though disappointing, is only what might have been anticipated. Of the forces enumerated in § 2, the components (iii.), (iv.), and (v.), as compared with (ii.), the direct radiation pressure, are of the same order as cometary velocities compared with the velocity of light. Their effect is therefore naturally far smaller, and can only become adequate to explain a conspicuous inequality when the gravitation constant itself is effectively diminished by the back pressure due to radiation. The forces which depend on the velocity of the comet are dissipative, as Professor Poynting notices, and therefore cumulative in effect. But though their influence may thus become of the greatest importance by the lapse of time, it is the conservative component of which evidence must first be expected from direct observation.

7. Since my earlier paper the question of evidence on this point has advanced, and we now know that as long ago as 1891 the late Dr August Svedstrup had been led by his labours on the motion of Comet 1886 I. to the conclusion that the constant of solar attraction was in this case effectively diminished. The result is the more remarkable that the author seems to have been uninfluenced by any hypothesis as to the action of radiation. In place of the Gaussian constant k, he found it necessary to use (A.N., 4062, p. 98)

$$K = k(1 - 0 \cdot 0000415)$$

which gives

$$\mu'/\mu = 1 - K^2/k^2 = 0 \cdot 000083$$

Hence

$$a\rho = \frac{3}{4} \cdot \frac{S}{U} \cdot \frac{b^2}{\mu} \cdot \frac{10^6}{83}$$

$$= 0 \cdot 89 \text{ gm./cm.}^2$$

if we adopt $S = 1{\cdot}75 \times 10^6$. This shows that if ρ is about 6, or something like the mean density of the Earth, the diameter of the particles composing the comet is of the order of 3 mm. This result confirms the hypothesis as to the physical constitution of comets on which this investigation rests, and supplies the first numerical estimate of the size of the meteoric bodies which constitute a cometary swarm.

8. Dr Svedstrup's discovery amply justifies the statement before made, that "it is unwarrantable to *assume* that the mean motion and the mean distance are related and not independent elements." This must be borne in mind in all future discussions of the definitive orbits of comets; otherwise a valuable means of gaining an insight into the constitution of these bodies will be neglected. It now appears exceedingly probable that in this direction lies the clue to the explanation of the anomalous motion of Encke's Comet, as suggested in § 3 of my earlier paper.

University Observatory, Oxford:
1906 *August* 18.

Note on a Mechanical Solution of Kepler's Equation.
By H. C. Plummer, M.A.

This note refers to the method of obtaining an approximate solution of Kepler's equation which was proposed by Dr Rambaut (*Monthly Notices*, vol. l. p. 301), and was subsequently described independently by myself (*Monthly Notices*, vol. lvi. p. 317), and again later by Herr B. Gonggrijp (*Ast. Nach.*, 3720). The principle of the method, which is based on the use of a trochoid, is to be found in Newton's *Principia*, book i. prop. 31.

Quite recently Dr Rambaut has proposed a new method which combines great mathematical elegance with considerable practical advantages (*Monthly Notices*, vol. lxvi. p. 519). His paper suggests that it may not be out of place if I describe some modifications in the application of the trochoid principle which occurred to me, by a curious coincidence, about the same time.

In the accompanying figure, A B C is a flat circular disc of suitable radius, mounted on a board so as to be capable of turning about its centre O. Fixed to the board and flush with the surface of the disc are wooden pieces Y Y, N N, which serve as guides to a scale S S, which, with bevelled edges, can slide between them. The point E, marked with a pointer, is the foot of the perpendicular from O on the lower edge of the scale. Attached near B, and passing round the edge of the disc, is a thin flexible metallic tape F, the other end of which is attached to the end of the scale S S. This tape is kept taut by another tape B C P passing round the disc in the opposite direction and fastened to a weak spring or, much better, to a hanging weight.

The rotation of the disc thus depends on and can be measured by the motion of the scale S S, a part of which, in length equal to half the circumference of the disc, is divided into 180° in such a way that the origin coincides with the point E when the radius O A is perpendicular to the scale. Then clearly the pointer E at any time marks the angle (< 180°) through which the disc has been turned from its initial position.

The initial radius O A is divided in fractions of itself (as shown, in tenths), so that a length O*e* can be taken corresponding to the eccentricity, the whole radius being unity. The lower edge of the guide N N is parallel to the scale S S, so that the edge of a T-square

T T can be kept perpendicular to the scale. The instrument can now be used to find an approximate solution of Kepler's equation, according to the following rule:—

Place the edge of the T-square at the reading M *on the scale* S S. *Move the scale and T-square together till the point e on* O A *which marks the eccentricity e falls on the edge of the T-square. Then read the position of the pointer* E *on the scale. The reading is the eccentric anomaly* E *corresponding to the mean anomaly* M.

For clearly

$$E - M = e \sin E \quad . \quad . \quad . \quad . \quad (1)$$

If M *lies between* 180° *and* 360°, *use the instrument according to the above rule to find* 360° − E *from* 360° − M.

The figure is drawn for $e = 0{\cdot}65$, showing $M = 35^\circ{\cdot}0$ and $E = 70^\circ{\cdot}0$. The rolling disc of the device formerly described, with its troublesome manipulation and liability to error, is now eliminated. The present instrument is nearly as simple to use as a slide-rule, and

it has the great advantage of being direct-reading. Two straight, uniformly divided scales O A and S S alone are necessary; and the accuracy of reading can be increased by placing a vernier instead of a simple pointer at E, and attaching another vernier to the T-square at M. The surface of the scale S S should be very slightly above the level of Y Y, N N and the disc. The T-square can then be kept at a fixed reading with one hand while the scale is being moved by the other. It would be still better if the head of the T-square moved along a groove in the scale itself. In such details of construction there is naturally much room for variety, but it is possible to make an instrument on these general lines which is both simple and efficient.

There are, however, three additional scales shown in the figure which may be briefly explained. The first is on the edge of the T-square, which is graduated in fractions of the radius of the disc. The origin of this scale is on a level with the lowest point of the disc. It is thus possible to read directly at the point e the height of this point above the lowest point of the disc, and this reading clearly corresponds to the quantity

$$1 - e \cos \mathrm{E} = \frac{r}{a} = \frac{d\mathrm{M}}{d\mathrm{E}} \quad . \quad . \quad . \quad (2)$$

This is the ratio of the radius vector to the mean distance, but its more important significance arises from the fact that it is the ratio of corresponding increments of the mean and eccentric anomalies. Thus when an approximate E (as found by mechanical means or otherwise) is known, and is found to lead to an error $d\mathrm{M}$ in M, the approximate error in the assumed E is found by dividing $d\mathrm{M}$ by the reading on the edge of the T-square at the point where it is crossed by the radius OA of the disc. In the case represented in the figure this reading should be 0·778.

Up to this point we have considered only practical modifications in applying the principles of a device formerly described. The graduation of one semicircle and the unequally divided scale on the radius O B, as shown in the figure, are due to an addition which, whatever its value, appears to be new. The object of this is to obtain by a direct reading an approximate value of the true anomaly. The graduations in the radius O B (numbered in the figure for $10\,e$) are made in such a way that corresponding to any eccentricity e a point f can be read off such that

$$\mathrm{O}f/\mathrm{OB} = \tan \tfrac{1}{2}\phi \quad \text{where} \quad \sin \phi = e$$

A thread C D is attached to the point C, the highest point of the disc. The disc being in the position already found, the thread is stretched over the point f and crosses the edge of the disc at v. Then the angle A O v or v is such that

$$\tan \tfrac{1}{2}v = \sqrt{\frac{1+e}{1-e}} \tan \tfrac{1}{2}\mathrm{E} \quad . \quad . \quad . \quad (3)$$

so that v is the true anomaly corresponding to the mean anomaly M.

The geometrical principle on which this very simple device is based need not be proved here.* An accurate value of the true anomaly can only be found by calculation from an accurate value of E. But there may be cases in which only a rough value is required, and in any case a value of this kind, when so easily obtained, may be useful in preventing any serious slip in the calculation from being overlooked. In the case represented in the figure the value of v should be $113^\circ{\cdot}3$.

The complete instrument thus provides the means of finding, for any value of the eccentricity between 0·1 and 0·9, approximate values of the eccentric and true anomalies and of the radius vector, when the mean anomaly or time is given. It thus solves in an approximate manner the whole problem of elliptic motion as expressed by the relations (1), (2), and (3), and the ease with which this can be done suggests that the instrument may have some slight educational as well as practical value.

1906 *September* 18.

Solar Parallax Papers. No. 5.

Examination of the Photographic Places of Stars published in the Paris Eros Circulars. By Arthur R. Hinks, M.A.

1. In a preceding paper, *Solar Parallax Papers, No.* 4 (*Monthly Notices*, 1906 June, lxvi. p. 481), it was shown that the photographic right ascensions of *repère* stars, published in Paris circulars 10 and 11, had little if any magnitude equation special to each observatory (with one exception); and the conclusion was drawn that it is unlikely that they have an absolute magnitude equation common to all. This applies only to stars down to a magnitude little fainter than 9th. In the present paper I propose to extend the search for photographic magnitude equation to the declinations of the *repère* stars, and to the fainter stars in both co-ordinates; and to look for systematic errors other than those depending on magnitude, so far as they can be detected by intercomparison of published results.

2. The stars measured upon Eros plates in accordance with M. Loewy's programme are divided into three classes:—

i. The *étoiles de repère*, extending right up to the edges of plates 2° square, and generally in the outer regions rather than near the centre.

ii. The *étoiles de comparaison*, which have been used as comparison stars in visual micrometric observations. They lie mostly in a narrow belt along the track of the planet, and extend right up to two edges of the plate.

* The principle is the same as that employed in "A Method of Mechanically Compensating the Rotation of the Field of a Siderostat" (*M.N.*, vol. lxi. p. 402). See § 2 for a proof which can be adapted without difficulty to the present case, and §§ 8 and 9 for an examination of the geometry of the principle.

Magnitudes.	Paris minus— Alg.	Bord.	Cat.	Green.	Hels.	North.	S. Fern.	Toul.	Mean of 7.
−6·2	(5) − ″·45	(2) + ″·14	(3) + ″·30	(3) − ″·05	(3) + ″·36	(3) + ″·06	(5) − ″·08	(5) − ″·51	(24) − ″·03
6·3−6·9	(6) − 17	...	(2) + 4	(5) + 7	(4) + 16	(5) + 22	(6) + 1	(6) − 3	(28) + 07
7·0−7·4	(15) − 32	(7) + 4	(9) + 12	(14) + 1	(9) − 12	(12) − 10	(15) − 10	(15) − 9	(81) − 4
7·5−7·9	(27) − 10	(12) + 11	(21) + 11	(20) − 1	(15) − 2	(20) − 3	(27) + 1	(27) − 7	(142) + 1
8·0−8·4	(60) 0	(24) + 5	(39) + 7	(47) + 3	(22) + 5	(33) 0	(53) + 13	(53) + 4	(271) + 3
8·5−8·8	(70) + 4	(40) 0	(43) + 3	(60) 0	(38) − 9	(46) − 2	(66) + 9	(67) + 1	(360) + 1
8·9−9·2	(45) + 7	(24) + 5	(31) + 13	(35) − 3	(20) − 4	(33) 0	(44) + 9	(44) + 5	(231) + 4
9·3−	(12) + 21	(9) + 4	(8) + 11	(8) 0	(10) + 4	(7) − 3	(13) + 24	(13) + 20	(68) + 10
Totals	(240) − ·01	(118) + ·04	(156) + ·08	(192) ·00	(121) − ·02	(159) − ·01	(229) + ·05	(230) + ·01	...
List II.									
·2	(5) − ·23	(2) + ·56	(4) + ·53	(3) − ·09	(5) + ·14	(3) + ·40	(5) + ·26	(3) + ·43	(25) + ·30
9	(7) − 12	(4) − 11	(7) − 16	(4) − 6	(3) + 31	(1) − 19	(7) + 29	(5) + 24	(31) − 1
4	(8) − 10	(4) − 19	(5) + 16	(8) + 1	(8) + 12	(2) − 10	(8) − 5	(7) − 7	(42) 0
9	(17) + 2	(6) − 8	(12) + 3	(12) + 8	(12) − 2	(5) − 20	(17) − 4	(11) − 4	(75) − 2
4	(18) − 4	(13) − 3	(15) 0	(16) − 10	(16) + 5	(9) + 10	(25) + 2	(18) − 2	(112) 0
8	(24) + 6	(15) + 10	(16) − 8	(16) + 3	(18) + 3	(8) − 4	(24) + 5	(17) + 6	(114) + 3
	(3) + 14	(3) − 16	(3) − 13	(3) + 6	(3) + 23	(1) + 27	(3) − 3	(3) − 12	(19) − 1
	...	...	...	...	...	...	...	...	...
	2) − ·01	(47) ·00	(62) + ·01	(62) − ·01	(65) + ·06	(29) + ·02	(89) + ·04	(64) − ·01	...

Note.—The number of stars contributing to each mean is given in brackets before it.

iii. The *étoiles du carré de* 20′, including all stars on the plate within a square of twenty minutes of arc, having the planet at the centre—the planet being always near the centre of the plate.

Unfortunately for the study of systematic errors, the places of the stars in these three categories have been published in three different ways in the Paris Circulars.

For the *étoiles de repère*, Tableau I. gives the mean place concluded from the whole number of plates.

For the *étoiles de comparaison*, Tableau III. gives, in some cases, collections of the separate individual results, but without indication of the plates from which they are derived; in other cases, means only.

For the *étoiles du carré* the Tableau II. gives the complete individual results, plate by plate.

It is therefore impossible to separate completely the proceeds of one plate from the rest, and this is a grave obstacle to the proper investigation of the results. For the present we are restricted to the study of mean places (except for a small central region). We shall therefore, in what follows, draw no distinction between classes II. and III., and shall refer to all the fainter stars as "comparison stars."

3. *Magnitude Equation in Photographic Declinations. Repère Stars.*—The comparison of the photographic declinations of *repère* stars, grouped according to magnitude, is presented in the same form as was the comparison of photographic R.A.'s in my last paper (*loc. cit.*, p. 483), but with the addition of a column for Paris minus mean of seven (Algiers being excluded).

For list I. stars, Algiers has a considerable magnitude equation in declination, as it had in R.A. San Fernando shows no very evident sign of it; but Toulouse has a well-marked magnitude equation, which is unexpected and very interesting. Bordeaux, Catania, Greenwich, Helsingfors, Northfield, and Paris show no serious divergence one from another, and we conclude that they are sensibly free from magnitude equation.

The comparison with list II. extends at present only to R.A. $3^h\ 17^m$ (corresponding to 1901 January 25). Up to that point there is little evidence of magnitude equation, even for Algiers.

4. *Magnitude Equation in Visual Declinations. Tucker's System.*—This comparison follows very closely on the lines of the preceding.

Comparison with the mean of six observatories (excluding Algiers, Toulouse, and San Fernando), and with the six individually, shows little or no evidence of any magnitude equation in Tucker's declinations, unless possibly for the brightest stars, for which material is insufficient.

And the various systems adopted by different observatories show no certain trace of relative magnitude equation when compared with Tucker.

We may conclude that the meridian circle declinations in general are practically free from personality depending on magnitude —an interesting and somewhat unexpected conclusion.

	System T. minus—									
	Alg.	Bord.	Cat.	Green.	Hels.	North.	Paris.	S. Fern.	Toul.	Mean of 6.
– 6·2	(7) – ″·42	(2) + ″·18	(5) – ″·07	(4) + ″·13	(4) + ″·32	(5) + ″·22	(5) + ″·12	(7) – ″·06	(7) – ″·33	(25) + ″·14
6·3 – 6·9	(7) – 17	...	(2) + 5	(5) + 11	(4) + 21	(5) + 27	(6) + 7	(7) + 12	(7) + 9	(22) + 15
7·0 – 7·4	(20) – 23	(7) + 7	(13) + 10	(18) + 7	(12) 0	(14) + 3	(15) + 6	(20) – 4	(20) – 7	(79) + 6
7·5 – 7·9	(35) – 6	(12) + 8	(25) + 18	(22) + 2	(17) 0	(28) – 2	(27) – 2	(35) – 4	(35) – 4	(131) + 4
8·0 – 8·4	(58) 0	(24) + 2	(44) + 6	(49) + 2	(26) + 4	(36) – 1	(53) – 2	(58) + 12	(57) + 1	(232) + 2
8·5 – 8·8	(74) + 6	(41) + 3	(51) 0	(60) + 1	(39) – 1	(47) + 3	(66) + 2	(77) + 12	(77) + 2	(304) + 1
8·9 – 9·2	(54) + 10	(24) + 2	(38) + 9	(38) 0	(23) – 9	(35) – 6	(44) – 2	(55) + 9	(55) + 3	(202) – 1
9·3 –	(12) + 13	(9) + 4	(9) + 17	(8) + 2	(10) – 1	(8) – 1	(13) – 4	(14) + 17	(14) + 16	(57) + 2
Totals	(267) ·00	(119) + ·04	(187) + ·07	(204) + ·02	(135) ·00	(178) + ·01	(229) ·00	(273) + ·08	(272) + ·01	...
List II.										
– 6·2	(5) – ·51	(2) – ·08	(4) + ·10	(3) + ·09	(5) + ·06	(3) + ·08	(5) – ·28	(5) – ·02	(3) – ·18	(22) – ·01
6·3 – 6·9	(7) – 25	(4) – 25	(7) – 25	(4) – 20	(3) + 3	(1) – 75	(7) – 13	(7) + 16	(5) – 39	(26) – 20
7·0 – 7·4	(9) – 17	(4) – 21	(6) + 17	(9) – 5	(8) + 2	(2) – 14	(8) – 5	(8) – 12	(7) – 14	(37) – 2
7·5 – 7·9	(17) – 8	(6) – 23	(12) – 10	(12) + 1	(12) – 6	(5) – 23	(17) – 14	(17) – 18	(11) – 12	(64) – 11
8·0 – 8·4	(18) – 20	(13) – 21	(15) – 15	(16) – 22	(16) + 2	(9) – 2	(19) – 16	(19) – 13	(18) – 16	(88) – 13
8·5 – 8·8	(21) 0	(15) 0	(16) – 15	(16) – 4	(19) – 3	(8) – 7	(24) – 6	(24) – 1	(17) – 2	(98) – 6
[illegible]	[illegible]	(3) – 43	(3) – 41	(3) – 21	(3) – 1	(1) + 3	(3) – 27	(3) – 30	(3) – 39	(16) – 23
	...	...	...	...	...	...	...	...	...	...
Totals	(63) – ·13	(47) – ·09	(63) – ·12	(63) – ·09	(66) – ·01	(29) – ·09	(83) – ·13	(83) – ·08	(64) – ·14	...

But the comparison of Tucker with the photographic results of Algiers and Toulouse exhibits the magnitude equation in the latter previously discovered.

The results of the comparison with list II. are too irregular to be of much service.

5. *Magnitude Equation in Photographic R.A. Comparison Stars.*—In extending to the fainter stars our search for magnitude equation, we meet with the difficulty that the magnitudes assigned to these stars at different observatories are discordant. For example, on tabulating several hundred comparison stars, we find that the magnitudes assigned at Paris are greater in the mean by $1^{m}\cdot6$ than those assigned to the same stars at Bordeaux, and greater by $0^{m}\cdot6$ than those assigned to the same stars on Catania plates, though these were measured at Paris. It would seem that the Paris "magnitudes" are really intensities of image on the plate, unreduced to any photometric scale; excellent, therefore, for investigating magnitude equation on plates treated individually, but of less merit when combined into means.

Because of these discordances in the assigned magnitudes, one must discuss the differences—Paris − Bordeaux, for example—in duplicate, once with the Paris magnitudes, and a second time with the Bordeaux. We find below the classification according to magnitude of the differences Paris − Bordeaux, and also of the differences Bordeaux − Paris (the same quantities with signs changed). The first classification is by Paris magnitudes, and the compartment $9^{m}\cdot9$–$10^{m}\cdot5$ contains 17 stars. The second division is by Bordeaux magnitudes, and the same compartment contains 101 stars. A result still more curious occurs in the comparison between Paris and San Fernando. The last group $12^{m}\cdot2$ and fainter contains 158 stars in the first classification by Paris magnitudes. On repeating it with the San Fernando magnitudes, all but two of these stars are thrown back into preceding groups. It is clear that there will be difficulties in expressing an error as a function of the magnitude.

6. The results here discussed cover nearly the same stretch of the planet's path as does list I. of the *repère* stars, and depend largely, though not entirely, upon stars common to the Paris and other series. The comparison is therefore somewhat incomplete. Quite early in the work it was realised that definitive corrections could not be derived from this material as it stands; that we should have to repeat the whole work in greater detail, and that it was not worth while to spend time in making the present preliminary discussion absolutely complete.

Photographic places of comparison stars have been published in Circulars 10 and 11 by Paris, Bordeaux, Catania, San Fernando, Toulouse, and Algiers. Each one of these six observatories has been compared with each of the others. But it is hardly necessary to give here all the results. It will be sufficient if we give the comparisons of Paris, San Fernando, and Algiers with one another and with the others.

The first set shows that there is no serious discordance between Paris and others, with the exception of San Fernando. To see if the apparent magnitude equation in San Fernando is real, we compare it with each of the others. They are unanimous in showing a relative magnitude equation, approximately linear, of about $0^s.009$ per magnitude. But we must notice that this is of opposite sign to that suspected in the *repère* stars. We shall return to this point in § 8.

The comparison of others with Algiers shows that the extraordinary magnitude equation of the *repère* stars is maintained through the fainter magnitudes. For stars of the sixth magnitude the Algiers R.A. is small by about $0^s{\cdot}03$; for the twelfth magnitude it is large by $0^s{\cdot}08$—the whole range equal to about 1″.

Table III.

Comparison Stars. Magnitude Equation in Photographic R.A.'s.

Paris minus—

	Bord.	Cat.	S. Fern.	Toul.	Alg.
m	s	s	s	s	s
− 9·2	(9) − ·001	(14) + ·016	(20) + ·009	(21) + ·001	(20) − ·005
9·3 − 9·8	(8) − 10	(14) − 3	(14) − 5	(17) − 9	(16) − 33
9·9 − 10·5	(17) − 2	(24) + 6	(46) + 9	(50) + 6	(42) − 32
10·6 − 11·2	(36) + 1	(73) + 2	(78) + 12	(79) 0	(71) − 48
11·3 − 12·1	(56) − 1	(153) − 4	(139) + 22	(129) + 6	(105) − 67
12·2 −	(93) − 7	(153) + 2	(158) + 21	(148) 0	(80) − 88
Totals	(219) − ·004	(431) + ·001	(455) + ·017	(444) + ·002	(344) − ·056

San Fernando minus—

	Bord.	Cat.	Paris.	Toul.	Alg.
m	s	s	s	s	s
− 9·2	(28) − ·007	(50) − ·001	(91) − ·008	(98) − ·004	(93) − ·031
9·3 − 9·8	(39) − 12	(50) − 16	(68) − 14	(68) − 8	(61) − 68
9·9 − 10·5	(72) − 20	(112) − 21	(113) − 17	(109) − 15	(101) − 86
10·6 − 11·2	(57) − 23	(112) − 24	(135) − 21	(96) − 12	(74) − 92
11·3 − 12·1	(16) − 39	(32) − 27	(46) − 34	(38) − 24	(24) − 108
12·2 −	(1) + 20	(1) + 40	(2) − 39	(4) + 2	(3) − 138
Totals	(213) − ·019	(357) − ·019	(455) − ·017	(413) − ·011	(356) − ·072

Algiers minus—

	Bord.	Cat.	Paris.	S. Fern.	Toul.
m	s	s	s	s	s
− 9·2	(25) + ·023	(41) + ·031	(76) + ·022	(79) + ·032	(81) + ·026
9·3 − 9·8	(31) + 57	(40) + 63	(63) + 50	(59) + 60	(63) + 54
9·9 − 10·5	(26) + 63	(37) + 83	(40) + 62	(46) + 66	(45) + 57
10·6 − 11·2	(42) + 66	(72) + 79	(76) + 69	(86) + 95	(84) + 82
11·3 − 12·1	(39) + 73	(39) + 87	(81) + 77	(80) + 96	(85) + 82
12·2 −	(2) + 30	(5) + 112	(8) + 89	(6) + 90	(10) + 88
Totals	(155) + ·062	(234) + ·071	(344) + ·056	(356) + ·072	(368) + ·062

Tables similar to the above for Bordeaux, Catania, and Toulouse show few features of interest. For Bordeaux minus others, the numbers fall slightly to the third group, and rise again. For Catania and Toulouse there is a slight upward tendency at the beginning and end. But the whole range is small, equivalent to about $0''\cdot2$; and, in light of what will follow, we cannot attribute it to magnitude equation.

7. *Magnitude Equation in Photographic Declinations. Comparison Stars.*—The procedure followed in discussing the R.A.'s has been repeated for the declinations without change. A selection of the more interesting results is given in Table IV.

Paris, Bordeaux, and Catania agree fairly well. We can hardly look upon the differences Paris – Bordeaux as significant of magnitude equation until we have examined the effects of the difference of adopted places in Bordeaux, which is a very incomplete series.

Toulouse diverges strongly from these three accordant series, and has a well-marked nearly linear magnitude equation in the declinations of comparison stars of about $0''\cdot1$ per magnitude.

San Fernando is a difficult case. We may summarise thus:—

Paris — San Fernando (P. magnitudes) shows a very slight M.E.
Bordeaux — San Fernando (B. magnitudes) shows decided M.E.
Catania — San Fernando (C. magnitudes) shows large M.E. of more than $0''\cdot1$ per magnitude.

On the other hand:

San Fernando – Paris (S.F. magnitudes) shows no M.E. whatever.
" – Bordeaux (S.F. magnitudes) shows none, or, at most, traces of a non-linear M.E.
" – Catania (S.F. magnitudes) shows a very small and uncertain M.E.

The discordance between the comparisons Catania — San Fernando and San Fernando – Catania, that is to say, between the figures grouped differently according as one accepts the Catania or San Fernando magnitudes, shows that no definitive corrections can be derived from the present work. The magnitude equation, if any, is doubtless entangled with other systematic errors of Catania or San Fernando, for which see later, § 14.

Algiers shows a considerable magnitude equation, as before, but it is not nearly so variable in the fainter magnitudes as in the brighter.

Table IV.

Comparison Stars. Magnitude Equation in Photographic Declinations.

Paris minus—

	Bord.	Cat.	S. Fern.	Toul.	Alg.
– 9·2	(8) + ″·03	(14) – ″·03	(21) – ″·09	(21) + ″·11	(20) – ″·04
9·3 – 9·8	(5) 0	(14) – 5	(14) + 22	(19) + 28	(18) + 22
9·9 – 10·5	(18) + 2	(25) + 1	(47) + 24	(48) + 28	(42) + 33
10·6 – 11·2	(37) + 9	(72) + 3	(78) + 20	(81) + 29	(68) + 29
11·3 – 12·1	(59) + 12	(153) + 2	(138) + 30	(126) + 37	(103) + 39
12·2 –	(93) + 4	(152) + 1	(157) + 28	(149) + 45	(93) + 46
Totals	(220) + ·07	(430) + ·01	(455) + ·26	(444) + ·36	(344) + ·35

San Fernando minus—

	Bord.	Cat.	Paris.	Toul.	Alg.
– 9·2	(29) – ″·10	(50) – ″·15	(93) – ″·24	(97) – ″·05	(93) – ″·03
9·3 – 9·8	(38) – 13	(52) – 13	(67) – 25	(69) + 4	(61) + 11
9·9 – 10·5	(73) – 24	(111) – 25	(119) – 23	(108) + 14	(101) + 18
10·6 – 11·2	(59) – 17	(113) – 29	(129) – 30	(98) + 16	(74) + 23
11·3 – 12·1	(15) – 16	(31) – 24	(45) – 25	(38) + 7	(24) + 25
12·2 –	(1) – 20	(1) – 10	(2) + 7	(4) + 7	(3) + 4
Totals	(215) – ·18	(358) – ·23	(455) – ·26	(414) + ·08	(356) + ·13

Toulouse minus—

	Bord.	Cat.	Paris.	S. Fern.	Alg.
– 9·2	(21) – ″·11	(45) – ″·20	(82) – ″·22	(74) + ″·05	(77) ″·00
9·3 – 9·8	(32) – 18	(44) – 21	(74) – 31	(70) – 5	(65) + 8
9·9 – 10·5	(72) – 32	(129) – 29	(129) – 35	(127) – 16	(112) – 1
10·6 – 11·2	(55) – 35	(98) – 35	(123) – 42	(113) – 8	(94) + 7
11·3 – 12·1	(12) – 61	(15) – 28	(36) – 56	(30) – 9	(20) 0
12·2 –	...	...	...	...	...
Totals	(192) – ·30	(331) – ·29	(444) – ·36	(414) – ·08	(368) + ·03

8. *Summary of Results for Photographic Magnitude Equation.*—We may now bring together the results for the *repère* and comparison stars.

Regrouping the *repère* stars into larger magnitude groups, to avoid the roughness due to paucity of numbers, we have the following table for the comparison of Toulouse, Algiers, and San Fernando, in which magnitude equation is discovered or suspected, with the mean of Paris, Catania, Bordeaux, in which it does not appear.

Table V.

Collected Results for Magnitude Equation.

R.A.	Toulouse minus Mean of P.C.B.		San Fernando minus Mean of P.C.B.		Algiers minus Mean of P.C.B.	
m	s		s		s	
− 7·4	+0·001		−0·015		−0·031	
7·5 − 8·4	0		− 10		− 6	
8·5 − 9·2	+ 1	−0·001 (s)	− 2	−0·005 (s)	+ 14	+0·025 (s)
9·3 − 9·8	5	+ 2	+ 10	− 14	+ 23	+ 57
9·9 − 10·5		0		− 19		+ 69
10·6 − 11·2		− 7		− 23		+ 73
11·3 − 12·1		− 6		− 33		+ 79
12·2 −	...		...			+ 77
Decl.						
− 7·4	+0·22 (″)		+0·14 (″)		+0·38 (″)	
7·5 − 8·4	+ 5		− 4		+ 8	
8·5 − 9·2	0	−0·18 (″)	− 6	−0·16 (″)	− 2	−0·16 (″)
9·3 − 9·8	− 15	− 23	− 18	− 17	− 16	− 24
9·9 − 10·5		− 32		− 24		− 28
10·6 − 11·2		− 37		− 25		− 38
11·3 − 12·1		− 48		− 22		− 44
12·2 −	...		...			− 35

The first of the two columns belonging to each observatory is derived from the *repère* stars, the second from the comparison stars. The roughness at the points where these overlap need not be taken too seriously, for the magnitudes of the first are Tucker's magnitudes, *i.e.* practically B.D. magnitudes, while those of the second are photographic magnitudes.

The conclusions, in brief, are as follows:—

Toulouse has very little magnitude equation in R.A., but a large one in declination, which is nearly linear.

Algiers has a very large magnitude equation in both co-ordinates; larger in R.A. than in Decl.; not strictly linear in either, but altering little for the faintest stars.

San Fernando appears to have a fairly small and decidedly non-linear magnitude equation in both co-ordinates.

In estimating the weight of these conclusions we must remember that there are other systematic errors involved with the magnitude equation, which have not yet been discussed; and that the [illegible] quantity upon which magnitude equation must depend is lik[illegible] be the number representing the intensity of the image up[illegible] plate, rather than the number more or less reduced to phot[illegible] scale which is derived from it and published as the ph[illegible] magnitude (except apparently at Paris). For these [illegible]

attempt will be made just yet to determine a set of definitive corrections.

9. For the same reasons it is premature to attempt to settle the question—What is the cause of these errors? But we may note in passing, that at Toulouse and at Algiers the photographic equatorial is mounted in the "English" style,* so that errors of the objective are not reversed on crossing the meridian; while the plates are reversed during measurement. One will in this case naturally look first for the error in the objective. At San Fernando the plates are not reversed during measurement, and the small non-linear magnitude equation may very well be due to this cause.

10. *Progressive Discordances. Repère Stars.*—If a photographic telescope gives results which are systematically wrong over a long series, there is naturally a strong probability that the cause is instrumental; and if so, the effect will most likely be a function of the magnitude. We have therefore dealt with magnitude equation first.

We have now to look for discordances of a semi-systematic character, varying from point to point along the path of the planet, —such discordances, for example, as might be due to roughness in the adopted places of the *repère* stars. We will look for them first in the concluded photographic places of the *repère* stars themselves.

Each *repère* star has been assigned to one or other of a series of groups on centres 2° apart along the orbit of the planet (alternate centres of list given in Paris Circular No. 3, p. 7, for the "special series" plates).

11. Consider in the first place the observatories which have reduced to a common fundamental system (that of M. Loewy), viz. Catania, Greenwich, Paris, and Toulouse. The group means of the differences of these series are given in the table on the next page.

In studying this table we must remember—

(*a*) That the Greenwich plates were reduced first of all to an independent system, and afterwards mean corrections were applied to each plate to reduce to Loewy's system.

(*b*) That the Toulouse system is not exactly the same as Loewy's, though for present purposes the difference is insignificant.

(*c*) That the plates taken at Catania were measured and reduced at Paris, and ought to be particularly concordant with the Paris plates.

(*d*) That the Tolouse declinations are slightly affected by magnitude equation.

(*e*) That stars appearing in the same group are not necessarily reduced with the same selection of *repère* stars, or even with any [illegible] to the two reductions.

[illegible] this information to the directors of the observatories [illegible] a little curious that, although the astrographic [illegible]t work for fifteen years, one cannot always find [illegible]e of mounting employed. Yet the question [illegible]versed on crossing the meridian is funda[illegible]ic error.

(*f*) On the other hand, that the mean photographic place of one star will often depend on a considerable range of *repère* stars grouped on different centres, and that the errors of individual adopted places will be smoothed out much more effectually than is the case on Astrographic Catalogue plates, with their centres at uniform distances.

Bearing these facts in mind, we may draw the following conclusions from Table VI. :—

(1) The systematic difference between Paris and Greenwich, taken over the whole series, is quite insensible. And we found that there was no trace of relative magnitude equation between

Table VI.

Repère Stars reduced to Loewy's System. Progressive Discordances.

Centre.	Date.	Paris minus— Greenwich.	Toulouse.	Catania.
	1900.	s ″	s ″	s ″
6	Oct. 2	(11) ·000 + ·03	(11) + ·005 + ·15	(11) − ·001 + ·02
8	,, 8	(15) + 3 − 2	(17) + 4 − 1	(17) + 3 0
10	,, 14	(11) − 13 + 4	(15) − 7 + 8	(13) + 22 + 20
12	,, 19	(15) 0 − 3	(20) 0 0	(14) − 3 + 3
14	,, 25	(11) + 1 0	(13) + 6 + 4	(12) + 26 + 6
16	Nov. 1	(14) + 13 + 1	(17) + 3 − 1	(16) + 11 + 10
18	,, 7	(16) − 9 0	(21) + 2 − 1	(21) − 6 + 6
20	,, 14	(16) + 3 − 1	(19) + 16 + 7	(13) + 2 + 1
22	,, 21	(16) − 7 + 4	(18) − 4 + 4	(12) − 8 + 18
24	,, 27	(15) + 3 − 1	(16) + 5 + 3	(13) + 17 + 21
26	Dec. 3	(7) + 4 − 21	(15) 0 + 2	(9) + 21 − 3
28	,, 9	(11) + 5 + 4	(11) + 2 0	(3) + 46 − 41
30	,, 14	(6) + 3 + 5	(7) + 15 − 20	(5) − 1 − 5
32	,, 18	(14) + 4 + 6	(14) + 4 + 3	(6) + 21 + 44
34	,, 22	(16) − 5 + 7	(16) + 1 + 3	...
36	,, 26	(16) − 4 + 2	(17) + 3 − 5	(1) − 18 + 40
Average without regard to sign		s ″ ·0048 ·040	s ″ ·0048 ·048	s ″ ·0137* ·147*
Mean		·0000 + ·028	+ ·0033 + ·072	+ ·0060 + ·086

Note.—The averages are formed from the group means given above, and sometimes the same star occurs in two groups. These duplicates have been expunged before the final means were taken, in which no star is reckoned more than once.

them. We may provisionally adopt the figures i[n] Paris—Greenwich as indicative of the degree of c

* Or, omitting the last four, which depend on few observ[ations]

are 0^s·0109 0″·082.

may be reached by two series made independently with different instruments and reduced to the same standard.

(2) The divergences from centre to centre seem to be due more to the roughness of the *repère* star places than to errors of measurement, or real errors of the photographs. This may be seen in the columns for Paris—Toulouse. The Toulouse plates are few in number compared with Greenwich and Paris. But the average discordance of the groups is nearly the same in Paris—Toulouse as it is in Paris—Greenwich.

(3) The Catania plates are still fewer in number, being practically only alternate plates of the special series. Although they were measured and reduced at Paris, the discordances from Paris are large. At first sight this might be attributed to small number of plates and want of averaging out errors by overlapping. But the discordances seem to be too large for any such simple explanation; notice especially centres 10, 14, 24, and 26. We will postpone discussion of them till we come to deal with the fainter stars.

Table VII.

Repère Stars not reduced to Loewy's System. Progressive Discordance from Paris of Photographic and Adopted R.A.'s.

Paris minus—

		San Fernando.		**Northfield.**		**Helsingfors.**	
Centre.	Date.	Phot.	Adopt.	Phot.	Adopt.	Phot.	Adopt.
	1900.	s	s	s	s	s	s
6	Oct. 2	(11) + ·004	+ ·009	...	...	...	...
8	,, 8	(17) + 13	+ 6	(13) + ·019	+ ·005	...	...
10	,, 14	(15) + 3	+ 9	(14) − 1	+ 8	...	...
12	,, 19	(20) + 10	+ 10	(15) + 12	+ 9	(8) + ·008	+ ·010
14	,, 25	(13) + 9	+ 11	(10) + 10	+ 5	(9) + 32	+ 10
16	Nov. 1	(17) + 11	+ 15	(11) 0	+ 10	...	...
18	,, 7	(21) + 15	+ 15	(14) − 1	+ 11	(18) − 3	+ 9
20	,, 14	(19) + 21	+ 9	(11) + 3	+ 7	(4) − 13	+ 10
22	,, 21	(18) + 13	+ 11	(10) − 3	+ 6	(13) + 7	+ 11
24	,, 27	(16) + 11	+ 11	(14) + 16	+ 8	(15) + 22	+ 17
26	Dec. 3	(15) − 1	0	(11) + 13	+ 11	(13) + 25	+ 14
28	,, 9	(11) + 5	+ 8	(8) + 2	+ 8	(12) + 18	+ 13
30	,, 14	(7) + 5	+ 3	...	...	...	...
32	,, 18	(14) + 5	− 1	(11) + 3	+ 7	...	...
[illegible]	,, 22	(16) − 3	+ 5	(14) + 4	+ 4	(11) + 9	+ 10
[illegible]	[illegible]	[illegible] 11	+ 3	(11) + 3	+ 6	(12) + 15	+ 8
		[illegible]	·0078	+ ·0057	+ ·0075	+ ·0120	+ ·0112

[illegible] de from Table VI. that it is possible [illegible] A.'s and declinations, from two [illegible] observatories—witness Paris

and Greenwich; but even under conditions which should make for accordance, one finds discordances unexpectedly big, as in the case of Catania.

12. So far we have dealt with series reduced to the same fundamental system. We must now examine how far difference of adopted system is likely to produce discordance.

We will take as examples San Fernando, Northfield, and Helsingfors. The comparison of R.A.'s, both deduced and adopted, is given in Table VII.

It will be seen at once that the difference in the adopted places is responsible for the principal part of the difference in the concluded photographic places. At San Fernando and Helsingfors the mean difference is exactly accounted for. At Northfield the agreement is less satisfactory, but the series is less complete.

13. The declinations have been treated in the same way, and give a very similar result. The mean discordances of the photographic declinations are due almost entirely to the difference of adopted places.

I think we may conclude that the policy of allowing any observatory to make its own system of adopted places for the *repère* stars has led to well-marked systematic discordances between some of the photographic series, of the order of $0^s.01$ and $0''.1$, which it will be worth while to eliminate.

14. *Comparison Stars. Progressive Discordances.*—We have now to examine how far the photographic places of the fainter stars differ from centre to centre. Our material is the same as that discussed above for magnitude equation. Each observatory has been compared with each of the others. The results have been studied by plotting in curves coloured to indicate the observatory of origin. These cannot be reproduced, and the tables which represent them numerically are not quite successful in showing up the features which are conspicuous in the diagrams. We can give only a few examples, and a general statement of results.

A typical section is that traversed by the planet between October 16 and November 4, which has been divided into blocks lettered K, L, M, N, O. Table VIII. gives the R.A. comparisons.

Since all these comparisons are differential, it is impossible to obtain the actual error of any series. But we sometimes can get a very good idea of which is in the wrong. Take, for instance, the comparison of Catania with others. In block K the Catania R.A.'s are about $0^s.04$ greater than all the others; in block L they nearly agree; in block M they are decidedly less; in block N they have risen again; and fall once more in O as compared with the others. I think that we may concl[illegible] se that the irregularity is mainly in the Catania o[illegible]

In this way it has been found po[illegible] idea of the degree of systemati[illegible] R.A.'s and declinations, of[illegible] no means reassuring. Let u[illegible]

Table VIII.

Comparison Stars. Photographic R.A. Progressive Discordances between Results from Different Observatories.

	Paris –	Bord. –	Cat. –	S. Fern. –	Toul. –
K. Oct. 16–19.					
	s	s	s	s	s
Paris	...	(14) + ·012	(29) + ·040	(14) – ·021	(12) + ·001
Bord.	(14) – ·012	...	(15) + 34	(13) – 20	(12) – 7
Cat.	(29) – 40	(15) – 34	...	(12) – 54	(9) – 37
S. Fern.	(14) + 21	(13) + 20	(12) + 54	...	(13) + 21
Toul.	(12) – 1	(12) + 7	(9) + 37	(13) – 21	...
L. Oct. 19–23.					
Paris	...	(28) + ·009	(16) – ·002	(19) – ·003	(26) – ·003
Bord.	(28) – ·009	...	(10) – 5	(17) – 10	(24) – 12
Cat.	(16) + 2	(10) + 5	...	(6) – 13	(8) – 12
S. Fern.	(19) + 3	(17) + 10	(6) + 13	...	(20) – 4
Toul.	(26) + 3	(24) + 12	(8) + 12	(20) + 4	...
M. Oct. 23–27.					
Paris	...	(29) – ·002	(36) – ·030	(39) – ·008	(32) + ·011
Bord.	(29) + ·002	...	(36) – 22	(36) – 29	(23) + 15
Cat.	(36) + 30	(36) + 22	...	(37) + 3	(23) + 39
S. Fern.	(39) + 8	(36) + 29	(37) – 3	...	(28) + 28
Toul.	(32) – 11	(23) – 15	(23) – 39	(28) – 28	...
N. Oct. 27–31.					
Paris	...	(31) – ·003	(38) – ·003	(32) – ·033	(19) – ·023
Bord.	(31) + ·003	...	(35) + 10	(35) – 38	(18) – 5
Cat.	(38) + 3	(35) – 10	...	(34) – 41	(21) – 19
S. Fern.	(32) + 33	(35) + 38	(34) + 41	...	(20) + 32
Toul.	(19) + 23	(18) + 5	(21) + 19	(20) – 32	...
O. Nov. 1–4.					
Paris	...	(20) – ·016	(33) – ·026	(21) – ·002	(22) – ·031
Bord.	(20) + ·016	...	(28) – 13	(21) + 21	(21) – 2
Cat.	(33) + 26	(28) + 13	...	(22) + 30	(24) + 7
S. Fern.	(21) + 2	(21) – 21	(22) – 30	...	(22) – 21
Toul.	(22) + 31	(21) + 2	(24) – 7	(22) + 21	...

15. Paris comes out decidedly the best. Only in a few cases ... systematically different from all the others.

... results are also good. The differences from other ...e very irregular, and to be attributed in the main ... not to Bordeaux.

Catania is an extremely interesting case. The discordances from Paris are quite extraordinary. The following table gives their values in the section where they are best determined:—

Table IX.

Comparison Stars. Progressive Discordances.
Paris minus Catania.

Block.	No. of Stars.	Mean Discordance in R.A.	Mean Discordance in Decl.
		s	"
E.	(8)	−·013	−·06
F.	(32)	− 18	− 5
G.	(12)	+ 32	− 9
H.	(30)	− 21	+ 24
K.	(29)	− 40	+ 12
L.	(16)	− 2	− 12
M.	(36)	+ 30	+ 12
N.	(38)	+ 3	− 19
O.	(33)	+ 26	− 7
P.	(39)	+ 2	+ 7
Q.	(41)	− 26	− 13
R.	(38)	+ 21	0
S.	(4)	0	+ 13
T.	(11)	+ 34	− 11
U.	(11)	− 40	+ 11
V.	(2)	− 25	− 5
X.	(7)	+ 74	− 2

The greatér part of the error is certainly in the Catania places. It is less in Decl. than in R.A. It does not correspond at all well with the rather large discordance between Paris and Catania for the *repère* stars, and cannot be due to roughness in the adopted places of the stars, for the places where it is greatest are generally those where Paris plates are most numerous.

Up to the present I have not succeeded in determining the form of this error, or in finding any explanation of it. That two series of plates, taken indeed with different telescopes, but measured and reduced by the same hands, should give results so remarkably discordant, could hardly have been expected by the most avowed disbeliever in the accuracy which is claimed, and generally with truth, for the photographic methods.

Toulouse and San Fernando exhibit great and very divergences from the others; but we have already sh these series are affected by rather large magnitude which in a climate of variable transparency must associated with semi-systematic mean errors of the

now considering. The only sure method of procedure in such a case seems to be a discussion for magnitude equation plate by plate, which is impossible unless the published material is amplified.

16. *Conclusion.*—The results of this discussion make it clear that the star places published in the Paris Circulars are by no means homogeneous, and that errors exist in some of the series which, uneliminated, would ruin the determination of the solar parallax. I have, however, full confidence that these difficulties can be overcome. By the great kindness of the directors of several observatories, the separate results from each of their plates have recently been placed at my disposal, and the discussion of this material is now proceeding at Cambridge, with results which I hope to communicate to the Society.

17. But our conclusions have, it seems to me, an interest wider than that of the particular problem in hand. Most of the photographic telescopes concerned have been engaged for years upon the Astrographic Chart and Catalogue. With that experience behind them, they undertook shares in the Eros co-operation; and several of the resulting series of star places are affected by errors much larger than are accounted tolerable in the Astrographic Catalogue—a disquieting result.

18. The work summarised in this paper was done concurrently with that described in paper No. 4, and the same acknowledgments are due to the Government Grant Fund of the Royal Society; and to Miss Julia Bell, who has carried out the greater part of the computation.

Cambridge Observatory:
1906 *November* 7.

On the Distribution of Energy in the Continuous Spectrum. By E. T. Whittaker, Sc.D., F.R.S., Royal Astronomer of Ireland.

The distribution of energy in the continuous spectrum of the "black body" at various temperatures, which is of interest astronomically from its application to the problem of solar and stellar temperatures, has been studied from the experimental side by many physicists, and the results obtained have been co-ordinated into the form of empirical laws of radiation. One such empirical law, which is due to Lummer and Pringsheim,* and closely represents the observations, is to the effect that the intensity in the part of the spectrum at wave-length λ, radiated by a black body at the absolute temperature T, is proportional to

$$T\lambda^{-4}e^{-\frac{c}{(\lambda T)^{1\cdot 3}}} \quad . \quad . \quad . \quad . \qquad \text{(I.)}$$

...t.

...rrence of T in this formula is not really ...oved by thermodynamical considerations

* ...*lls. Verh.*, 2, 12*a*, 103–180 (1900).

that if the energy between wave-lengths λ and $\lambda + d\lambda$, when the body is at temperature T_0, is $\psi(\lambda)d\lambda$, then the corresponding quantity when the body is at temperature T is

$$\left(\frac{T}{T_0}\right)^5 \psi\left(\frac{T\lambda}{T_0}\right) d\lambda \quad . \quad . \quad . \quad \text{(II.)}$$

Lummer and Pringsheim's formula is evidently constructed to satisfy this condition, and what is really empirical in the formula is therefore the mode of occurrence of λ when T is constant; in other words, the distribution of energy in the spectrum at some one definite temperature is taken from observation, and the distribution of energy in spectra at all other temperatures can then be deduced from thermodynamical principles.

For the above formula (I.) no theoretical justification has yet been found. But one feature, common to it and to all the rival formulæ which have been suggested since the results of observations in the extreme infra-red have been available, is that the distribution of energy in the region of long wave-lengths is proportional to

$$T\lambda^{-4} d\lambda \quad . \quad . \quad . \quad . \quad \text{(III.)}$$

In other words, the curve of intensity I in the spectrum must approximate to the curve $I = CT\lambda^{-4}$ in the ultra-red where C is a constant.

This result (III.) may now be regarded as a well-established result of observation;* and it becomes important to inquire whether it can be explained on theoretical grounds.

Several writers have discussed this question by the aid of assumptions regarding the nature of the radiating mechanism in the black body. Lord Rayleigh † suggested the application of the Boltzmann-Maxwell doctrine of the partition of energy among the different modes of vibration. The difficulty here lies in the doctrine itself, which is not free from uncertainties, and would give results inconsistent with observation if applied to the shorter wave-lengths.

Planck ‡ has attacked the matter from the point of view of a distinctive theory of the mechanism of radiation. The radiating body is supposed to contain a great many electrical vibrators, each having its own period of free vibration, and exchanging energy with the æther and the material molecules. By discussion of such a system, Planck derives the law of radiation

$$\frac{c_1 \lambda^{-5}}{e^{\frac{c_2}{\lambda T}} - 1} d\lambda ;$$

* It is further confirmed by the observations of Rubens and Kurlbaum, *Ann. d. Phys.*, iv. p. 649 (1901).

† "Remarks upon the Law of Complete Radiation," *Phil.* [illegible] (1900). Lord Rayleigh's paper was written before the [illegible] been experimentally verified, and therefore constit[illegible] from theory. See also Jeans, *Phil. Mag.* (6) x. 9[illegible]

‡ Ueber das Gesetz der Energieverteilung im [illegible] *Phys.*, iv. 553 (1901).

where c_1 and c_2 are constants. This formula evidently satisfies law (III.), and is indeed in good accord with observation for the shorter wave-lengths also.

Later, Lorentz * gave a different view of the mechanism of radiation: he explains the emission of a metal by means of the heat-motion of its free electrons, which are regarded as moving to and fro with a velocity of agitation increasing with the temperature, and frequently striking against the molecules. He shows that such a system would emit the longer radiations in accordance with formula (III.) above.

The object of the present paper is to show that formula (III.) can be established on theoretical grounds quite apart from any assumptions as to the mechanism of radiation; that in fact it is a necessary consequence of the laws of thermodynamics, together with the usual assumptions regarding the nature of the white light.

Natural radiation is now generally understood to consist of a succession of discrete disturbances or "pulses" in the æther, which are not co-ordinated as regards phase, and each of which consists of compensating positive and negative parts, so that the curve representing a pulse has the total area of those portions of it which are below the axis equal to the total area of those portions which are above the axis. By the agency of a prism or grating, a single pulse of this kind is drawn out into trains of periodic disturbances, the dispersive apparatus in fact performing a resolution of the pulse which corresponds to that furnished analytically by Fourier's integral; and it is this resolution which constitutes spectroscopy.

Suppose, then, that a pulse in æther is represented by $f(x-ct)$, where x denotes distance measured in the direction of propagation of the pulse, t denotes time, and c is the velocity of light in æther. We shall express the discrete character of the pulse by supposing that $f(x-ct)$ is zero except when $x-ct$ lies between the limits a and b.

Then Fourier's resolution can be written

$$f(x-ct)=\frac{1}{\pi}\int_0^{\infty}dn\int_a^b\cos\{n(x-ct-\mu)\}f(\mu)d\mu$$

So when the pulse is spectroscopically resolved, the elements with wave-lengths between λ and $\lambda+d\lambda$ in æther will be (writing $2\pi/\lambda$ for n, and also for convenience writing $a+y$ for μ)

$$\frac{2d\lambda}{\lambda^2}\int_0^{b-a}\cos\left\{\frac{2\pi}{\lambda}(x-ct-a-y)\right\}f(a+y)dy$$

Now throughout the range over which the integration is taken, y is small compared with λ if the wave-length is taken so far in the ... that λ is large compared with the extent of the pulse. ...fore expand the cosine in ascending powers of $\frac{y}{\lambda}$ and

* ... and Absorption by Metals of Rays of Heat of great ...s of the *Amsterdam Academy of Sciences* (English

retain only the leading terms of the expansion. Retaining for the present the first two terms, the preceding expression becomes

$$\frac{2d\lambda}{\lambda^2}\cos\left\{\frac{2\pi}{\lambda}(x-ct-a)\right\}\int_0^{b-a}f(a+y)dy$$
$$+\frac{2d\lambda}{\lambda^2}\frac{2\pi}{\lambda}\sin\left\{\frac{2\pi}{\lambda}(x-ct-a)\right\}\int_0^{b-a}yf(a+y)dy$$

The first of these integrals vanishes in consequence of the condition that the pulse consists of compensating positive and negative parts; the second integral depends on the particular form of the pulse, but is independent of λ, and will in general have a finite value different from zero, which we shall denote by C. The spectroscopic element of the pulse with wave-lengths between λ and $\lambda+d\lambda$ is therefore

$$\frac{4\pi C d\lambda}{\lambda^3}\sin\left\{\frac{2\pi}{\lambda}(x-ct-a)\right\}$$

Now if in any disturbance the spectroscopic element with wave-lengths between λ and $\lambda+d\lambda$ is

$$f(\lambda)\,.\,d\lambda\,.\,\sin\left\{\frac{2\pi}{\lambda}(x-ct-\epsilon)\right\},$$

it is known * that the energy radiated in this interval is proportional to $\lambda^2\{f(\lambda)\}^2 d\lambda$. So in the present case the energy radiated with wave-lengths between λ and $\lambda+d\lambda$ is proportional to $\lambda^{-4}d\lambda$. As the various pulses are supposed to be entirely unco-ordinated as regards phase, this law which holds for each of them individually will hold also for their aggregate; and therefore, in the radiation emitted by any body, the energy in the part of the spectrum between λ and $\lambda+d\lambda$ is proportional to $\lambda^{-4}d\lambda$ in the region of longer wave-lengths.

From this result, by an application of the thermodynamical theorem (II.) above, we immediately deduce the consequence that the radiation of a body at temperature T is, in the ultra-red, proportional to $T\lambda^{-4}d\lambda$.

Law (III.) is thus established as a direct consequence of the nature of white light, without reference to the mechanism of radiation in the radiating body.

On the Resolving Power of Spectroscopes. By E. T. Whittaker, Sc.D., F.R.S., Royal Astronomer of Ireland.

The resolving power of a spectroscope is, according to the usual definition, the value of $\frac{\lambda}{\delta\lambda}$, where $\delta\lambda$ is such that the lu centre of the spectral line of wave-length $\lambda+\delta\lambda$ falls on t minimum of intensity of the line of wave-length λ. It i

* *Cf.* Lord Rayleigh, *Phil. Mag.*, xxvii. 460 (188

known theorem that if θ denotes the deviation of light of wave-length λ, so that $\frac{d\theta}{d\lambda}$ measures the dispersion caused by the spectroscope, and if a denotes the breadth of the beam of parallel light of the wave-length λ as it emerges from the dispersive apparatus, then the resolving power of the instrument is $a\frac{d\theta}{d\lambda}$.

The resolving power of a spectroscope can also be discussed in connection with the modern view of the nature of luminous disturbance as a succession of pulses. This has been done by several writers.* In the case of the grating, it is evident that a single incident pulse is broken up into as many separate pulses as there are spacings in the grating: these separate pulses, as they issue from the grating, are no longer in the same wave-front, and they will consequently follow each other at regular intervals to the focus of the observing telescope. In this way we can establish for the grating a theorem to the effect that the number of alternations, in the disturbance which is formed from a single original pulse, is equal to the resolving power of the instrument.

The case of dispersion by a prism presented greater difficulties, as it seemed somewhat mysterious that alternations should be formed at all in this case. But the discussions of Professors Schuster and Ames and Lord Rayleigh † make it evident that the difference between the wave-velocity and the group-velocity of luminous disturbance in the material of the prism will cause a single incident pulse to be spread out into an alternating disturbance; and the working out of this idea verifies the same theorem for prisms as has been stated above for gratings, namely, that the number of pulses into which a single incident pulse is spread out is equal to the resolving power of the instrument.

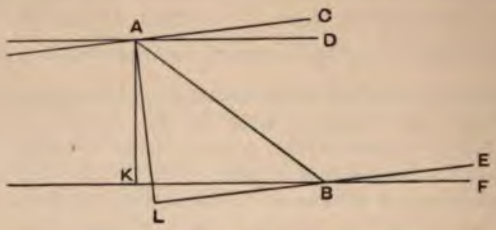
It becomes, therefore, of interest to determine whether this theorem can be established in the most general case by a proof which will be independent of the nature of the dispersive apparatus, and consequently applicable to every type of spectroscope. The object of the present note is to communicate such a proof.

Suppose, then, that a pulse, which for our present purpose we may regard as a thin plane sheet of disturbance moving perpendicularly to its own plane, is incident on any dispersive apparatus. For simplicity we shall suppose that the Fourier analysis of the pulse will yield radiations of wave-lengths λ and $\lambda + d\lambda$ only. At incidence the *pulse-front*, or plane on which the disturbance exists, is the same as the *phase-front*, or plane parallel to which the λ-disturbance is in the same phase; but after emergence from the dispersive apparatus this will no longer in general be the case. ...hen, that ϕ denotes the angle between the pulse-front and ... emergence; and let $d\theta$ denote the angle between the

...igh, Larmor, Schuster, and Ames.

...*tics*, p. 325 (1904); Ames, *Astrophysical Journal*, ...*Mag.*, x. p. 401 (1905).

phase-fronts for the λ radiation and the $\lambda + d\lambda$ radiation, so that $d\theta$ is the dispersion. If, now, A C, B E in the figure are two successive crests of the λ-disturbance at emergence, and A D, B F



two successive crests of the $\lambda + d\lambda$ disturbance, then the pulse-front will pass through the points A, B, where the crests reinforce each other. Thus if A K, A L are perpendicular respectively to B F and B E, we have

$$AK = \lambda, \qquad AL = \lambda + d\lambda, \qquad K\hat{A}L = d\theta, \qquad ABK = \phi$$

and so from the figure

$$AB \sin \phi = \lambda, \qquad AB \sin (\phi + d\theta) = \lambda + d\lambda$$

Subtracting these two equations, we have

$$AB \cos \phi \; d\theta = d\lambda$$

and dividing this into the first of them, we have

$$\tan \phi = \lambda \frac{d\theta}{d\lambda}$$

This equation, which is applicable to any dispersive apparatus, shows that the tangent of the angle between the pulse-front and the phase-front at emergence is equal to the product of the wave-length into the dispersion.

Since the resolving power is the product of the dispersion into the breadth of the emergent beam, it follows from this equation that the resolving power is equal to the product of $\frac{1}{\lambda} \tan \phi$ into the breadth of the emergent beam, *i.e.* it is equal to the product of $\frac{1}{\lambda} \sin \phi$ into the breadth of the emergent pulse. But $\frac{\lambda}{\sin \phi}$ is evidently A B in the figure, *i.e.* it is the interval between two successive maxima of intensity in the pulse-front; and thus the resolving power is equal to the quotient of the bre emergent pulse by the interval between two successi it; that is, the resolving power is equal to the num maxima of disturbance in the emergent pulse-fr be proved.

Extended Nebula near 26 *Ceti*. By Dr Max Wolf, Assoc. R.A.S.

On four plates taken with the Bruce 16-inch telescope in September I find an extended nebula, which attracts attention both by its situation and its appearance.

Practically all extended nebulosities are found in or near the Milky-way, but this object is about 70° distant from the plane of the Milky-way. Its densest part is situated at

$$\alpha = 0^h\ 57^m\cdot4 \qquad \delta = +1^\circ\ 20' \qquad (1885\cdot0)$$

Round this is spread faint nebulous matter of varying intensity. At several points small clouds of greater density shine forth. Further out, the intensity is so feeble that it is impossible to exactly define its limits. It seems that these will be materially altered by lengthening the exposure. My exposures extend to four hours. With these the nebulous cloud extends about 40′ in declination and about 30′ in R.A. The three B.D. stars

	h m	
+1°.191	α = 0 56·4	δ = +1°24′
1 .200	57·0	1 27
1 .201	57·3	1 32

are all involved in the cloud.

Southwards it reaches nearly to the B.D. star

$$+0^\circ.207 \qquad 0^h\ 56^m\cdot9 \qquad +1^\circ 0'$$

but it cannot be certainly detected up to the star. Especially towards the west the nebulous matter promises to go much further.

Under the microscope the brighter parts of the mass are filled with numerous very small spots and short trails, so that the appearance is very different from that presented by the Milky-way nebulæ. It seems possible that this object is a multitude of very small planetary nebulæ collected in a cluster, which a more powerful instrument than mine may perhaps resolve.

Astrophysical Observatory, Heidelberg:
1906 *October.*

Observations of the Satellite of Neptune from Photographs taken at the Royal Observatory, Greenwich, between 1905 *December* 19 *and* 1906 *April* 25.

(*Communicated by the Astronomer Royal.*)

measures of position-angle and distance ot
re made from photographs taken with the
ᵔpson equatorial. The occulting
years. The photographs were

taken by Messrs Davidson, Edney, or Melotte, and were measured in a position micrometer in direct and reversed positions by Messrs Davidson and Edney. The tabular positions with which comparison is made were computed from the data given in the *Connaissance des Temps*, based on Dr Hermann Struve's elements, the eccentricity of the orbit being neglected.

A discussion of these residuals gives the following differences in the sense Tabular–Observed to Dr Hermann Struve's elements

$$du = -0^{\circ}{\cdot}87 \quad dN = -1^{\circ}{\cdot}16 \quad dI = +0^{\circ}{\cdot}17 \quad da = +''{\cdot}090$$

giving for the epoch 1906·2

$$a = 16''{\cdot}181 \quad N = 188^{\circ}{\cdot}71 \quad I = 116^{\circ}{\cdot}51.$$

NEPTUNE AND SATELLITE.

Position-angle and Distance from Photographs taken with the 26-inch Refractor.

Date and G.M.T.					Position-angle.			Distance.		
					Observed.	Tabular.	T.-O.	Observed.	Tabular.	T.-O.
1905	d	h	m	s	°	°	°	"	"	"
Dec.	19	10	58	6	115·48	115·58	+0·10	15·02	15·46	+0·44
1906										
Jan.	13	11	7	27(*a*)	(45·38)	45·03	(−0·35)	(13·32)	12·93	(−0·39)
	13	11	35	28	42·43	43·68	+1·25	12·46	12·81	+0·35
	19	12	13	11(*b*)	32·04	32·62	+0·58	11·80	12·00	+0·20
	19	12	26	45	29·73	30·75	+1·02	12·01	11·89	−0·12
	22	11	8	9	212·11	211·36	−0·75	12·14	11·92	−0·22
	22	11	34	27(*c*)	208·37	209·88	+1·51	11·56	11·84	+0·28
Feb.	3	7	59	26	200·43	201·90	+1·47	11·63	11·43	−0·20
	3	8	30	0	199·21	200·05	+0·84	11·25	11·37	+0·12
	3	9	0	31	198·52	198·18	−0·34	11·09	11·30	+0·21
	3	9	29	57	195·08	196·34	+1·26	11·11	11·24	+0·13
	3	9	54	56	193·08	194·78	+1·70	11·14	11·20	+0·06
	12	9	44	45	359·23	358·54	−0·69	10·88	11·00	+0·12
	12	10	9	58(*g*)	356·21	356·90	+0·69	10·95	11·01	+0·06
	12	11	7	42	352·68	353·18	+0·50	10·83	11·05	+0·22
	14	10	2	25	245·45	246·53	+1·08	15·03	15·09	+0·06
	14	10	27	31	244·40	245·66	+1·26	14·98	15·00	+0·02
	14	10	54	48	243·99	244·69	+0·70	14·79	14·88	+0·09
	14	11	16	4(*b*)	242·63	243·94	+1·31	14·82	14·80	−0·02
	15	9	13	7(*d*)	174·71	174·91	+0·20	11·06	11·02	−0·04
	20	9	44	29	239·21	240·83	+1·62	14·41	14·44	+0·03
	20	10	11	26	238·34	239·81	+1·47	14·32	14·33	+0·01
	20	11	14	39(*f*)	236·88	237·38	+0·50	14·22	14·07	−0·[illegible]
	23	10	36	42(*h*)	54·10	55·38	+1·28	13·47	13·84	+[illegible]
	23	11	8	31(*g*)	53·25	54·08	+0·83	13·19	13·76	+[illegible]
Mar.	2	9	35	20(*e*)	326·92*	327·27	+0·35*	11·38*	11·99	[illegible]

* Half weight.

NEPTUNE AND SATELLITE—*continued.*

Date and G.M.T.	Position-angle. Observed.	Position-angle. Tabular.	Position-angle. T.-O.	Distance. Observed.	Distance. Tabular.	Distance. T.-O.
1906 d h m s	°	°	°	"	"	"
2 10 2 6(c)	324.13	325.85	+1.72	12.05	12.09	+0.04
2 10 31 44	324.89	324.30	−0.59	12.01	12.20	+0.19
2 10 54 35	322.23	323.13	+0.90	12.10	12.28	+0.18
3 10 13 17	272.44	273.23	+0.79	16.32	16.57	+0.25
3 10 46 39	271.03	272.31	+1.28	16.50	16.59	+0.09
3 11 15 41(c)	269.96	271.50	+1.54	16.29	16.59	+0.30
7 9 50 59(c)	42.14	41.69	−0.45	12.73	12.49	−0.24
7 10 18 28(c)	39.38	40.33	+0.95	12.27	12.38	+0.11
7 10 44 20(h)	(34.29)	39.03	(+4.74)	(12.54)	12.28	(−0.26)
7 11 13 13	36.85	37.55	+0.70	12.11	12.17	+0.06
12 8 37 32	87.88	88.49	+0.61	16.84	16.50	−0.34
12 9 6 27	86.84	87.68	+0.84	16.65	16.48	−0.17
12 9 47 24	85.73	86.53	+0.80	16.97	16.45	−0.52
22 10 25 50	192.30	194.89	+2.59	11.03	10.96	−0.07
22 10 50 41	192.73	193.32	+0.59	11.03	10.92	−0.11
29 9 20 53(j)	113.24	114.16	+0.92	14.71	14.88	+0.17
29 9 50 37(f)	111.06	113.17	+2.11	15.16	14.99	−0.17
Apr. 3 8 38 41(k)	178.90	179.05	+0.15	11.10	10.70	−0.40
3 9 5 39(d)	176.31*	177.32	+1.01*	10.77*	10.71	−0.06*
3 9 36 35(c)	174.96	175.31	+0.35	10.97	10.73	−0.24
3 9 59 32(j)	(169.80)	173.84	(+4.04)	(11.50)	10.75	(−0.75)
4 8 8 17	109.36	110.75	+1.39	15.11	15.18	+0.07
4 8 41 8	108.17	109.70	+1.53	15.21	15.28	+0.07
4 9 7 7	106.93	108.88	+1.95	15.22	15.36	+0.14
4 9 31 40	107.38	108.11	+0.73	15.24	15.43	+0.19
6 8 2 7(e)	355.38	355.69	+0.31	10.42	10.70	+0.28
6 8 31 33	352.96	353.80	+0.84	10.75	10.73	−0.02
6 9 4 38	352.31	351.68	−0.81	10.72	10.76	+0.04
6 9 36 11	351.09	349.67	−1.42	10.75	10.80	+0.05
9 8 2 57(h)	169.98	169.96	−0.02	10.67	10.77	+0.10
9 8 38 56	165.97	167.68	+1.71	10.97	10.82	−0.15
9 9 8 12(e)	165.54*	165.86	+0.32*	10.99*	10.87	−0.12*
10 8 13 36	103.34	105.12	+1.78	15.50	15.64	+0.14
10 8 40 55(j)	103.50	104.01	+0.51	15.53	15.68	+0.15
14 9 21 33	234.79	235.96	+1.17	13.79	13.52	−0.27
19 8 49 48	275.68	276.44	+0.76	16.09	16.07	−0.02
25 8 53 59(f)	269.24	271.41	+2.17	16.01	16.11	+0.10

(a) Image of Satellite imperfect.
(b) Neptune near edge of shutter.
(c) Diffused.
(d) Very diffused.
(e) Extremely diffused.
(f) Faint.
(g) Faint a
(h) Very faint and diffused.
(j) Extrem
(k) Satellite touching fringe of secondary spectrum.

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Observations of Comet c 1905, from Photographs taken with the 30-inch Reflector of the Thompson Equatorial.

(*Communicated by the Astronomer Royal.*)

The following positions of Comet *c* 1905 were obtained from photographs taken with the 30-inch reflector.

Exposures of 4^m and 5^m were given on December 8 and 9, from 1^m to 3^m on December 19, and $\frac{1}{2}^m$ for the remainder.

The plates were measured in the astrographic micrometer. Six reference stars were taken where possible, situated as symmetrically as possible about the comet. The positions of the stars were taken from the catalogues of the Astronomische Gesellschaft, or from Karlsruhe Observations 1885·0, or the Radcliffe Catalogue 1890.

Date and G.M.T.		Apparent R.A.	Apparent Decl.	Log Δ.	Corr. for Parallax. R.A.	Corr. for Parallax. Decl.
	1905.					
	d h m s	h m s	° ′ ″		s	″
Dec.	8 17 12 41	14 31 18·99	+20 5 4·6	0·1475	−0·25	+4·0
	9 17 8 3	14 36 10·66	+19 36 56·7	0·1409	−0·25	+4·1
	9 18 7 51	14 36 23·48	+19 35 45·6	0·1409	−0·21	+3·8
	19 17 30 37	15 30 26·34	+13 46 40·0	0·0903	−0·27	+4·9
	19 17 58 6	15 30 33·11	+13 45 52·8	0·0903	−0·25	+4·8
	1906.					
Jan.	6 18 40 17	17 34 40·56	− 3 10 49·9	0·0425	−0·28	+6·3
	7 18 46 53	17 42 39·12	− 4 21 40·3	0·0425	−0·28	+6·4
	9 18 46 6	17 58 56·01	− 6 46 9·6	0·0441	−0·29	+6·4
	10 18 39 27	18 7 16·25	− 7 59 46·0	0·0457	−0·29	+6·4

Royal Observatory, Greenwich :
1906 *November* 9.

Observations of Comet a 1906, from Photographs taken with the 30-inch Reflector of the Thompson Equatorial.

(*Communicated by the Astronomer Royal.*)

The following positions of Comet *a* 1906 were obtained from photographs taken with the 30-inch reflector, with exposures of from 3 minutes on January 30 to 20 minutes on April 14.

The plates were measured in the astrographic micrometer, and the position of the comet is deduced from the means of six reference stars whose places were derived from the catalogues of the Astronomische Gesellschaft except on February 20, when Greenwich Observations for the catalogue for 1900·0 were used.

	Date and G.M.T.	Apparent R.A.	Apparent Decl.	Log Δ.	Corr. for Parallax. R.A.	Corr. for Parallax. Decl.
	1906.					
	d h m s	h m s	° ′ ″		s	″
Jan.	30 15 51 44	16 16 11·57	+53 33 2·3	9·9992	−0·52	+1·7
Feb.	5 9 34 14	16 4 27·17	64 26 20·7	9·9738	−0·55	+7·3
	7 12 21 16	15 55 42·81	68 44 52·1	9·9688	−1·08	+3·3
	7 12 33 59	15 55 39·88	68 45 55·9	9·9688	−1·08	+3·0
	12 8 54 41	15 5 55·23	78 39 7·4	9·9658	−1·52	+5·3
	20 11 59 4	7 22 4·42	81 18 18·6	9·9912	+1·57	−3·2
	23 9 53 13	6 27 35·17	76 25 27·1	0·0087	+0·63	−3·2
Mar.	2 11 42 46	5 50 22·43	65 15 23·4	0·0589	+0·71	+0·9
	3 11 54 51	5 48 20·29	63 50 44·0	0·0669	+0·68	+1·3
	3 12 5 54	5 48 19·32	63 50 6·5	0·0669	+0·69	+1·5
	5 10 26 39	5 45 21·95	61 16 16·6	0·0828	+0·50	+0·3
	5 10 47 26	5 45 20·91	61 15 10·5	0·0828	+0·54	+0·6
	6 9 22 21	5 44 16·01	60 3 59·8	0·0909	+0·37	−0·3
	6 9 34 49	5 44 15·31	60 3 21·0	0·0909	+0·39	−0·1
	22 11 33 10	5 41 35·36	45 3 57·7	0·2135	+0·32	+2·9
	28 8 6 18	5 43 51·45	41 26 8·2	0·2567	+0·18	+1·3
	29 8 23 28	5 44 19·54	40 52 35·6	0·2634	+0·19	+1·5
	29 8 43 4	5 44 20·1[illegible]	40 52 10·0	0·2634	+0·21	+1·6
Apr.	14 8 33 51	5 53 38·5	+33 59 21·1	0·3602	+0·17	+1·9

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Observations of Comet b 1906, from Photographs taken with the 30-inch Reflector of the Thompson Equatorial.

(*Communicated by the Astronomer Royal.*)

The following positions of Comet *b* 1906 were obtained from photographs taken with the 30-inch reflector, with exposures of from 5 minutes to 10 minutes.

The plates were measured in the astrographic micrometer.

The position of the comet is referred to the mean of six reference stars whose places were derived from the catalogues of the Astronomische Gesellschaft.

Date and G.M.T. 1906.	Apparent R.A.	Apparent Decl.	Log Δ.	Corr. for Parallax R.A.	Corr. for Parallax Decl.
d h m s	h m s	° ′ ″		s	″
Mar. 6 10 26 11	11 34 43·86	+1 43 34·2	9·6760	−0·43	+14·2
6 10 39 49	11 34 43·60	+1 43 34·9	9·6760	−0·39	+14·2
7 11 46 41	11 34 16·60	+1 45 5·1	9·6850	−0·16	+13·8
7 11 56 1	11 34 16·32	+1 45 5·4	9·6850	−0·13	+13·8
22 12 30 1	11 27 51·41	+2 7 36·4	0·3777	+0·04	+ 2·8
29 11 2 13	11 25 13·63	+2 16 47·7	0·3861	0·00	+ 2·7
Apr. 3 10 30 21	11 23 35·88	+2 22 10·9	0·3935	−0·01	+ 2·7
3 10 44 27	11 23 35·70	+2 22 10·7	0·3935	0·00	+ 2·7
4 10 15 46	11 23 18·33	+2 23 6·0	0·3951	−0·01	+ 2·7
4 10 29 25	11 23 18·17	+2 23 6·5	0·3951	0·00	+ 2·7
9 10 15 22	11 21 59·65	+2 26 51·8	0·4035	0·00	+ 2·6
9 10 25 21	11 21 59·48	+2 26 53·6	0·4035	+0·01	+ 2·6
10 9 5 13	11 21 46·70	+2 27 22·8	0·4053	−0·04	+ 2·6
14 10 5 30	11 20 59·65	+2 29 2·4	0·4128	+0·01	+ 2·6
19 10 2 19	11 20 19·44	+2 29 29·1	0·4229	+0·02	+ 2·5
20 9 48 56	11 20 13·93	+2 29 21·0	0·4250	+0·01	+ 2·5
25 9 53 27	11 19 59·15	+2 27 29·9	0·4356	+0·03	+ 2·4
26 9 52 27	11 19 58·79	+2 26 53·1	0·4378	+0·03	+ 2·4
27 9 47 3	11 19 59·34	+2 26 12·3	0·4401	+0·03	+ 2·4
28 10 15 16	11 20 0·87	+2 25 25·2	0·4423	+0·04	+ 2·4

Royal Observatory[illegible]
1906 *Nov*[illegible]

Cometary Observations at the Liverpool Observatory. By W. E. Plummer.

Continued from Vol. LXIV. p. 783.

Comet a 1904, Brooks = Comet 1904 I.

Greenwich Mean Time of Observation.		☄ − ✱ R.A.	No. of Comp.	Apparent R.A. of Comet.	☄ − ✱ Decl.	No. of Comp.	App. Decl. of Comet.	Log. Factor of Parallax in α.	Log. Factor of Parallax in δ.	Star of Comp.
1904	h m s	m s		h m s	′ ″		° ′ ″			
Apr. 17	10 26 8·0	− 3 25·73	10	16 56 26·89	− 3 4·8	5	+44 42 59·7	−9·6782	+0·6068	1
19	10 22 17·8	− 3 19·46	10	16 50 49·17	− 2 46·7	5	+45 57 45·1	−9·6823	+0·5753	2
24	10 29 30·2	− 1 15·17	12	16 35 2·70	+ 5 20·0	4	+48 54 52·9	−9·6729	+0·4404	3
24	10 29 30·2	− 0 54·31	12	16 35 2·57	− 8 8·6	4	+48 54 52·5	−9·6729	+0·4404	4
30	10 3 30·1	− 46·41	10	16 13 6·52	+11 18·1	5	+52 1 36·4	−9·6798	+0·3768	5
May ·2	9 37 11·0	+ 2 59·71	8	16 5 8·41	− 3 1·8	4	+52 56 10·4	−9·7010	+0·3269	6
3	9 14 40·7	+ 10·65	10	16 1 3·75	− 6 57·0	5	+53 21 48·0	−9·6202	+0·3623	7
5	9 40 16·3	− 32·66	10	15 52 29·40	− 7 20·8	5	+54 11 11·2	−9·6887	+0·1872	8
5	9 40 16·3	+ 1 49·36	10	15 52 29·17	+ 1 37·1	5	+54 11 12·3	−9·6887	+0·1872	9
7	10 1 31·8	+ 3 23·39	12	15 43 38·84	+ 5 35·1	4	+54 55 55·9	−9·6216	+9·9586	10
9	9 59 20·2	+ 3 39·69	8	15 34 39·03	− 1 29·9	4	+55 35 46·7	−9·5975	+9·8024	11
9	9 59 20·2	− 3 38·88	8	15 34 38·82	+ 1 19·7	4	+55 35 47·0	−9·5975	+9·8024	12
14	10 50 2·3	+ 7 9·17	10	15 12 21·62	− 8 46·9	5	+56 55 30·3	−9·2589	−9·5884	13
15	9 54 0·4	+ 1 36·80	8	15 6 49·23	+ 3 10·7	4	+57 7 27·9	−9·4829	−8·8546	13
18	9 59 4·1	+ 1 16 28	10	14 52 48·17	+ 4 7·6	5	+57 37 35·0	−9·3573	−9·5961	14
18	9 59 4·1	− 1 20·56	10	14 52 48·31	+11 41·9	5	+57 37 34·3	−9·3573	−9·5961	15
19	9 36 22·5	+ 0·98	12	14 48 13·50	− 3 2·3	4	+57 45 19·2	−9·4252	−9·4944	16

Comet 1904 *I.*—(*continued*).

Greenwich Mean Time of Observation.		☌ − ✱ R.A.	No. of Comp.	Apparent R.A. of Comet.	☌ − ✱ Decl.	No. of Comp.	App. Decl. of Comet.	Log. Factor of Parallax in α.	Log. Factor of Parallax in δ.	Star of Com
1904	h m s	m s		h m s	′ ″		° ′ ″			
May 20	9 52 11·2	− 1 40·36	12	14 43 34·11	+ 7 32·9	4	+57 52 8·4	−9·3041	−9·6985	17
24	9 40 16·0	+ 3 13·47	Ret.	14 25 31·80	+ 4 51·6	Ret.	+58 8 54·7	−9·1520	−9·8133	18
25	10 35 13·9	+ 1 21·09	12	14 20 58·62	+ 3 53·6	4	+58 10 44·7	+8·6157	−9·9565	19
25	10 35 13·9	− 4 49·66	12	14 20 58·22	− 15·7	4	+58 10 45·2	+8·6157	−9·9565	20
28	10 22 11·0	+13 26·09	Ret.	14 8 15·90	− 1 7·3	Ret.	+58 10 40·9	+8·8711	−9·8589	21
June 3	10 39 47·2	+ 52·07	12	13 44 54·06	+ 5 27·1	4	+57 49 45·0	+9·3941	−9·3404	22
4	10 55 30·1	+ 4 25·02	12	13 41 17·60	+ 2 23·3	4	+57 44 4·4	+9·5223	−8·8915	23
4	10 55 30·1	− 2 44·52	12	13 41 17·47	− 13·0	4	+57 44 4·9	+9·5223	−8·8915	22
5	10 50 57·0	+ 56·75	10	13 37 49·33	− 3 57·0	5	+57 37 44·1	+9·4937	−9·1129	23
12	10 40 15·8	+ 9·38	Ret.	13 16 3·93	+13 35·9	Ret.	+56 40 52·4	+9·5971	+9·6205	24
21	10 30 19·9	− 1 0·53	Ret.	12 54 29·20	− 5 52·1	Ret.	+55 6 0·3	+9·6721	+0·1008	25
28	10 40 35·2	+ 1 2·37	10	12 42 3·99	+ 7 2·8	5	+53 44 29·1	+9·7214	+0·3488	26
29	10 25 4·0	− 26·92	10	12 40 34·67	− 4 36·6	5	+53 32 49·8	+9·7088	+0·3196	26
	11 16 10·2	− 1 12·26	10	12 30 34·59	− 4 34·9	5	+51 58 14·0	+9·7520	+0·5886	27
	11 3 58·4	+ 35·26	10	12 29 34·67	+ 19·0	5	+51 46 45·1	+9·7486	+0·5771	28
	[illegible] 3 58·4	+ 23·64	10	12 29 34·50	+ 14·4	5	+51 46 46·2	+9·7486	+0·5771	29
	[illegible] 4 0·3	+ 0·21	12	12 28 36·53	− 3 16·6	4	+51 35 8·9	+9·7486	+0·5916	30
	[illegible] 4 0·3	+ 2 26·30	12	12 28 36·41	+ 4 18·3	4	+51 35 9·3	+9·7486	+0·5916	31
	[illegible] 3 36·0	+ 4 58·53	10	12 26 49·91	− 1 44·3	5	+51 12 13·3	+9·7465	+0·6115	32
	[illegible] 16·2	+ 31·48	10	12 23 5·92	− 7 14·7	5	+50 16 30·2	+9·7418	+0·6108	33
	[illegible]	[illegible] 7·37	Ret.	12 21 51·87	− 6 35·7	Ret.	+49 54 39·8	+9·7355	[illegible]	34

Encke's Comet.

Date	Time	☌ − ✱ R.A.	No. of Comp.	Apparent R.A. of Comet.	☌ − ✱ Decl.	No. of Comp.	App. Decl. of Comet.	Log. Factor of Parallax in α.	Log. Factor of Parallax in δ.	Sta of Comp.
	h m s	m s		h m s	′ ″		° ′ ″			
[illegible]	[illegible] 4 11·0	+ 2 24·25	Ret.	21 30 51·33	+ 7 7·2	Ret.	+ 12 18 29·4	+ 9·1571	+ 0·7669	35
[illegible]	[illegible] 3 18·2	+ 38·31	10	20 57 46·42	− 5 14·6	5	+ 7 4 19·5	+ 9·3820	+ 0·8134	36
[illegible]	[illegible] 3 18·2	+ 1 8·55	10	20 57 46·60	+ 3 16·9	5	+ 7 4 22·2	+ 9·3820	+ 0·8134	37
[illegible]	[illegible] 32 18·5	− 1 9·60	12	20 54 7·02	+ 2 10·9	4	+ 6 28 27·4	+ 9·3525	+ 0·8151	38
[illegible]	6 10 40·9	+ 1 22·82	12	20 46 37·15	+ 3 10·5	4	+ 5 14 46·6	+ 9·3119	+ 0·8205	39
[illegible]	6 10 40·9	+ 1 32·16	12	20 46 36·94	+ 7 49·4	4	+ 5 14 47·1	+ 9·3119	+ 0·8205	40
10	6 27 18·0	− 5·00	Ret.	20 38 49·57	− 2 13·1	Ret.	+ 3 57 40·7	+ 9·4040	+ 0·8303	41
13	6 0 35·3	− 1 4·50	10	20 26 43·85	− 4 56·4	5	+ 1 57 29·2	+ 9·3980	+ 0·8391	42
14	5 27 11·4	− 1 22·82	10	20 22 25·30	+ 3 40·9	5	+ 1 16 19·4	+ 9·3452	+ 0·8415	43
17	5 16 40·0	+ 2 51·96	10	20 9 14·62	+ 9 42·9	5	− 0 56 23·0	+ 9·3766	+ 0·8510	44
17	5 16 40·0	+ 14·66	10	20 9 14·60	+ 55·3	5	− 0 56 24·9	+ 9·3766	+ 0·8510	45

Comet e 1904 (Borrelly) = Comet 1905 II.

Date	Time	☌ − ✱ R.A.	No. of Comp.	Apparent R.A. of Comet.	☌ − ✱ Decl.	No. of Comp.	App. Decl. of Comet.	Log. Factor of Parallax in α.	Log. Factor of Parallax in δ.	Sta of Comp.
1905 Jan. 4	6 27 30·0	− 7·71	Ret.	1 21 24·40	− 47·6	Ret.	− 5 10 46·9	+ 7·1072	+ 0·8731	46
8	6 29 49·8	− 3 44·87	10	1 27 16·12	− 3 3·0	5	− 1 56 40·5	+ 8·3042	+ 0·8578	47
9	6 42 39·1	− 17·42	10	1 28 49·16	− 2 44·3	5	− 1 8 6·4	+ 8·6384	+ 0·8535	48
9	6 42 39·1	+ 10·99	10	1 28 49·11	+ 7 40·8	5	− 1 8 9·3	+ 8·6384	+ 0·8535	49
11	7 22 40·3	+ 2 6·08	Ret.	1 32 0·60	+ 53·8	Ret.	+ 0 28 48·2	+ 9·0415	+ 0·8447	50
26	7 40 16·0	+ 1 3·20	Ret.	1 59 3·37	− 2 52·5	Ret.	+ 11 50 25·1	+ 9·2625	+ 0·7750	51

Comet a 1905 (*Giacobini*) = *Comet* 1905 *III*.

Greenwich Mean Time of Observation.		☌ − ✱ R.A.	No. of Comp.	Apparent R.A. of Comet.	☌ − ✱ Decl.	No. of Comp.	App. Decl. of Comet.	Log. Factor of Parallax in α.	Log. Factor of Parallax in δ.	Star of Comp.
1905	h m s	m s		h m s	′ ″		° ′ ″			
Mar. 30	9 4 32·1	− 30·92	Ret.	5 58 43·36	+ 5 3·6	Ret.	+15 54 11·1	+9·4520	+0·7677	52
Apr. 3	9 15 11·9	+ 40·20	...	6 14 17·70	+ 8 0·5	...	+20 44 56·0	+9·4800	+0·7384	53
8	9 14 0·4	− 1 25·20	...	6 35 32·70	+ 3 8·0	...	+26 33 40·2	+9·4957	+0·6880	54

Comet b 1905 (*Schaer*) = *Comet* 1905 *V*.

Greenwich Mean Time of Observation.		☌ − ✱ R.A.	No. of Comp.	Apparent R.A. of Comet.	☌ − ✱ Decl.	No. of Comp.	App. Decl. of Comet.	Log. Factor of Parallax in α.	Log. Factor of Parallax in δ.	Star of Comp.
Nov. 20	10 2 35·0	− 16·84	Ret.	23 54 42·79	−17 17·8	Ret.	+62 10 59·8	+9·5901	+0·3411	55
[illegible]	10 2 4·9	− 47·87	8	23 38 17·92	+ 5 59·2	4	+37 29 45·3	+9·4356	+0·4862	56
[illegible]	[illegible] 14 45·1	+ 49·38	10	23 35 53·63	− 4 1·8	5	+30 31 42·7	+9·5383	+0·6656	57
[illegible]	[illegible]	− 3 58·11	8	23 31 1·84	− 9 51·0	4	+ 9 23 8·0	+9·2930	+0·7972	58
[illegible]	[illegible]	− 1 52·46	Ret.	23 30 43·19	− 4 13·1	Ret.	+ 7 1 59·2	−8·8229	+0·8027	59

Comet b 1906 (*Kopff*) = *Comet* 1905 *IV*.

Greenwich Mean Time of Observation.		☌ − ✱ R.A.	No. of Comp.	Apparent R.A. of Comet.	☌ − ✱ Decl.	No. of Comp.	App. Decl. of Comet.	Log. Factor of Parallax in α.	Log. Factor of Parallax in δ.	Star of Comp.
[illegible]	[illegible] 2 16·0	− 2 32·18	10	11 34 44·57	− 2 30·5	5	+ 1 43 32·8	−9·3744	+0·8398	60
[illegible]	[illegible] 43 28·9	− 3 23·27	10	11 33 53·51	+ 24·2	5	+ 1 46 27·3	−9·3945	+0·8399	60
[illegible]	9 43 28·9	+ 5 39·07	10	11 33 53·11	− 1 45·0	5	+ 1 46 26·2	−9·3945	+0·8399	61
15	9 27 11·3	+ 3 2·01	8	11 30 50·94	+ 5 30·8	4	+ 1 56 59·3	−9·3656	+0·8386	62
18	8 44 9·4	− 22·08	10	11 29 35·01	+ 40·3	5	+ 2 1 29·3	−9·4257	+0·8394	63
20	10 22 0·5	− 1 14·11	10	11 28 42·99	+ 3 45·7	5	+ 2 4 34·7	−9·1218	+0·8358	63
18	9 31 5·0	− 34·93	8	11 20 25·61	+ 6 24·4	4	+ 2 29 39·3	−8·4235	+0·8324	64
10	9 26 53·1	+ 17·99	8	11 20 14·52	− 7 15·3	4	+ 2 29 22·1	−8·3151	+0·8325	65
8	9 16 24·2	+ 24·06	10	11 20 1·22	− 30·8	5	+ 2 25 25·9	+8·0789	+0·8329	66

Stars of Comparison. Comet 1904 I.

Date. 1904	Star's Designation or Authority for Place.	Mean R.A. Equinox of Year. h m s	Corrections to Mean R.A. s	Mean Declination Equinox of Year. ° ′ ″	Correction to Mean Declination. ″	No. of Reference.
Apr. 17	A.G.Z. Bonn, No. 10909	16 59 51·06	+1·56	+44 46 11·9	− 7·4	1
19	" " No. 10841	16 54 7·01	+1·62	46 0 39·0	− 7·2	2
24	" " No. 10664	16 36 16·05	+1·82	48 49 38·6	− 5·7	3
24	" " No. 10656	16 35 55·16	+1·72	49 3 6·1	− 5·0	4
30	A.G.Z. Harvard, No. 4954	16 13 50·87	+2·06	51 50 21·9	− 3·6	5
May 2	" " No. 4911	16 2 6·59	+2·11	52 59 14·9	− 2·7	6
3	" " No. 4903	16 0 50·96	+2·14	53 28 47·4	− 2·4	7
5	" " No. 4869	15 52 59·88	+2·18	54 18 33·6	− 1·6	8
5	" " No. 4862	15 50 37·63	+2·18	54 9 36·7	− 1·5	9
7	" " No. 4822	15 40 13·24	+2·21	54 50 21·4	− 0·6	10
9	A.G.Z. Hels. Gotha, No. 8408	15 30 57·10	+2·24	55 37 16·3	+ 0·3	11
"	" " " No. 8461	15 38 15·43	+2·27	55 34 27·4	− 0·1	12
14	" " " No. 8240	15 5 10·20	+2·25	57 4 14·4	+ 2·8	13
18	" " " No. 8172	14 51 29·71	+2·18	57 33 23·2	+ 4·2	14
18	" " " No. 8183	14 54 6·69	+2·18	57 25 48·2	+ 4·2	15
19	" " " No. 8149	14 48 10·35	+2·17	57 48 16·8	+ 4·7	16
20	" " " No. 8117	14 45 12·33	+2·14	57 44 30·4	+ 5·1	17
24	" " " No. 7966	14 22 16·29	+2·04	58 3 56·9	+ 6·2	18
25	" " " No. 7949	14 19 35·56	+1·97	58 6 44·2	+ 6·9	19

Comet 1904 *I.—(continued).*

Date. 1904	Star's Designation or Authority for Place.	Mean R.A. Equinox of Year. h m s	Corrections to Mean R.A. s	Mean Declination Equinox of Year. ° ′ ″	Correction to Mean Declination. ″	No. of Reference.
May 25	A.G.Z. Hels. Gotha, No. 7988	14 25 45·86	+2·02	58 10 54·4	+ 6·5	20
28	,, ,, ,, No. 7774	13 54 48·05	+1·76	58 11 40·3	+ 7·9	21
June 3	,, ,, ,, No. 7709	13 44 0·44	+1·55	57 44 8·9	+ 9·0	22
[illegible]	,, ,, ,, No. 7657	13 36 51·08	+1·50	57 41 31·9	+ 9·2	23
	,, ,, ,, No. 7525	13 15 53·44	+1·11	56 27 6·2	+10·3	24
	,, ,, ,, No. 7397	12 55 28·95	+0·78	55 11 42·0	+10·4	25
	[illegible] Z. Harvard, No. 4136	12 41 1·05	+0·57	53 37 16·5	+ 9·8	26
	,, ...	...	+0·54	...	+ 9·9	26
	,, ,, No. 4104	12 31 46·54	+0·31	52 2 39·8	+ 9·1	27
	,, ,, No. 4097	12 28 59·14	+0·27	51 46 17·2	+ 8·9	28
	,, ,, No. 4098	12 29 10·59	+0·27	51 46 22·9	+ 8·9	29
	,, ,, No. 4095	12 28 36·07	+0·25	51 38 16·7	+ 8·8	30
9	,, ,, No. 4092	12 26 9·86	+0·25	51 30 42·2	+ 8·8	31
11	,, ,, No. 4076	12 21 51·20	+0·18	51 13 49·3	+ 8·3	32
16	,, ,, No. 4079	12 22 34·33	+0·11	50 23 37·0	+ 7·9	33
18	,, ,, No. 4075	12 21 44·42	+0·08	50 1 7·8	+ 7·7	34

Encke's Comet.

Date. 1904	Star's Designation or Authority for Place.	Mean R.A. Equinox of Year. h m s	Corrections to Mean R.A. s	Mean Declination Equinox of Year. ° ′ ″	Correction to Mean Declination. ″	No. of Reference.
26	A.G.Z. Leipzig I., No. 8555	21 28 25·02	+2·06	+12 10 56·4	+25·8	35
5	,, ,, II., No. 10528	20 57 6·34	+1·77	7 9 11·5	+22·6	36
	,, ,, II., No. 10523	20 56 36·28	+1·77	7 0 42·7	+22·6	37

te.	Star's Designation or Authority for Place.	Mean R.A. Equinox of Year. h m s	Correction to Mean R.A. s	Mean Declination Equinox of Year. ° ′ ″	Corrections to Mean Declination. ″	No. of Reference.
..04. Dec. 6	A.G.Z. Leipzig II., No. 10508	20 55 14·87	+1·75	+ 6 25 54·2	+22·3	38
8	Paris No. 28996 & Leip. II. No. 10391 ...	20 45 12·63	+1·70	+ 5 11 14·6	+21·5	39
8	A.G.Z. Leipzig II., No. 10387	20 45 3·08	+1·70	5 6 36·3	+21·4	40
10	A.G.Z. Albany, No. 7242	20 38 52·92	+1·65	+ 3 59 33·2	+20·6	41
13	Paris No. 28383 & Albany No. 7168 ...	20 27 46·75	+1·60	+ 2 2 6·2	+19·4	42
14	A.G.Z. Albany, No. 7129	20 23 46·54	+1·58	+ 1 12 20·2	+18·3	43
17	θ Aquilæ (Nautical Almanac)	20 6 21·12	+1·54	− 1 6 23·4	+17·5	44
17	A.G.Z. Nicolajew, No. 5097	20 8 58·40	+1·54	− 0 57 37·7	+17·5	45
		Comet 1905 *II.*				
1905. Jan. 4	Schjellerup, No. 442	1 21 32·24	−0·13	− 5 9 50·7	− 8·6	46
8	A.G.Z. Nicolajew, No. 308	1 31 1·08	−0·09	− 1 53 29·4	− 8·1	47
9	,, ,, No. 300	1 29 6·70	−0·12	− 1 5 14·3	− 7·8	48
9	Bonn Beob. vi. −1° 207	1 28 38·22	−0·10	− 1 15 42·3	− 7·8	49
11	A.G.Z. Nicolajew, No. 303	1 29 54·63	−0·11	+ 0 28 1·8	− 7·4	50
26	A.G.Z. Leipzig I., No. 612	1 58 0·23	−0·06	+11 53 23·0	− 5·4	51
		Comet 1905 *III.*				
Mar. 30	A.G.Z. Berlin A., No. 1883	5 59 14·15	+0·13	+15 49 18·2	−10·7	52
Apr. 3	,, ,, B., No. 2304	6 13 37·38	+0·12	+20 37 4·5	− 9·0	53
8	A.G.Z. Cambridge, No. 3449	6 36 57·76	+0·14	+26 30 39·2	− 7·0	54

Comet 1905 *V.*

Date. 1905	Star's Designation or Authority for Place.	Mean R.A. Equinox of Year. h m s	Corrections to Mean R.A. s	Mean Declination Equinox of Year. ° ′ ″	Correction to Mean Declination. ″	No. of Reference.
Nov. 20	A.G.Z. Hels. Gotha, No. 14580	23 54 55·25	+4·38	+62 27 46·5	+31·1	55
23	Bonn vol. vi., No. 4882	23 39 2·66	+3·13	+37 23 16·9	+29·2	56
,,	A.G.Z. Leiden, No. 10043	23 35 1·41	+2·84	+30 35 17·3	+27·2	57
	,, ,, No. 11721	23 34 57·39	+2·56	+ 9 32 37·9	+21·1	58
	A.G.Z. Leipzig II., No. 11713	23 32 33·15	+2·50	+ 7 5 52·1	+20·2	59

Comet 1905 *IV.*

Date.	Star's Designation or Authority for Place.	Mean R.A. Equinox of Year.	Corrections to Mean R.A.	Mean Declination Equinox of Year.	Correction to Mean Declination.	No. of Reference.
[illegible]	A.G.Z. Albany, No. 4335...	11 37 15·64	+1·11	+ 1 46 11·3	− 8·0	60
[illegible]	,, ,, ...	...	+1·14	...	− 8·2	60
[illegible]	,, ,, No. 4300...	11 28 12·87	+1·17	+ 1 48 19·3	− 8·1	61
[illegible]	,, ,, No. 4299...	11 27 47·75	+1·18	+ 1 51 37·1	− 8·6	62
[illegible]	,, ,, No. 4308...	11 29 55·90	+1·19	+ 2 0 57·7	− 8·7	63
[illegible]	,, ,, ...	...	+1·20	...	− 8·7	63
[illegible]	,, ,, No. 4271...	11 20 59·44	+1·10	+ 2 23 23·5	− 8·6	64
[illegible]	,, ,, No. 4265...	11 19 55·46	+1·07	+ 2 36 45·9	− 8·5	65
[illegible]	,, ,, No. 4264...	11 19 36·16	+1·00	+ 2 26 4·9	− 8·2	66

Auroræ observed in the Parish of Delting, Shetland, from September 1905 *to September* 1906. By the Rev. Alex. C. Henderson, B.D.

1905.

Sept.	3.	No unusual features noted.
"	22.	Visible from 8.40 p.m. till 9.25 p.m.
"	25.	Beginning about 7.50 p.m.
"	26.	No unusual features noted.
Oct.	7.	A *curtain* aurora, 8.50 p.m.
"	28.	Auroral light shining through clouds.
"	29.	Auroral light again, through clouds.
*Nov.	15.	Crimson, with green streamers.
"	23.	No unusual features noted.
Dec.	29.	" "
"	30.	" "

1906.

Feb.	15.	A broad, white arch.
"	16.	No unusual features noted.
"	22.	Extending over half of the sky.
"	24.	Greenish-white.
"	25.	Greenish-white.
"	26.	No unusual features noted.
Mar.	13.	" "
"	17.	" "
"	24.	At 9 p.m.
"	26.	A bright, but ill-defined arch.
Apr.	10.	A broad arch, with streamers.
"	28.	A beautiful display, chiefly between 10 and 11 p.m. Here follows a gap, caused by the absence of night in this latitude.
Aug.	14.	Aurora, and simultaneous shower of Perseid meteors, from 10 p.m. till 1 a.m. on 15th.
"	15.	Diffused aurora behind cloud.
Sept.	15.	Between 10 and 11 p.m.
"	16.	9 p.m., the streamers shooting up from horizon through Ursa Major, and afterwards forming an arch.
"	17.	*Reported* to me as being crimson and yellow. I did not observe *this* aurora.
"	22.	Began in the NNE at 8.30 p.m. (first *yellow*, afterwards light *green*). At 9.30 p.m., flickering clouds extended across the zenith, and there were crackling noises.
"	23.	Greenish-white.

The Manse of Delting, from which these auroræ were observed, is in lat. 60° 24′ ¾ N ; long. 1° 19′ ½ W ; longitude in time, 5 min. 18 sec.

* At 7.40 p.m. the crimson aurora extended in a broad band across Altair, to the south of it. At 9.42 p.m. the magnificent green streamers

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXVII. DECEMBER 14, 1906. No. 2

W. H. MAW, Esq., PRESIDENT, in the Chair.

Major Alexander Davidson Fleming, Artillery Mansions, 75 Victoria Street, London, S.W.,

was balloted for and duly elected a Fellow of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Edgar T. Adams, 5 Warkworth Street, Cambridge (proposed by E. T. Whittaker);
Robert Jonckheere, Observatoire Stella, Roubaix, France (proposed by Camille Flammarion);
John Stewart, Chief Officer, R.M.S. "Empress of China," The Willows, Wallasey, Birkenhead (proposed by E. B. Knobel); and
Samuel Veevers, Normanton, Kimberley Drive, Great Crosby, near Liverpool (proposed by R. C. Johnson).

Seventy-seven presents were announced as having been received since the last meeting, including, amongst others :—

W. Bramsen, Japanese Chronological Tables, presented by E. B. Knobel; Optical Convention, Proceedings of the Meeting May–June, 1905, presented by the Committee; Oxford Astro- ·talogue, vol. i., presented by the University Observatory, \. Parkhurst, Researches in Stellar Photometry, pre- ·uthor; Royal Observatory, Greenwich, Astronomical presented by the Observatory; Royal Observa- Hope, Annals, vols. x., xii., Astrographic ·d by the Observatory.

Astrographic Chart: 20 charts, presented by the Royal Observatory, Greenwich, and 19 charts presented by the San Fernando Observatory. Photographs of the spectrum of Mira Ceti, presented by the Rev. W. Sidgreaves.

On the Possibility of Improving the Places of the Reference Stars for the Astrographic Catalogue from the Photographic Measures. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. On each of the plates taken for the Astrographic Catalogue there are certain stars of which meridian observations have been made, and the constants of the plate are found by using these recorded places. But the places are often defective, from errors of observation and accumulated proper motions; and the errors are indicated by the residuals found on comparing the photographic measures (reduced with the plate-constants found by using all the stars) with the individual meridian places. But these residuals cannot be taken as satisfactory corrections to the adopted places, because the plate-constants, having been found from faulty meridian places, are themselves faulty.

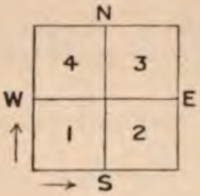


Fig. 1.

2. Taking only a single plate, if we correct the adopted places by the residuals found, and then solve for plate-constants again, we shall get precisely the same constants as before, and the residuals now will be all zero; but the improvement is of course only fictitious, and no real advance has been made. We need not, however, restrict ourselves to a single plate. Every star occurs on at least two plates, and we get at least two different residuals for it. Moreover, the stars in the four quarters of any plate will, as a rule, be on four different overlapping plates. Shall we then adopt as corrections to the original places the means of the residuals for each star (usually two, but sometimes more), and then determine the plate-constants afresh? There is no *prima facie* objection to this course; but it is que[illegible] whether it is the best possible, for the following reason.

3. Call the residuals determined fro[illegible] from other plates B: it is propose[illegible] portion $\frac{1}{2}$A is simply non-eff[illegible]

that if the residuals A be applied to the original places, and the constants redetermined, they will be the same as before; in other words, if we determine the *corrections* to a, b, c by solving a series of equations such as

$$\Delta a.x + \Delta b.y + \Delta c = \mathrm{A}$$

we shall get $\Delta a = 0$, $\Delta b = 0$, $\Delta c = 0$. Naturally we shall get the same zero result if we write $\frac{1}{2}$A instead of A on the right of all the equations. Hence it may be argued that we may as well save ourselves the trouble of introducing the system of residuals A into the equations at all, since they do neither good nor harm; with one possible qualification. Some stars occur on more than two plates, and if we form the mean of all plates, the residuals A will have a factor $\frac{1}{3}$ or $\frac{1}{4}$ in some cases instead of $\frac{1}{2}$. But there is clearly no systematic advantage to be gained by such exceptions, and we need not discuss them in detail.

4. The question therefore arises whether it is not better to exclude the residuals A altogether, since they do neither good nor harm in improving the plate-constants. Should not the corrections to the originally adopted places be determined from residuals B entirely?—*i.e.* instead of neglecting the $\frac{1}{2}$A as useless, and retaining the $\frac{1}{2}$B, should we not substitute B simply? or the mean of several B residuals if they are available? The answer to this question can only be arrived at by considering in some detail what the residuals represent.

5. For each star we write down two equations of the form

$$ax + by + c - \mathrm{X} = 0, \quad dx + ey + f - \mathrm{Y} = 0$$

and it will be sufficient to consider the first of these equations only, since the procedure is the same for both. In this equation x and y are the co-ordinates of the star, a, b, c the plate-constants to be determined, X the difference between the measured and calculated standard co-ordinates of the star. There will be as many equations as there are known stars on the plate, and they may be solved by least squares or any equivalent process. At Greenwich and Oxford the labour of least squares has been avoided by taking the mean of all the equations for the S and N halves of the plate and for the E and W halves; *i.e.* the four equations

$$\begin{aligned}
\{[1] + [2]\}/(n_1 + n_2) &= 0 \\
\{[4] + [3]\}/(n_4 + n_3) &= 0 \\
\{[2] + [3]\}/(n_2 + n_3) &= 0 \\
\{[1] + [4]\}/(n_1 + n_4) &= 0
\end{aligned}$$

have been formed; where [1] represents the sum of all the essions

$$ax + by + c - \mathrm{X}$$

of the plate marked 1 in fig. 1: and n_1 is the his portion. Next, the constant c is eliminated

by subtracting the second equation from the first and the fourth from the third, and we get the pair

$$\{[1]+[2]\}/(n_1+n_2)-\{[4]+[3]\}/(n_4+n_3)=0$$
$$\{[2]+[3]\}/(n_2+n_3)-\{[1]+[4]\}/(n_1+n_4)=0$$

from which to determine a and b.

6. If the stars are equally distributed in the four quadrants, so that $n_1=n_2=n_3=n_4$, these equations reduce to

$$[1]=[3] \quad \text{and} \quad [2]=[4]$$

The discussion thus divides itself into two parts: first, the consideration of what happens when the stars are uniformly distributed; and secondly, the effect of irregular distribution.

7. We shall take uniform distribution first; and we may take the very simplest case of it, viz. when there are just four stars placed at the centres of the four quadrants. If the side of the plate be $4s$, the co-ordinates of these stars referred to the centre of the plate as origin will be (see fig. 1)

1	$x=-s$	$y=-s$
2	$x=+s$	$y=-s$
3	$x=+s$	$y=+s$
4	$x=-s$	$y=+s$

and the equations for finding a and b become

$$-2s.a-2s.b=X_1-X_3$$
$$+2s.a-2s.b=X_2-X_4$$

Thus

$$4s.a=-X_1+X_2+X_3-X_4 \quad . \quad . \quad . \quad (1)$$
$$4s.b=-X_1-X_2+X_3+X_4 \quad . \quad . \quad . \quad (2)$$

and for c we add all four equations, so that

$$4c=X_1+X_2+X_3+X_4\,. \quad . \quad . \quad . \quad (3)$$

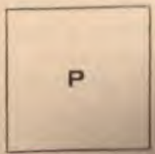
8. The residual X_1 is reduced, with these values of a, b, and c, to

$$X_1-(ax+by+c)$$

which on putting $x=-s$, $y=-s$, and the values of a, b, c given above becomes

$$\tfrac{1}{4}(X_1-X_2+X_3-X_4)=+I, \text{ say} \quad . \quad . \quad . \quad (4)$$

9. The residual for the third quadrant is also reduced to $+I$



while the residuals for the second and fourth quadrants become $-\mathrm{I}$. This quantity I, therefore, cannot be removed by a linear solution.

Its effect is to alter the square P (fig. 2) into the trapezium Q; and it may be produced, not by erroneous adopted places, but by a "tilt" of the plate; or, in other words, by our assuming the wrong plate-centre to which to refer our standard co-ordinates. It is known that this modifies ξ, η into expressions of the form

$$\xi' = \frac{(1+a)\xi + b\eta + c}{1 - k\xi - l\eta}$$

$$= (1+a')\xi + b'\eta + c' + k\xi^2 + l\xi\eta \text{ approx.}$$
$$\eta' = d'\xi + (1+e')\eta + f' + k\xi\eta + l\eta^2 \quad ,,$$

where (k, l) are the co-ordinates of the true centre. The terms which give rise to an I effect are the $\xi\eta$ terms, which are positive in the first and third quadrants and negative in the second and fourth. But it is known from experience of plates, in parts of the sky where stars are numerous, that no very large part of I is due to tilt in this way.

10. It is perhaps also as well to verify that the same I term would be given by the method of least squares, at any rate in the simple case at present under consideration. Solving the equations

$$\begin{aligned} -as - bs + c &= \mathrm{X}_1 \\ +as - bs + c &= \mathrm{X}_2 \\ +as + bs + c &= \mathrm{X}_3 \\ -as + bs + c &= \mathrm{X}_4 \end{aligned}$$

by least squares, the normal equations are

$$\begin{aligned} 4s^2a &= s(-\mathrm{X}_1 + \mathrm{X}_2 + \mathrm{X}_3 - \mathrm{X}_4) \\ 4s^2b &= s(-\mathrm{X}_1 - \mathrm{X}_2 + \mathrm{X}_3 + \mathrm{X}_4) \\ 4c &= (\mathrm{X}_1 + \mathrm{X}_2 + \mathrm{X}_3 + \mathrm{X}_4) \end{aligned}$$

which are precisely the same as those obtained in § 7. Hence the constants and residuals obtained are the same, and we still get the residuals $+\mathrm{I}$, $-\mathrm{I}$, $+\mathrm{I}$, $-\mathrm{I}$ for the four quadrants where

$$4\mathrm{I} = \mathrm{X}_1 - \mathrm{X}_2 + \mathrm{X}_3 - \mathrm{X}_4$$

11. It is convenient to have a name for the quantity I, and the name "inconsistency" will be adopted in the present paper for convenience. When I is not zero, *no* linear solution will fit the plate. We can find the value of I before applying any linear solution at all, since it is not altered by the application of linear terms.

12. Now it is clear that the procedure indicated in §§ 2 and 3, [illegible]ing the means between residuals from different plates, will [illegible] the "inconsistency" for any plate to zero. [illegible] even this is not certain. Calling the [illegible] I_0, and of the four overlapping plates

$+I_1$, $-I_2$, $+I_3$, $-I_4$, then the process of taking the mean of residuals substitutes for I_0 the quantity

$$\tfrac{1}{4}\{\tfrac{1}{2}(I_0+I_1)+\tfrac{1}{2}(I_0+I_2)+\tfrac{1}{2}(I_0+I_3)+\tfrac{1}{2}(I_0+I_4)\}$$
$$=\tfrac{1}{2}I_0+\tfrac{1}{8}(I_1+I_2+I_3+I_4)$$

The first term represents a certain gain, since I_0 is halved; but the second term may either increase or diminish the first numerically. On the average we shall perhaps not advance beyond halving the original inconsistencies. We can, of course, proceed to a second approximation and halve them again; and to a third. And by using the invariant property of the inconsistency we can make these approximations without performing the solutions on the plates if we simply have the inconsistencies for all the plates tabulated before us. It is even easy to summarise this process of approximation algebraically and combine several steps in one. But probably it will be better in practice to watch the process numerically. The end to be attained is the reduction of the inconsistencies to small quantities, zero for choice. And the question is suggested, Is there any simple process for making them all zero *en bloc*? If we knew the correct places of the stars and had perfect photographic measures, the inconsistencies would be all zero. The measures are not perfect, but the errors of measurement may be assumed small compared with the quantities given below; and the problem is to find a set of places for the standard stars to fit them as closely as possible.

13. We may still suppose the standard stars to be arranged in exact rows and columns, four on each plate, and each one common to two plates. Let us represent a set of advantageous corrections to the adopted places of these stars by the subjoined scheme :—

$$\begin{array}{cccccc}
a_1 & a_2 & a_3 & a_4 & \cdots \\
b_1 & \boxed{\begin{array}{cc} b_2 & b_3 \\ c_2 & c_3 \end{array}} & & b_4 & \cdots \\
c_1 & & & c_4 & \cdots \\
d_1 & d_2 & d_3 & d_4 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots
\end{array}$$

so that the four stars on one particular plate, for instance, require the corrections b_2, b_3, c_2, c_3. Then if we reduce the inconsistency of every plate to zero we have a series of equations of the form

$$-b_2-c_3+b_3+c_2=4[B_2C_3] \quad . \quad . \quad . \quad (5)$$

where the symbol $[B_2C_3]$ is used to represent the inconsistency.

14. Let n be the numb ates, and therefore of such equations. There are four on each plate, and therefore altogether 4 n star images ch star occurs on two plates, there will be only er. Hence we have n equations among 2 n re any numb ways of satisfying them mption for plate we get n etic determinate.

15. Suppose we make the assumption

$$b_2+c_3+b_3+c_2=0 \quad . \quad . \quad . \quad . \quad (6)$$

that is, that the mean correction to the places on any plate is zero, or the plate-constant is undisturbed. This seems a natural assumption to try, at any rate in the first instance. Then, combining the two equations (5) and (6) for the same plate, we get

$$b_2+c_3=-2[B_2C_3] \quad (b_3+c_2)=2[B_2C_3] \quad . \quad . \quad (7)$$

that is, the inconsistency is divided equally between pairs of opposite quadrants.

16. Now if we follow the diagonal line of stars $a_1\ b_2\ c_3\ d_4$ we have a series of equations giving a_1+b_2, b_2+c_3, c_3+d_4, and so on. Hence if we know any one correction a_1, we find successively $b_2\ c_3\ d_4$ and all the rest.

17. It is to be noted that the case is not the same for the adjacent diagonals in the same sense $b_1\ c_2\ d_3$ . . . . and $a_2\ b_3\ c_4$ . . . . There are, according to the scheme adopted for the Astrographic Catalogue, no plates running with corners in this direction—no plate $c_2\ c_3\ d_3\ d_2$ for instance. In covering the sky twice over, we select two only out of four possible ways of covering it; for considering any single plate $b_2\ c_2\ c_3\ b_3$, let us characterise it by the star at the left-hand top corner, in this case b_2. We can cover the sky completely by plates b_2, b_4, b_6, ... d_2, d_4, d_6 . . . . and so on, without using systems in which $b_3\ c_2\ c_3$ are represented at all. And by the scheme adopted for the Astrographic Catalogue, the overlapping plates would be the system c_3, including plates $c_1\ c_3\ c_5$ . . . . , e_1, e_3, e_5 . . . . but not including c_2 or b_3. The systems b_3 and c_2 would overlap the others in a different way. Had these additional plates been taken, we should have had 2 n equations for inconsistency of type (5) instead of n only; and we could have determined the corrections to the star places completely from these alone, without any additional assumptions.

18. The absence of the plates of type b_1 and c_2 prevents the formation of equations for b_1+c_2, c_2+d_3, etc., but we get b_1 and c_2 from the diagonals of the other sense, $a_2\ b_1$ and $a_4\ b_3\ c_2\ d_1$, and so on. Hence if we know or assume all the a's, we can find all the other terms. Or similarly if we know or assume $a_1\ b_1\ c_1\ d_1$ . . . . we could follow the diagonals from these as starting-points and find all the quantities.

19. But it will throw light on the validity of the assumption of equations (6) if we take an actual example. We have hitherto been assuming an ideal distribution of stars, one and one only at the centre of each quadrant. In passing to an actual example, we must replace each ideal star by a number of actual stars scattered the quadrant, and for a single residual we substitute the several.

these changes will be understood without further ex- the following example closely represents an actual

instance which occurred in the usual course of reduction of plates taken at Oxford in zone $+29°$, at R.A. $4^h\ 7^m$, where the stars are few in number. The original solution for plate-constants gave residuals for some stars differing by quantities such as $0·006 = 1''·8$ and $0·009 = 2''·7$ from the residuals found from adjacent plates. A new solution for plate-constants was accordingly made by least squares without removing these anomalies; and various assumptions as to error in scale value, etc. were tried without success.

21. In the light of the considerations above advanced, the inconsistencies of this plate and of overlapping plates were calculated with the following approximate results (the precise definition of

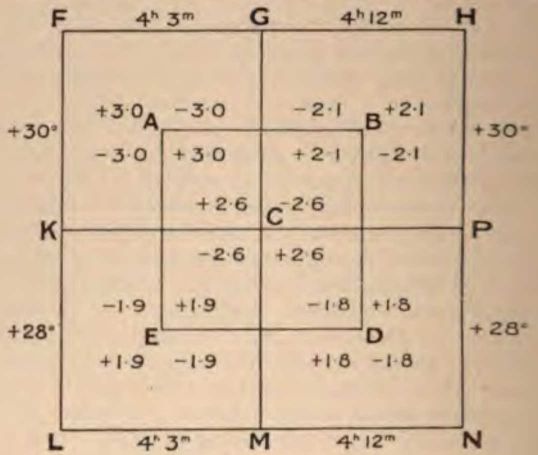


FIG. 3.

inconsistency for stars casually distributed has not yet been settled, and hence the results are provisional only):—

Zone $+30°$, For R.A. $4^h\ 3^m$ $I = -3·0$, For R.A. $4^h\ 12^m$ $I = +2·1$
Zone $+29°$, For R.A. $4^h\ 7^m$, $I = -2·6$
Zone $+28°$, For R.A. $4^h\ 3^m$ $I = +1·9$, For R.A. $4^h\ 12^m$ $I = +1·8$

The unit for I is $0''·3$ or ·001 of a réseau interval. It will be seen that the plates overlap by 4^{min} in one direction and 5^{min} in the other, and are thus not strictly divided into quarters; but let us neglect this for the present. The values for I indicate that the mean residuals (observed—adopted) in the quadrants are disposed as in fig. 3.

22. In the quadrant AC the agreement of the two plates is good but in none of the other quadrants is it at all sat
linear solution can improve it, for such solutions
and it will at once be understood why ... by l
altering the scale value by some assum

23. We must begin by *assuming* so
outside quadrants. In default of

illustration merely, let us assume *zero* corrections for the top row of quadrants FA, AG, GB, and BH.

Then the correction to AC must be $+6{\cdot}0$, in order to take up the $+3{\cdot}0$ and $+3{\cdot}0$ in the two quadrants FA, AC.

The correction to CD will be

$$+2{\cdot}6+2{\cdot}6-6{\cdot}0=-0{\cdot}8$$

and to DN $$-1{\cdot}8-1{\cdot}8+0{\cdot}8=-2{\cdot}8$$

Similarly following the diagonal HL we get corrections to

$$\begin{aligned} &HB=0\\ &BC=+2{\cdot}1+2{\cdot}1+0=+4{\cdot}2\\ &CE=-2{\cdot}6-2{\cdot}6-4{\cdot}2=-9{\cdot}4\\ &EL=+1{\cdot}9+1{\cdot}9+9{\cdot}4=+13{\cdot}2 \end{aligned}$$

The increase in this last set is alarming, and suggests a revision of the original assumptions; but we will first finish the example. The central plate is now to be revised as follows:—

We correct the adopted places of the stars in the four quadrants by the following quantities

$$\begin{matrix} +6{\cdot}0 & +4{\cdot}2\\ -9{\cdot}4 & -0{\cdot}8 \end{matrix}$$

and then solve the equations for scale values and orientation again. The effects on the constants a and b are shown in equations (1) and (2) of § 7; that on c was arranged to be zero. But what immediately concerns us is that when this new linear solution is applied, the mean residuals in the four quadrants will now be zero; for we saw in § 7 that the effect of the corrections above tabulated, taken by themselves, is to leave us with four numerically equal residuals of the value

$$\pm\tfrac{1}{4}(-9{\cdot}4+4{\cdot}2-6{\cdot}0+0{\cdot}8)=\pm 2{\cdot}6$$

and these will just neutralise the residuals shown by the original solution. Indeed, it is the basis of the method to reduce these residuals to zero. The same will be true of other plates, and hence the discrepancies between adjacent plates will disappear.

24. This will only be true in the mean, *i.e.* at the centre of each quadrant. For stars away from these points errors in a and b will still introduce discrepancies, but they will be naturally smaller. One thing at least is clear: in default of some system of correction of the kind above indicated, there are bound to be discrepancies between adjacent plates in thinly starred regions, due

r of measurement nor reduction, which cannot
change of the linear solution based on the
places. To get improvement we must alter
one another; and at first sight this seems
it more drastic than any other modifi-
—taking the mean of the residuals

25. As regards this latter process, we may note in passing how ineffective it may be in helping us to remove discrepancies between plates. Recurring to fig. 3 (p. 114) we see that the corrections suggested by taking the means from adjacent plates are as below, those suggested by observing the inconsistency being put alongside for comparison :

By "means"		By "inconsistency"	
+2·8	−0·3	+6·0	+4·2
−0·3	+0·4	−9·4	−0·8

The set obtained by "means" would leave us, after a new solution, with residuals of numerical value

$$\pm\tfrac{1}{4}(-0{\cdot}3-0{\cdot}3-2{\cdot}8-0{\cdot}4)=\pm 0{\cdot}9$$

for this plate, which is certainly less than the $\pm 2{\cdot}6$ we started with; but to get rid of the discrepancies altogether we apparently require quite a different set of corrections.

26. We return now to the assumptions made in deriving these corrections, which can obviously be improved. Let us first consider the effect of a different set of starting-points in the top row of quadrants. The diagonal H B C E L, for instance, gives us the series

$$0,\ +4{\cdot}2,\ -9{\cdot}4,\ +13{\cdot}2$$

which is becoming alarmingly large. How much of the increase is due to the assumption of zero as a starting-point? It is easily seen that if we start with $+a$, the effects on the series will be

$$+a,\ -a,\ +a,\ -a$$

and thus if we put $a=+6{\cdot}6$ we get the series

$$+6{\cdot}6,\ -2{\cdot}4,\ -2{\cdot}8,\ +6{\cdot}6$$

Thus, if the sum of one set of alternate terms in a diagonal differs from that of the other set, we can reduce the difference to zero.

27. Consider next the other set of assumptions represented by the equation (6) of § 15,

$$b_2+c_3+b_3+c_2=0$$

If we make the alternative assumption

$$b_2+c_3+b_3+c_2=4k\ . \qquad (8)$$

then combining with the equation for inconsistency

$$-b_2-c_3+b_3+c_2=4[B_2C_3]$$

we get

$$b_3+c_2=2\{[B_2C_3]+k\}$$
$$b_2+c_3=2\{-[B_2C_3]+$$

If we have started by assuming a.

b's; and equations (9) and (10)

point all the other corrections in two diagonals will be modified by the quantity k, with alternately positive and negative signs. Thus if the two diagonals originally read

$$\begin{array}{cccc} 0·0 & & & 0·0 \\ & +6·0 & +4·2 & \\ & -9·4 & -0·8 & \\ +13·2 & & & -2·8 \end{array}$$

we can add the same quantity, say $+5·1$, to both $-9·4$ and $-0·8$ provided we subtract it from $+13·2$ and $-2·8$, and add it to the terms next to these, and so on. The few terms given above will then read

$$\begin{array}{cccc} 0·0 & & & 0·0 \\ & +6·0 & +4·2 & \\ & -4·3 & +4·3 & \\ +8·1 & & & -7·9 \end{array}$$

28. It is clear that we have ample facilities for devising sets of corrections to the original places which shall make overlapping plates accordant; the question now seems to be how to limit our choice. It seems probable that in practice the limitations will arise from the necessity of satisfying known conditions for particular stars. Running down a diagonal in the manner indicated, we shall come to stars with well-determined places and known proper motions, and the proper correction to the adopted place will be known within narrow limits. Such a method cannot be discussed in general terms, but will be special in each particular case; though experience of several cases may suggest some general remarks. But it seems clear that it will serve a useful purpose to calculate the "inconsistency" of every plate, to exhibit the results in diagrammatic form, and then formulate an empirical set of corrections. This work has been put in hand at Oxford, but will naturally take a little time to complete.

29. Meanwhile, it seems desirable to publish this note for the following reason. We have formed at Oxford ledgers of the corrections to the standard stars from all the plates on which they occur, and have done much work in examining the larger discrepancies. In many cases, of course, they have been traced to mere numerical errors (comparatively seldom to errors in measurement), but a sensible number have persistently defied explanation, and yet are so large as to suggest an error. It is now realised that these are due to "inconsistency" of the plate as above, and that the time spent in looking for errors or for better plate-constants was
ime lost. Before continuing the examination of large
, we propose now to tabulate the inconsistency for each
ow many discordances can be removed by suitable
ems possible that a note of warning may save
ho may have encountered similar puzzling
rhaps a fair excuse for publishing an

30. [Paragraphs 30 to end added December 3.] The above paragraphs (except 11 and 12 which have been expanded) were circulated in proof to several astronomers, and various criticisms received. From some of these I recognise that undue prominence has been given to the methods indicated in §§ 13 to 27. It was not intended to represent them as satisfactory (this should be clear from §§ 28 and 29); but in that case they might have been curtailed.

31. Further, it is well remarked by Professor Dyson that there may be independent methods of improving the plate-constants, *e.g.* we may be able to assume that a and e, the constants for scale value, are known from other plates; while b and d, the orientation constants, strengthen one another.

32. But the main point suggested for consideration is untouched. To make the point clear, I venture to restate it as follows, modifying the statement so as to take account of § 31.

(*a*) Find by any method the best linear solution for a plate. Denote the mean residuals in the four quarters by x_1 x_2 x_3 x_4; and let

$$4\mathrm{I} = x_1 - x_2 + x_3 - x_4$$

Then I will be the same whatever linear solution is applied, or before any is applied at all; and if I is not zero, *no* linear solution will fit the *adopted* places.

(*b*) If we regard $x_1\ x_2\ x_3\ x_4$ as the proper corrections to the adopted places of the stars, then after applying them the new value of I will be zero; and we have a set of adopted places which can be fitted by a linear solution, viz. the solution already found.

(*c*) But we must take account of overlapping plates, and these do not indicate, in general, the same corrections: they give, say, x'_1, x'_2, x'_3, x'_4. Shall we then adopt as corrections

$$\tfrac{1}{2}(x_1 + x'_1),\ \tfrac{1}{2}(x_2 + x'_2),\ \tfrac{1}{2}(x_3 + x'_3),\ \tfrac{1}{2}(x_4 + x'_4)\ ?$$

This procedure has the obvious advantage that we have only one correction for each star instead of two discordant ones; but it has the serious disadvantage that the inconsistency for each plate is not made zero.

Hence the corrections cannot be regarded as final. When we apply them to the adopted places and make new solutions for each plate we shall have the same situation as at first: the residuals from overlapping plates for the same star will not agree.

(*d*) We can, of course, repeat the process: again take means of the residuals, and again find new cor
third approximation and a fourth; and w
the process is not endless. A *necessary co*
last found satisfactory places is that the v
should be zero, or at least so small that it
ways—tilt, defective measures, etc.

have no guarantee that the inconsistencies will be reduced by this method: they may even be increased.

(*e*) We can only find by actual trial whether this is or is not the case. When we have tabulated the inconsistencies, we can very quickly test the effect of taking means of overlapping plates without performing any actual solutions. If the process is convergent, we can adopt the results to which it leads just as satisfactorily as we can take means between two plates in the first instance: if the process is not convergent, the first step is as wrong as any other; and we detect places where new meridian observations are *imperative* if any advance is to be made. Needless to remark, additional meridian observations will be always welcome and helpful; but they may not be immediately forthcoming.

Pogson's Observations of U Geminorum. Edited by H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In 1904 April Mr Joseph Baxendell, of Southport, put into the writer's hands a number of MSS., notebooks and charts, representing the work of his uncle, Mr N. R. Pogson, on Variable Stars. Mr Baxendell found that he had not the leisure necessary to arrange the material for publication, and expressed the hope that the editing of Knott's and Peek's observations (*Memoirs R.A.S.*, vols. lii. and lv.) might be extended to Pogson's. A small grant from the Government Grant Fund enabled me to get the original notebooks copied out, as a safeguard against loss or injury, and the charts were photographed by Mr Bellamy and the originals deposited with the Royal Astronomical Society. But as yet I have made but little headway with the actual copy for press. The material requires more study than might be expected, some of it being very scrappy.

2. A recent announcement of a prize question on the variable U Geminorum, by the University of Utrecht, produced some inquiries for Pogson's original observations of this curious variable, and turned my thoughts in a new direction. Instead of trying to deal with the whole material at once, which would require at least several weeks' continuous attention, could the observations of individual stars be published separately? As an experiment, the observations of U Geminorum have been collected as below, and their immediate publication will in any case serve the good purpose of putting this valuable early material in the hands of those now undertaking a discussion of this remarkable star.

3. Pogson began to look for this star soon after Hind's discovery [illegible]55 December 15, and his first observation of it was dated [illegible]h 26, when he makes this note:—

[illegible]riable subject to strange fluctuations at intervals of [illegible]ds, a[illegible] to the extent of 4 mags. The [illegible] steady, not at all twitching like

the variable. The phenomenon (which was quite new to me) was watched for above half-an-hour with powers 54, 65, and 93. At times it quite vanished, and then surpassed the comparison star *a*.

On the next night, 1855 March 27, he notes:

Far from steady, but the pulsations much less marked than last night.

And he seems to have looked in vain for a repetition of the fluctuations of his first observation. Thus he notes on—

1857 April 15. A fine sky, but definition rather unsteady. U is certainly less steady than the neighbouring stars of less magnitude, but not in the marked manner previously observed.

1859 February 16. No extraordinary appearance different to neighbouring stars, except that it was rather less sharply defined than they were; the colour was a leaden white, not at all red.

With these exceptions the notes made are not of great interest. Many of them are simply "fine sky," or "moonlight," or "passing clouds"; and it seems unnecessary to encumber the record with them. Those likely to prove of value are here collected.

1857 April 7. The south 93 certainly brighter than the north.

1857 April 18. In a splendid sky, just suspected.

1857 October 30. Well compared: certainly on the rapid increase.

1864 September 28. Star 124 very little less than 113.

1866 April 16. Well seen and compared at 8 p.m. About 10½ p.m. decidedly brighter.

1868 November 17, 18. Either U or star 142 of Baxendell seen.

1870 January 17. Bright yellow; moon totally eclipsed.

4. The telescopes used by Mr Pogson at various times and places were as follows:—

At the Radcliffe Observatory, Oxford, to end of 1858.

Designation.	Aperture in inches.	Focus in inches.	
E	7·2	120	Equatorial at Radcliffe Observatory.
R	2·2	30	Ramsden portable telescope.
Dd	3·8	42	Dollond portable telescope.
SL	3·8	60	"Smythian" or "Lee": acquired from Adm. Smyth by Dr Lee of Hartwell, and lent by him to Mr Pogson in 1857 (Speculum … Taken back to … Mr Pogson and … Apparen… him. Oct. 4, 1…

At the Hartwell Observatory, 1859 Jan. 1 to 1860 Dec. 31.

SL	3·8	60	As above.
H	5·9	102	Hartwell Equatorial.
Da	8·2	...	Mr Dawes' Equatorial at Haddenham.
B	10·0	144	Mr Barclay's Equatorial at Leyton.

At Madras, 1861 Feb. 8 onwards.

L	6·0	...	Lerebours Equatorial.
D	3·5	...	Dollond telescope.
S	8·0	...	Simms Equatorial.
SL	3·8	60	Smythian or Lee.

5. Many of the observations of U Geminorum in the first ten years are records of invisibility, with estimated superior limits; and to save space, these are collected in Table I.

TABLE I.

Dates in the Years 1856–1865 *when U Geminorum was looked for without success.*

Date.	Tel. Power.	Inferred.	Date.	Tel. Power.	Inferred.
1856.			1856.		
Jan. 2	E 54	<12	Nov. 26	E 93	$<13·5$
27	R 40	<11	29	,,	$<13·5$
	Maximum here.		Dec. 1	,,	$<13·5$
			15	E 54	$<12·0$
Apr. 2	?	$<13·5$	24	R 40	$<11·0$
4	?	<12	27	E 54	$<12·7$
10	?	<12	29	E 93	$<13·5$
12	?	$<d+0·5$	1857.		
16	?	$<e+0·5$	Jan. 14	E 93	$<13·0$
20	R	$<11·0$	15	E 65	
24	E		16	,,	
May 10	,,	$<e+10$	27	,,	
13	,,	$<d$	29	,,	
20	,,	$<d$	31	,,	
22	,,	$<\begin{cases} f+1·3 \\ e+1·7 \end{cases}$	Feb. 13	,,	
		$<11·0$	14	,,	
		$<11·0$	16	,,	
			20	,,	
			23	,,	
			26	,,	

TABLE I.—*continued.*

Date.		Tel. Power.	Inferred.
1857.			
Feb.	28	E 65	
Mar.	3	,,	< *d*
	4	E 95	< *f*
	12	,,	< 13·0
	13	,,	< 13·0
	17	,,	< 13·5
	20	,,	< 13·5
		Maximum here.	
Apr.	18	E 95	< 13·5
May	5	E 65	< 11·5
Aug.	28	SL 90	< 11·5
Sept.	17	,,	< 12·0
	18	,,	< 12·6
Oct.	1	SL 50	< 11·0
	7	,,	< 11·0
	8	,,	< 10·7
	13	,,	< 12·5
	16	,,	< 12·4
	27	,,	< 12·3
		Maximum here.	
Nov.	16	SL 50	< 12·5
	18	E 95	< 13·5
	28	E 54	< 13·0
Dec.	19	,,	< 13·5
1858.			
Jan.	9	SL 50	< 12·3
	11	,,	< 12·5
	13	,,	< 12·5
	18	,,	< 12·5
	20	,,	< 12·5
	21	,,	< 12·5
	23	,,	< 12·1
	25	,,	< 11·5
	31	,,	< 11·3
Feb.	4	,,	< 12·7
	7	,,	< 12·5
	9	,,	< 12·7
	15	,,	< 12·1
	16	,,	< 12·7

Date.		Tel. Power.	Inferred.
1858.			
Feb.	19	SL 50	< 12·5
	21	,,	< 11·0
	28	,,	< 12·3
Mar.	5	,,	< 12·7
	8	,,	< 12·6
	9	E 96	< 13·5
	11	SL 50	< 12·5
	17	E 96	< 13·5
	21	SL 50	< 11·0
	22	E 54	< 11·0
	26	,,	< 11·2
Apr.	1	SL 50	< 12·0
	10	E 65	< 13·6
	13	,,	< 13·0
	15	,,	< 11·0
	17	,,	< 11·0
	23	,,	> 11·5
	30	,,	< 13·5
May	3	SL 50	< 11·5
	6	E 65	< 11·0
	7	,,	< 12·5
	11	,,	< 13·0
	19	B 41	< 11·3
Sept.	12	SL 50	< 12·5
Oct.	8	E 65	< 13·5
Nov.	2	H 50	< 13·5
	9	SL 74	< 12·5
		Maximum here.	
Dec.	2	SL 100	< 12·5
	26	SL 50	< 12·0
1859.			
Jan.	5	H 50	< 13·5
	7	,,	< 13·0
	12	,,	[illegible]
	22		
	26		
	28		
Feb.			

TABLE I.—*continued.*

Date.	Tel. Power.	Inferred.
1859.		
Mar. 2	H 118	<13·5
7	H 50	<13·0
30	SL 50	<12·5
May 3	,,	<12·5
12	SL 74	<10·7
14	,,	<10·7
16	SL 50	<11·0
21	SL 74	<11·0
23	SL 50	<11·0
Aug. 29	H 66	<12·4
Sept. 4	SL 74	<12·1
Nov. 7	H 118	<13·0
21	H 50	<11·8
28	H 50	<13·0
30	H 118	<12·9
Dec. 1	,,	<13·0
3	,,	<13·0
5	,,	<12·9
8	,,	<12·2
14	,,	<13·0
21	,,	<13·0
26	H 84	<13·1
1860.		
Jan. 6	H 84	<12·3
16	,,	<13·0
20	,,	<13·0
27	,,	<13·0
31	,,	<12·8
Feb. 9	H 66	<12·6
13	H 50	<13·0
19	,,	<13·0
27	H 66	<12·7
29	H 50	<12·7
Mar. 9	SL 52	<11·3
[illegible]	74	<12·3
[illegible]	[illegible]	<13·0
[illegible]	[illegible]	<13·0
[illegible]	[illegible]	<13·0
[illegible]	[illegible]	[illegible]·7
1860.		
	Maximum here.	
Aug. 22	H 50	<12·0
Sept. 12	H 52	<12·7
18	,,	<12·5
25	H 50	<12·5
30	SL 20	<10·5
Oct. 2	H 50	<11·0
3	H 50	<12·0
11	,,	<12·5
20	,,	<13·0
Nov. 1	H 50	<11·5
15	,,	<13·0
22	,,	<12·5
Dec. 20	B 40	<11·0
1861.		
Feb. 9	L 66	<13·0
13	,,	<13·0
14	D	<12·5
16	L 66	<13·5
28	,,	<13·0
Mar. 14	,,	<13·0
17	D	<12·0
18	,,	<12·3
22	L 66	<12·5
30	,,	<13·0
31	,,	<13·0
Apr. 1	,,	<13·0
2	,,	<13·0
5	,,	<13·0
9	D	<12·5
14	,,	<12·0
May 16	L 66	<10·5
	Maximum here.	
Oct. 10	L 62	<10·5
27	SL 50	<12·4
29	L 62	<13·0
Nov. 15	L 77	<12·5
16	L 62	<12·5
18	,,	<12·5
23	,,	<12·5

TABLE I.—*continued.*

Date.	Tel. Power.	Inferred.
1861.		
Nov. 27	L 77	< 13·0
Dec. 3	L 62	< 13·2
6	SL 50	< 12·7
9	L 77	< 13·2
14	L 62	< 12·8
17	,,	< 12·8
23	L 77	< 13·1
	Maximum here.	
1862.		
Jan. 30	L 66	< 13·0
Feb. 8	SL 50	< 12·0
Mar. 22	,,	< 12·5
27	L 66	< 13·0
Apr. 4	,,	< 13·0
9	,,	< 12·5
17	,,	< 12·9
23	,,	< 12·5
24	,,	< 13·0
May 1	,,	< 13·0
20	,,	< 13·0
23	,,	< 13·0
Oct. 6	,,	< 12·7
16	L 62	< 12·7
Nov. 4	SL 74	< 11·5
9	SL 52	< 11·0
10	L 66	< 11·7
25	,,	< 13·0
Dec. 31	,,	< 12·8
1863.		
Jan. 6	,,	< 12·5
12	,,	< 13·0
21	,,	< 13·0
Feb. 6	,,	< 12·8
12	,,	< 13·0
22	,,	< 13·0
Mar. 7	,,	< 12·7
12	[illegible]	[illegible]
23	[illegible]	[illegible]
	Maxi[illegible]	
May 5	[illegible]	[illegible]

Date.	Tel. Power.	Inferred.
1863.		
Oct. 7	SL 50	< 12·5
28	L 66	< 12·5
Nov. 11	,,	< 13·0
22	,,	< 12·8
26	,,	< 12·5
Dec. 7	,,	< 13·2
22	,,	< 12·7
	Maximum here.	
1864.		
Jan. 2	L 164	< 13·5
25	L 63	< 12·5
Feb. 15 to	,,	< 13·0
29	,,	< 12·9
Mar. 5	,,	< 12·8
May 25	,,	< 13·0
	Maximum here.	
Oct. 4	L 164	< 13·0
Dec. 6	L 63	< 13·0
	Maximum here.	
1865.		
Jan. 14	L 63	< 12·8
25	,,	< 13·1
Feb. 4	SL 52	< 12·3
14	SL 74	< 12·0
25	SL 52	< 12·5
Mar. 2	,,	< 12·4
19	L 63	< 12·7
21	,,	< 13·0
24	,,	< 13·0
	Maximum here.	
Sept. 26	L 63	< 12·7
30	L 70	< 13·0
Oct. 10	L 63	< 13·0
?	,,	< 13·0
Nov. 4	,,	< 12·7
10	L 164	< 13·0
30	L 63	< 12·8
Dec. ?	,,	< 13
14	,,	< [illegible]
[illegible]	L	[illegible]

6. *Comparison Stars.*—Pogson kept a small MS. book labelled "Comparison Stars for Variables," in which usually the upper half of the page is devoted to those for one star and the lower half to those of another. Thus p. 17 is shared by R Cancri and V Piscium. But the whole of the opposite page (16) is devoted to the stars for U Geminorum, and some of the records are squeezed in margins, having obviously been added. The stars first adopted were *a*, *b*, *d*, *e*, *f*, *g*; then *c*, *h*, *e*, *l*, *m* were added in the margins, and finally new measures of *e*, *f*, *g*, *h* and the star *n* were squeezed in. No dates are given of the separate observations. When ten measures of any star had been secured, the mean was taken, and the star was thenceforward denoted by this magnitude, omitting the decimal point. Thus star *a* is often called 88, even in the original notebooks; and where it is called *a* in the notebooks it is copied out as 88. Sometimes the adopted magnitude of a star has been changed, and thus the same star is designated by different numbers at different times. Thus the magnitudes of *b* and *c* were at first taken as 9·3, and both stars designated by this number. Later the revised magnitudes were found $b=92$, $c=94$; and ultimately they were again changed to 93. Hence *the record of the original notebook is given in Table III. whenever possible*; but in some cases a doubt is inevitable. The original notebooks are not available before 1859. Search has been made for them, by the kind permission of the Radcliffe observer, at the Radcliffe Observatory, but so far without success.

There are some curious corrections and deletions, notably in the cases of stars *e*, *g*, and *h*, where ten large readings have been systematically struck out; but there seems to be no reason to restore them against Pogson's own judgment, though they are added at the end of the table. Those for *e* all come between the first and second series retained: those for *g* and *h* are interspersed among the separate members of the first set of ten retained.

[Table II.

TABLE II.

Pogson's Observations of Comparison Stars.

	a		*b*		*c*		*d*		*e*		*f*		
	9·0	9·0	9·3	9·5	9·5	9·5	10·2	10·2	11·1	10·8	11·4	11·3	[illegible]
	8·9	8·7	9·4	9·2	9·5	9·2	10·2	10·4	10·8	10·8	11·0	11·3	[illegible]
	8·7	9·0	9·1	9·6	9·0	9·5	10·2	10·4	11·0	10·7	11·3	11·2	[illegible]
	8·9	8·7	9·3	9·0	9·4	9·3	10·4	10·5	10·9	10·8	11·4	11·4	[illegible]
	9·0	8·8	9·1	9·2	9·5	9·5	10·5	10·3	10·6	10·9	11·0	11·2	[illegible]
	8·6	8·8	9·2	9·2	9·3	9·2	10·1	10·6	10·5	11·0	11·0	11·3	[illegible]
	8·6	8·7	9·2	9·2	9·3	9·1	10·3	10·5	10·9	10·6	11·3	11·5	[illegible]
	9·0	8·7	9·1	9·1	9·5	9·0	10·2	10·3	11·0	10·8	11·6	11·4	[illegible]
	8·7	8·5	9·2	9·4	9·2	9·4	10·1	10·5	10·7	10·6	11·2	11·2	[illegible]
	9·0	8·5	9·5	9·4	9·5	9·5	10·4	10·4	10·9	10·7	11·3	11·4	[illegible]
Means	8·84	8·74	9·24	9·28	9·37	9·32	10·26	10·41	10·84	10·77	11·25	11·32	[illegible]
Adopted	8·8		9·3		9·3		10·3		10·8		11·3		

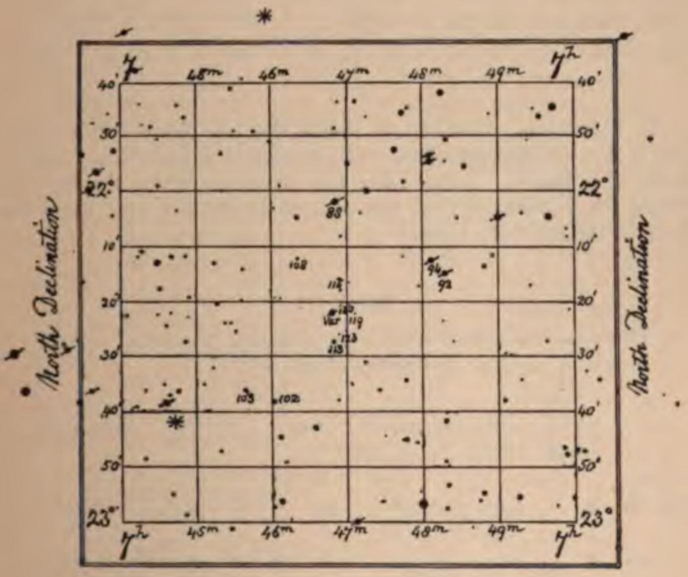
	g		*h*		*l*	*m*	*n*	Readings struck out *e*	*g*	[illegible]
	12·0	11·7	12·3	11·8	13·0	11·2	10·0	11·1	12·5	[illegible]
	12·0	11·6	12·2	12·5	13·0	11·3	10·1	10·8	12·1	[illegible]
	12·0	11·5	12·5	12·0	13·0	11·3	10·3	10·9	12·3	[illegible]
	12·0	12·0	12·2	12·4	13·0	11·5	10·2	11·1	12·2	[illegible]
	12·0	11·9	12·4	12·2	13·1	11·2	10·2	11·5	12·2	[illegible]
	12·0	11·9	12·0	12·3	13·1	11·3	10·3	11·1	12·3	[illegible]
	11·7	11·9	12·0	12·3	13·0	11·5	10·1	11·2	12·3	[illegible]
	12·0	12·0	12·4	12·5	12·9	11·4	10·2	11·0	12·6	[illegible]
	11·9	11·8	12·1	12·1	13·0	11·4	10·2	11·1	12·1	[illegible]
	12·0	11·9	12·3	12·5	12·9	11·3	10·1	11·1	12·5	[illegible]
Means	11·96	11·82	12·24	12·26	13·00	11·34	10·[illegible]	[illegible]	[illegible]	[illegible]
Adopted	11·9		12·3		13·0	11·3	[illegible]	[illegible]	[illegible]	[illegible]

7. From examination of Table II. the following identifications are suggested for the numbers by which Pogson indicates stars :—

88 or 89 = *a*; 92 = *b*; 93 = *b* or *c*;
94 = *c*; 102 = *n*; 103 = *d*; 108, 110, or 111 = *e*;
112 = *f*; 113 = *f* or *m*; 115 = *m*; 119, 120, or 121 = *g*;
123, 124, or 126 = *h*; 130 or 131 = *l*; 137 = ?

Pogson's diagram is reproduced with this paper, and the approximate positions given below are read from it. On reference to Mr Knott's diagram on p. 94 of *Memoirs R.A.S.*, vol. lii., it will be seen that he used practically the same stars, and a comparative table is given below.

U Geminorum. Var. 5.—



Range: 9th to below 14th Magnitude.
Period. 97 Days.

Pogson.				Knott.			
Diagram.							
No.	R.A.	Dec.	Letter.	Letter.	Mag.	R.A.	Dec.
	h m s	° ′				h m s	° ′
Var.	7 46 48	22 22	U	U	...	7 47 5	22 20
88	46 49	22 2	*a*	*a*	8·6	7 47 18	22 1
92	48 19	22 15	*b*	*c*	9·2	7 48 31	22 13
94	48 8	22 13	*c*	*b*	9·3	7 48 25	22 11
103	45 38	22 37	*d*	*d*	10·3	7 45 53	22 34
108	46 22	22 13	*e*	*e*	10·6	7 46 38	22 10
113	46 48	22 28	*f*	*f*	11·2	7 47 3	22 26
119	47 6	22 23	*g*	*g*	12·3	7 47 22	22 20
123	46 53	22 27	*h*	*h*	12·3	7 47 12	22 24
130	46 53	22 21	*l*	*k*	13·3	7 47 10	22 18
113	46 52	22 17	*m*	...	...	...	...
102	46 2	22 38	*n*	...	...	...	...
137	?	?	*k*?	*l*	13·7	7 47 0	22 22

8. This being premised, it seems the best course now to give simply *the original notebook record*, and this has been done in Table III. below. After 1865 the occasions when the star was looked for without success are comparatively few, and have been given in the ordinary form.

Table III.

Observations of U Geminorum near maximum 1856–1865; *and all observations* 1866–1881.

Date.	Tel.	Power.	Comparisons.
1856.			
Mar. 26	E	?	Est. 9·7 = 88 + 7 = 93 + 3 = 93 + 2
27	E	54	88 + 13 = 93 + 8 = 93 + 6 = 113 − 6
29	E	54	Est. 11·0 = 93 + 17
1857.			
Apr. 7	E	95	88 + 8 = 93 + 5 = 103 − 6
8	E	54	88 + 8 = 93 + 5 = 103 − 5
11	E	54	88 + 6 = 93 + 3 = 103 − 7
1857.			
Apr. 14	E	54	103 + 7 = 110 − 4 = 113 − 16
15	E	95	110 + 3 = 113 + 2 = 121 − 7 = 124 − 10
16	E	95	121 = 124 − 3
18	E	95	Invis. = < 13·5
Oct. 30	SL	50	9·7 ...
1858.			
Nov. 17	SL	50	93 + 5 = 103 − 3
18	SL	50	103 + 7 = 110 − 3 = 113 − 5

TABLE III.—*continued.*

Date.		Tel.	Power.	Comparisons.
1859.				
Feb.	16	H	50	$a+6=b=c-2$ $=d-8$
	17	SL	85	$a+3=b-2=c-2$
	22	H	50	$a+2=b-3=c$
	23	H	50	$b+3=c+1=d-7$
	24	H	50	$b+6=c+5=d-4$
	25	H	50	$c+15=d+6=f-4$
	27	H	118	$f+10=g-1=h-4$
1860.				
Apr.	22	SL	50	$88+2=93-3$
	24	SL	50	$a+3=b+2=c-2$
	26	Da	66	$88+6=93$
	27	SL	50	$88+10=93+5$
	28	SL	50	$93+8$
	30	H	52	$103+7=110-2$
May	1	H	52	$113+5=121-4$
	2	Da	?	$121+4=130-7$
1861.				
May	2	L	66	$88+1=92-2$
	3	L	66	$88+7=92+5$ $=103-13$
	4	L	66	$88+4=92$
	5	L	66	$88+4=93$
	6	L	66	$88+8=93+6$
	7	L	66	$88+7=92+5$
	8	L	66	$88+5=93+2$
	9	L	66	$88+9=93+6$
	10	L	66	$88+11=93+8$ $=92+3$
	12	L	66	$88+15=92+13$ $=93+7=103-7$
	14	L	66	$103+3=110-5$
	16	L	66	Invis. $=<10{\cdot}5$
Dec.	[illegible]	L	[illegible]7	Invis. $=<13{\cdot}1$
[illegible]				[illegible]$+2=110-5$
				[illegible]$93-2$

Date.		Tel.	Power.	Comparisons.
1862.				
Jan.	2	SL	50	$88+2=93-3$
	4	SL	50	$88+5=93-1$
	5	SL	50	$88+7=93+1$ $=103-5$
	6	SL	50	$88+10=93+3$ $=103-6$
	8	L	62	$93+6=103-2$
	10	SL	50	$103+2=110-3$
	11	SL	50	$110-1=113-4$
	18	L	77	$130+2$
1863.				
Apr.	8	L	66	$94+2=103-8$
	9	L	66	$93+7=103-4$
	10	L	66	$94+6=103-7$
	11	L	66	$88+12=92+8$ $=103-4$
	12	L	66	$94+11=103-1$ $=110-9$
	13	L	66	$103+4=110-2$
	14	L	66	$110+6=113+2$ $=124-6$
	15	L	66	$124+8=130-2$
	17	L	164	$137+1$
Dec.	31	L	66	$124+2=130-4$
1864.				
Sept.	27	L	164	$110+6=112+2$ $=121-5$
	28	L	164	$121-1=124-5$ $=113+7$
	30	L	164	$130-1$
Oct.	4	L	164	Invis. $=<13{\cdot}1$
1865.				
Jan.	9*	L	63	$103+1=110-7$
	14*	L	63	Invis. $=<12{\cdot}8$

[illegible]ok is written "Looked in vain for U [illegible]), but the moon was too bright to see [illegible]rs." Is not observation of Jan. 9

TABLE III.—*continued.*

Date.	Tel.	Power.	Comparisons.
1865.			
Apr. 18	L	70	130 − 2
20	L	70	94 + 2 = 103 − 8
21	L	63	94 + 6 = 103 − 5
22	L	63	94 + 9 = 103 − 4
23	L	63	94 + 14 = 103 + 1 = 110 − 4
25	L	63	124 + 5 = 130 − 2
26	L	63	130 + 5
1866.			
Jan. 4	L	63	Invis. = < 12·5
14	L	70	Invis. = < 13·0
17	L	63	88 + 5 = 94 − 1 = 92 + 2
18	L	63	94 + 2 = 103 − 10
19	L	63	88 + 5 = 92 + 2 = 94 − 1
20	L	63	92 + 3 = 94 + 1 = 103 − 11
21	L	63	94 + 3 = 103 − 8
22	L	63	94 + 3 = 103 − 8
23	L	63	94 + 4 = 103 − 8
24	L	63	94 + 5 = 103 − 7
25	L	63	94 + 5 = 103 − 8
26	L	63	94 + 5 = 103 − 6
27	L	63	94 + 7 = 103 − 4
28	L	70	94 + 7 = 103 − 3
31	L	70	124 + 6 = 131 − 2
Feb. 1	L	164	Invis. = < 13·0
2	L	164	130 + 3 = 137 − 3
Mar. 21	L	63	Invis. = < 12·5
Apr. 6	L	63	Invis. = < 13·0
{ 16	L	70	*h* + 8 = 130 − 4
{ 16	L	70	*h* + 3 = 130 − 7
17	L	63	94 + 4 = 103 − 5
18	L	63	94 + 5 = 103 − 5
19	L	63	94 + 7 = 103 − 3 = 108 − 10
20	L	63	94 + 11 = 103 + 2 = 108 − 4
21	L	70	108 + 5 = 112 + 2 = 115 − 3
1866.			
Apr. 22	L	70	124 + 4
23	L	164	Invis. = < 12·8
May 14	63	63	Invis. = < 12·8
Nov. 3	S	45	Invis. = < 13·0
Dec. 25	S	86	Invis. = < 13·5
1867.			
Jan. 17	L	63	Invis. = < 13·0
23	L	63	Invis. = < 13·0
Mar. 8	S	86	Invis. = < 13·5
Oct. 26	L	63	Invis. = < 13·0
Dec. 14	S	86	88 + 10 = 93 + 5 = 103 − 6
15	S	40	93 + 5 = 103 − 8 = 88 + 10
16	S	40	93 + 7 = 103 − 3
17	S	86	*n* − 1 = 103 − 2
18	S	86	108 = *m* − 5
19	S	86	113 + 3 = 112 + 1 = 119 − 7
20	S	86	*h* + 1 = 130 − 6
21	S	86	130 + 1
22	S	86	130 + 5 = 137 − 1
23	S	163	equal 137
1868.			
Jan. 21	S	86	Invis. = < 13·7
Mar. 11	S	86	Invis. = < 13·0
Apr. 9	S	86	Invis. = < 13·7
18	S	86	Invis. = < 13·5
Nov. 17	S	105	137 + 3
18	S	86	137 + 3
Dec. 9	S	86	Invis. = < 13·5
1869.			
Jan. 6	L	66	Invis. = < 13·0
Feb. 20	S	86	93 + 4 = [illegible]
21	S	86	93 [illegible]
23	S	86	9[illegible]
24	S	86	[illegible]
27	S	1[illegible]	[illegible]
Oct. 30	S	[illegible]	[illegible]

TABLE III.—*continued.*

Date.	Tel.	Power.	Comparisons.
1870.			
Jan. 17	S	86	94+2=103−5
19	S	86	94+4=103−7
22	S	86	94+5=103−4
23	S	86	103−1
25	S	86	103+7=108+4 =113−2
26	S	86	119+3=123
27	S	86	130+1
Nov. 24	S	86	Invis.=<13·7
1872.			
Jan. 17	S	105	Invis.=<13·5
22	L	66	Invis.=<12·5
27	L	166	Invis.=<12·7
Feb. 1	L	166	Invis.=<13·7
6	L	66	Invis.=<13·5
11	L	66	Invis.=<13·7
16	S	...	Invis.=<14·0
21	S	105	93+15=103+2 =113−9
22	S	45	89+5=93
23	S	45	89+3=92
24	S	45	89+3=92−2 =94−4
25	S	45	89+5=92+2 =94
26	S	45	92+4=94+2 =103−9
27	S	45	92+3=94+2 =103−8
28	S	45	92+3=94+4 =103−8
29	S	45	92+6=94+4 =103−7
Mar. 1	S	45	92+7=94+6 =103−6
2	S	45	92+8=94+7 =103−4
3	S	45	92+12=94+10 =103−3
4	S	45	93+12=103−2 =108−6
5	S	45	103+2=108−4
6	S	90	108+4=*113+1*
1872.			
Mar. 7	S	90	123+3=130−6
8	S	90	130+2=138−6
9	S	90	=138
10	S	90	Suspected=<14·0
12	L	63	Invis.=<13·0
1873.			
Feb. 15	L	53	Invis.=<12·5
Mar. 27	L	53	Invis.=<12·7
1874.			
Jan. 14	L	106	Invis.=<13·0
Feb. 18	L	63	Invis.=<13·0
1875.			
Jan. 15	L	63	88+8=94+4 =102−7
16	L	63	88+10=94+4 =103−7
18	S	73	88+10=94+5 =102−4
19	L	63	88+12=94+5 =102−4
20	S	73	94+6=102−3
22	S	73	94+7=102−3
24	S	73	108+1=113−3
26	S	102	123+6=130−2
27	S	102	130+2=137−7
Feb. 1	S	99	130+7=137−2
?	S	120	<14·5 (suspected)
Dec. 18	L	67	Invis.=<13·0
1876.			
Jan. 13	S	89	Invis.=<13·0
1877.			
Jan. 4	L	61	Invis.=<12·7
Mar. 11	S	73	Invis.=<13·5
May 9	L	61	Invis.=<13·0
Oct. 13	S	73	Invis.=<13·0
1880.			
Jan. 2	S	72	Invis.=<13·0
1881.			
Mar. 23	L	65	Invis.=<12·7

Stellar Parallax Papers, No. 3.

The Parallax of Eight Stars, from Photographs taken at the Cambridge Observatory by Arthur R. Hinks, M.A., and the writer. By Henry Norris Russell, Ph.D.

The following results are derived from plates taken at Cambridge by Mr A. R. Hinks and the writer, and measured and discussed by the latter in the course of his work as a research assistant of the Carnegie Institution. A full description of the methods of observation and reduction is given on pp. 775–800 of the *Monthly Notices* for June 1905.

Table I. gives the relative parallax of these stars with respect to comparison stars averaging about the 9th magnitude. The last column but one gives the number of comparison stars for each series, and the preceding column the number of plates in the series. The same comparison stars were used for Nos. 2 and 3, which appear on the same plates, and similarly for Nos. 7 and 8. The two bright stars β and η Cassiopeiæ were photographed with a colour-screen, which reduced their photographic brightness by about $5\frac{1}{2}$ magnitudes.

TABLE I.

Ref. No.	Star.	R.A. 1900.0	Dec. 1900.0	Mag.	P.M.	Parallax.	Plates.	Comp. Stars.	
		h. m.	° ′		″	″			
1	β Cassiopeiæ	0 3·8	+58 26	2·4	0·55	+0·082±0·009	5	9	±
2	Groombridge 34	0 12·6	+43 27	7·9	2·82	+0·250±0·012	6	9	±
3	26 Andromedæ	0 13·5	+43 15	5·9	0·03	−0·026±0·041	6	9	±
4	η Cassiopeiæ	0 42·9	+57 18	3·6	1·20	+0·188±0·021	7	8	±
5	o Ceti	2 14·3	− 3 26	Var.	0·24	+0·136±0·035	7	9	±
6	Lalande 25372	13 40·7	+15 27	8·5	2·32	+0·221±0·019	8	9	±
7	Berlin B 5072	14 21·1	+24 6	9·0	1·42	+0·067±0·040	7	7	±
8	Berlin B 5073	14 21·1	+24 6	9·1	1·42	+0·000±0·029	7	7	±

The last column gives the probable error of a co-ordinate of each star, resulting from one plate, as derived from the least-square solution for the parallax. If we divide the stars into three classes according to their effective photographic brightness, we have three stars with faint images, Nos. 5, 7, and 8, four of moderate brightness, Nos. 1, 2, 4, and 6, and one star, No. 5, whose images are somewhat large and diffuse. The average values of the probable error of one plate for these three groups are $\pm 0''\cdot065$ for the faint stars, $\pm 0''\cdot027$ for those of [illegible] ness, and $\pm 0''\cdot069$ for the bright star.

This emphasises the importance of proper [illegible] wish to secure highly accurate plates. It w[illegible] if the exposures for the three faint stars [illegible] usual length (five minutes), had been l[illegible]

Stars brighter than the 6th magnitude were, as a rule, excluded from our working list, unless observed with a colour-screen. The large probable error for No. 3 (which was only measured because it happened to be on the plates of No. 2) justifies this course, while the two good results obtained with the colour-screen show that it affords a satisfactory way of avoiding the over-exposure difficulty.

Several of these stars have already been observed for parallax. The results, so far as known to the writer, are as follows:—

β Cassiopeiæ.

$\pi = +0''\cdot15 \pm 0''\cdot02$	Pritchard, photography.
$\pi = +0\cdot14 \pm 0\cdot03$	Kostinsky, absolute declinations.
$\pi = +0\cdot10 \pm 0\cdot03$	Flint, meridian transits.

Groombridge 34.

$\pi = +0\cdot29 \pm 0\cdot025$	Auwers, diff. of R.A., micrometer.
$\pi = +0\cdot31 \pm 0\cdot034$	Flint, transits.

η Cassiopeiæ.

$\pi = +0\cdot10 \pm 0\cdot05$	from distances	O. Struve,
$\pi = +0\cdot37 \pm 0\cdot10$	,, pos. angles	filar micrometer.
$\pi = +0\cdot20 \pm 0\cdot06$	from distances	Schweizer,
$\pi = +0\cdot14 \pm 0\cdot08$	,, pos. angles	filar micrometer.
$\pi = +0\cdot44 \pm 0\cdot04$	Davis, from Rutherfurd photographs.	
$\pi = +0\cdot34 \pm 0\cdot04$	Flint, transits.	
$\pi = +0\cdot18 \pm 0\cdot010$	Peters, heliometer.	

Lalande 25372.

$\pi = +0\cdot43 \pm 0\cdot065$	Flint, transits.
$\pi = +0\cdot17 \pm 0\cdot043$	Elkin, heliometer.

Flint's results have received large corrections for systematic personal error, and the Rutherfurd photographs of η Cassiopeiæ, being taken at widely different hour angles, are affected to a considerable but unknown degree by atmospheric dispersion. If we reject the last, give Flint's values half weight, and the heliometer results double weight, we obtain mean values for the parallax of the four stars, from which the results of the present paper differ by $0''\cdot05$, $0''\cdot05$, $0''\cdot00$ and $0''\cdot00$ respectively.

If we assume that our results are responsible for two-thirds of the discrepancies, their average probable error would be $\pm 0''\cdot020$, including the effects of any systematic error. It would therefore the latter must be very small.

ion of the residuals for the comparison stars shows evidence of parallax or proper motion except for 9·2 which appears on the plates of No. 2, and oper motion of about $+0''\cdot3$ in x. By parallax of this star should be about

0″·029, while the average parallax of stars of the ninth magnitude is 0″·006.

We may therefore assume that the mean parallax of our comparison stars is 0″·009 for No. 2, and 0″·006 for the rest of our series. Adding this to the relative parallaxes of Table I., we obtain from the resulting absolute parallaxes the following values of the absolute magnitudes of the stars (*i.e.* their magnitudes if at such a distance that their parallax was 0″·10) and of their light in terms of the Sun's, and their velocities at right angles to the line of sight.

Groombridge 34 and η Cassiopeiæ are double, and data are given for both components, and for Mira at maximum (mag. 3·5) and minimum (9·5).

TABLE II.

No.	Star.		Absolute Mag.	Light.	Cross-velocity. Ast. units per year.	Cross-velocity. Kilometres per sec.
1	β Cassiopeiæ		2·1	23·	6·2	29
2	Groombridge 34	A	10·0	0·016	10·9	52
		B	12·6	0·0015		
4	η Cassiopeiæ	A	5·1	1·5	6·2	29
		B	9·1	0·04		
5	ο Ceti	Max.	4·3	3·0	1·7	8
		Min.	10·3	0·011		
6	Lalande 25372		10·3	0·011	10·2	49
7	Berlin B 5072		(7·0)	(0·25)	(35)	(170)
8	Berlin B 5073		(7·1)	(0·23)		

Nos. 7 and 8, which are 45″ apart, have a common proper motion,* and are no doubt physically connected. The mean of the observed parallaxes, $+0''\cdot033 \pm 0''\cdot025$, has therefore been taken as the true value for both stars, but it is clear that little reliance can be placed upon the numbers calculated from it. It is, however, probable that these stars are fainter than the Sun, and are moving across the line of sight faster than any of the others in the table.

The negative result for 26 Andromedæ is less than its probable error, and indicates that the parallax of this star is insensible. It was measured because the Bonn A. G. catalogue gives its proper motion in R.A. as $+0^{s}\cdot016$, which is ten times the true value. If its actual cross-velocity is equal to the mean of that of the other stars (excluding Nos. 7 and 8), its parallax would be only 0″·004. It is double (OΣ 5), and its 10th magnitude companion
fixed in 240°, 6″·1—which should be the case if its
small.

η Cassiopeiæ is a well-known binary. Fro

* Bellamy, *M. N.*, Dec. 1899, p. 1:

† *Mem. R.A.S.*, vol. lvi. p. 16.

we have $a = 8''.51$ P=233 years, which with our parallax gives the mean distance of the components as 44 astronomical units. The actual distance varies from 58 to 30, and the combined mass is 1·6 times that of the Sun. According to Lewis, the bright star is twice as massive as the fainter one, so that it very nearly equals the Sun in mass as well as in light.

Groombridge 34 has a physical companion, which was measured by Auwers* in 1865, and is shown on some of the plates of the present series, as is also a fainter companion nearer the principal star.

The writer's measures, made this month with the 23-inch equatorial of the Halsted Observatory at Princeton, show that this is only an optical companion. They give, for the co-ordinates of the companions, relative to the principal star,

		B $10^m.5$		C $11^m.5$	
		x	y	x	y
Auwers	1866·23	$+31''.2$	$+23''.4$		
Plate 391	1904·98	+32·3	+21·6	$+32''.0$	$-13''.3$
Halsted Obs.	1906·78	+32·2	+21·5	+26·6	−14·2
	Proper motion of A in interval			+ 5·1	+ 0·7

This companion affords an unusually good opportunity for the micrometric determination of the parallax of the other two stars.

The actual separation of A B is at least 150 astronomical units —four times that of η Cassiopeiæ—so that we may expect the period of the system to be some thousands of years.

The results for Mira are also of interest. If the other long-period variables resemble it, and their maximum rather than their minimum light is comparable with the Sun's, the brighter ones must have easily measurable parallaxes, and it would pay to observe them.

Campbell and Stebbins † find that the radial velocity of Mira is constant, and equal to +63 km. per sec. As its cross-velocity is only 8 km., it is moving almost directly away from us—in a direction making an angle of only 7° with the line of sight.

If the present value of the parallax is correct, it follows that Mira was nearest the Sun about 110,000 years ago. It was then in Ursa Major, and had a parallax of $1''.1$ and a proper motion of $15''$ per annum. If its intrinsic brightness varied between the same limits as at present, it was of the 5th magnitude at minimum, and at maximum was as bright as Sirius.

[...]ks are due to Professor Lovett of Princeton for [...] to continue the work here.

* [...]*rlin Akad. der Wissenschaften*, 1867, p. 23.
† [...]*urnal*, vol. 18, p. 341.

[...]*ry*:

Hansteen's Eclipse at Stiklastad, 1030 *August* 31.

By P. H. Cowell.

The record states that the eclipse occurred during the battle at Stiklastad at which Olaf the Fat was killed, and the date assigned to the battle is 1030 July 29. There are two reasons for supposing this date to be in error: the narrative mentions a dark night following the battle, whereas on July 29 the sun never sinks as much as 9° below the horizon of Stiklastad; also, Olaf was canonised, and July 29 became his festival and in time the supposed day of his death. Now August 31 was already at that time assigned to another saint. The foregoing arguments are Hansteen's (*Ast. Nach. Ergänzungsheft*, p. 49). Dr Dreyer (*The Observatory*, 1895 October, p. 363), quoting Prof. Konrad Maurer, decides against the later date for the battle. His arguments, as far as they are based on the day of the week, do not appear to me conclusive, as the chronicler may easily have counted backwards. But in any case we have a description of a total eclipse, that it was beyond the power of the chronicler to invent, and the eclipse is stated to have occurred at Stiklastad.

The eclipse has therefore been worked up. Considering its comparatively recent epoch, it might have turned out not to discriminate between the present tables and my formulæ. The very contrary is the case. My formulæ leave 3″ of the northern end of the Sun's diameter uncovered (indicating the possibility of small errors that I am quite prepared to admit). Hansen's tables and the present tables, which are practically the same for the epoch 1030, shift the Moon 35″ further south relatively to the Sun. The low altitude of the Sun makes this correspond to about 100 miles on the Earth's surface.

Outline of Calculations.

$$T = -7{\cdot}69303 = \quad 1030 \text{ Aug. } 31 + 28^{\circ}{\cdot}53 \text{ G.M.T.}$$

$$\begin{array}{rl} g = 286^{\circ}\ 12'\ 41''{\cdot}8 & \qquad L' = 165^{\circ}\ 0'\ 9''{\cdot}8 \\ \omega = 88\ 35\ 37{\cdot}2 & \qquad \pi' = 266\ 18\ 7{\cdot}1 \\ -\Omega = 207\ 10\ 12{\cdot}3 & \end{array}$$

Inequalities of Moon's Longitude :—

A. Solar terms over 20″		−	16 752″·7
B.	,,	1·35 −	69·8
C.	,,	0·15 −	5·0
Figure of Earth terms		+	4·9

Moon's Latitude and Sine Parallax :—

A. Solar terms over 10″		+	3 089″·5	+	3 500
B.	,,	0·55 +	27·0	+	[illegible]
C.	,,	0·05 −	1·0	−	[illegible]
Figure of Earth terms		−	3·9		

Movements in Julian century $\div 10^6$:—

In geocentric elongation in longitude	$1681''\cdot 6$
In geocentric latitude	$165\cdot 3$

Stiklastad $11^\circ\ 35'$ E; $63^\circ\ 38'\cdot 8$ geocentric latitude; $1-\rho = 0\cdot 00268$

Parallax in longitude $+210''\cdot 7$; in latitude $-3122''\cdot 8$

Movements in Julian century $\div 10^6$ as seen from Stiklastad:—

In elongation in longitude	$1480''\cdot 3$
In latitude	$44\cdot 5$

Hence if $0\cdot 09\Delta F$, ΔD are the corrections required by the latitude and difference of longitudes, then the latitude at apparent conjunction in longitude is

$$-8'' + 0\cdot 09\Delta F - 0\cdot 03\Delta D$$

The difference of apparent semi-diameters is about $5''$.

Note on the Approaching Return of Halley's Comet.
By A. C. D. Crommelin.

It is well known that Dr A. J. Ångström published in 1862 a paper entitled "Sur deux inégalités d'une grandeur remarquable dans les apparitions de la comète de Halley" (Actes de la Société royale des Sciences d'Upsal, Sér. III., t. iv.). In this paper he discusses the observed perihelion passages, as determined by Dr Hind, from B.C. 11 to A.D. 1835, and deduces the mean period of the comet, 76·93 years, this period being affected by two large inequalities of amplitudes 1·5 years, 2·3 years, periods 2650 years, 782 years. He has found theoretical arguments which will satisfy these periods, viz. 13 ☄ - 2 ♃, ♃ + ♄ − 9 ☄, where the symbols denote the mean annual motions of comet, Jupiter, Saturn. The amplitudes have been obtained by observation, and in no case does the error in the formula as compared with the observed time of perihelion exceed 1 year; in most cases it is less than half a year. Dr Ångström does not claim that these two are the only inequalities, merely that they are the most important ones.

I do not think it is so generally known that the time of the next perihelion passage, as deduced from Dr Ångström's curve, is altogether different from that published by Count de Pontécoulant. I deduce from the curve 1913·08 for the time of the next passage, whereas that given by Count de Pontécoulant is 1910·37, a discordance of 2·7 years.

We are not, of course, justified in assuming that M. Pontécoulant's

result is erroneous, solely because it differs from Dr Ångström's curve. For exact numerical computation is entitled to far more weight than an empirical method. But it is difficult not to feel some slight uneasiness about the matter, when we consider the extreme length and intricacy of the calculations, and the possibility that some important numerical error may have escaped detection. It is surely a *desideratum* that the perturbations should be independently computed, and I hope that some mathematicians may be actually engaged in the work; if not, there is still time for someone with the necessary leisure and ability to undertake it. Before the 1835 return there were at least five independent computations of the orbit—those of Damoiseau, Pontécoulant, Lehmann, Rosenberger and Lubbock; and it is difficult to understand why an equal amount of interest is not shown in the approaching return. Pontécoulant's result was published in 1864, and doubtless he regarded it as certain that there would be numerous investigations when the time drew nearer, so that he may well have given somewhat less attention to the next return than he gave to that of 1835. This is borne out by the fact that there are certainly some slips or misprints (not having seen his manuscript, I cannot say which) in his paper as printed in *Comptes Rendus de l'Académie*, tome lviii. Thus on p. 826 he gives T as 1910 May 24·37, Paris civil time (I have verified that this is the correct value from the figures and formula that he quotes), but on p. 828 T is given as May 16·95; this is merely an error of copying, and is corrected in a note in a subsequent number of *Comptes Rendus* (p. 915 of the same volume). The *Connaissance des Temps*, however, quotes the latter result converted into astronomical reckoning; while Dr Hind (*Encyc. Brit.*, 9th edit., vol. vi. p. 193) quotes the former, which is unquestionably the right one to take. Another case is that of the perturbation in the eccentricity. This is made up of three terms due to Jupiter, Saturn, and Uranus, the separate terms and their sum being given as follows on p. 827:—

Planètes.		Altération de l'excentricité $\int de$.
♃		− ·00529752
♄		− ·001104850
♅		+ ·000040625
	Total,	− ·005655833

It is at once obvious that the total does not agree with the separate terms as printed, nor am I able to suggest any conjectural emendation which will bring them into harmony.

This value of the total is used in deducing the eccentricity as given in the elements for 1910, and it is noteworthy that both the eccentricity and perihelion distance for that epoch show a striking discordance from those in earlier apparitions, as this table shows.

Year.	Eccentricity.	Perihelion Distance.
1531	0·9684	0·568
1607	0·9669	0·584
1682	0·9677	0·583
1759	0·9676	0·5845
1835	0·9674	0·5866
1910	0·9617	0·6872

The change in the distance is a full tenth of the Earth's distance from the Sun. I do not assert that such a change is impossible, but it is certainly desirable that it should be verified, as I strongly suspect that there is a confusion in the position of the decimal point, and that the actual change is only $\frac{1}{10}$ of that given.

I have prepared a diagram showing the portion of the comet's orbit inside the orbit of Jupiter. The numerals I., II., III., IV., V., VI., indicate the positions of Jupiter at the instant of the comet's perihelion passage in 1531, 1607, 1682, 1759, 1835, 1910, while the same numerals with suffixes A, B indicate Jupiter's positions when the comet passed the points A and B. The distances of A, B from Jupiter's orbit are 1·0, 1·7 respectively. The

Orbits of Jupiter, Earth, Halley's Comet, showing positions of Jupiter, 1531, 1607, 1682, 1759, 1835, 1910.

numbers near the numerals I., II., etc., indicate the Jovian perturbation of the comet's mean anomaly in the revolution following that passage. It will at once be seen that the figure opposite V. is abnormally large, although the approach to Jupiter was less close than for passage III. This increases the suspicion that the perturbations for the present revolution may be too large; indeed, in the case of the perturbation in eccentricity the suspicion rises almost to a certainty.

I do not know whether Pontécoulant's calculations are still in existence; the article in *Comptes Rendus*, vol. lviii., does not mention where they are to be found. Assuming that they are still in existence, a careful re-examination of all the figures might suffice instead of an entirely independent computation of the perturbations. For there is no question whatever of Pontécoulant's ability, merely a suspicion of possible numerical slips. The inconsistency that I have adduced above is of itself sufficient to demand such a re-examination, before the results can be received with confidence. One can hardly imagine a greater loss of prestige to astronomy than that which would arise if there were a notable error in the prediction of this return of the comet, after the wonderful success achieved in 1759 and 1835.

Assuming that the date 1910 May is correct for the perihelion passage, I think the complete failure of the Ångström curve is not without a warning to us. We have here a curve which admirably fits 25 successive passages, and yet the first time it is used to predict a return it breaks down utterly, the error being almost 3 years, or three times the largest previous error. This is a decidedly startling fact, and indicates the danger of using terms of an empirical nature in lunar or planetary tables. Where such are used, it is at least desirable that means should be provided for readily effecting their removal from the calculated positions.

The Proper Motion of Castor. By A. C. D. Crommelin.

In the determinations that have hitherto been made of the proper motion of Castor, it does not appear that any effort has been made to separate true proper motion from orbital motion; the values given have merely been deduced from the meridian observations of the principal star, from the time of Bradley to the present day, on the assumption of uniform rectilinear motion. This assumption was fairly justifiable for the 130 years from 1755 to 1885, during which the curvature of the orbital path was slight; it is now rapidly increasing, so that the orbital moti[illegible] 1906 is at right angles to that in 1820, and in a fe[illegible] accepted proper motion will become entirely erroneou[illegible] as yet scarcely a long enough arc covered by the [illegible] vations to make a reliable determination of th[illegible] from them. An attempt is made by Mr Lewis

the recently published Memoir on the Struve Double Stars (vol. lvi. p. 215); but while the collection of meridian observations and their reduction to a common epoch is of great interest and value, it seems to me that their determination of the mass-ratio (brighter star 20 times the fainter) is vitiated by the implicit assumption that Auwers' proper motion may be taken as the true proper motion of the centre of gravity of the system. I believe that it is the apparent resultant motion of the brighter star, and consequently it is only to be expected that its application should bring this star to relative rest. The inference that the brighter star is far the more massive I consider unsound. I had reached this conclusion before I saw the discussion of this system from spectroscopic observations by Dr Heber D. Curtis in *Lick Observatory Bulletin* No. 98. On the assumption that the two minor systems (each star being a spectroscopic binary) are coplanar with the major system, he reaches the conclusion that the mass of a_1 (the fainter component) is 6 times that of a_2. The assumption is of course tentative, but the resulting mass-ratio is probably more reliable than any that can be at present obtained from the meridian observations. I decided to adopt it as a trial hypothesis, and to see how it would work. Using the Greenwich observations collected on p. 215 of the Struve Memoir, the position of the centre of gravity reduced to 1890 with Auwers' proper motions and corrections, comes out as follows :—

Epoch of Catalogue.	Position of Centre of Gravity on the above Assumption of Masses. R. A.	N.P.D.
	h m s	° ′ ″
1840	7 27 ...	57 52 16·99
1845	34·459	16·51
1850	34·544	16·99
1860	34·520	17·35
1864	34·491	16·86
1872	34·490	17·90
1880	34·547	17·91
1890	34·601	18·87
1900	34·566	18·57

From these I deduce a correction to the assumed proper motion of

	s	″
	+ ·0016	+ ·041
Auwers' value is	− ·0151	+ ·079
Newcomb's being	− ·0144	+ ·082

Hence the concluded proper motion of the centre of gravity of the system is $-{}^{s}{\cdot}0135$, $+{}^{\prime\prime}{\cdot}120$.

If the above mass-ratio is correct, we should expect a much more definite curvature in the position of a_2 than of a_1. It is

fortunate that this is the case, as observations of a_2 are much more numerous. I have accordingly reduced the following observations of a_2 to 1890, with the proper motion $-^s{\cdot}0135$, $+''{\cdot}120$.

Positions of a_2 reduced to 1890.

		h	m	s	°	′	″	
Bradley .	1757	7	27	34·88	57	52	21·64	Adopt $34^s{\cdot}83$ $21''{\cdot}6$
T. Mayer .	1757			34·74			21·54	
Piazzi .	1800			34·84			17·8	
Greenwich	1840			34·99			17·78	
,,	1845			34·94			17·24	
,,	1850			34·93			16·86	
,,	1860			34·93			16·43	
,,	1864			34·90			15·86	
,,	1872			34·88			15·73	
,,	1880			34·91			15·60	
,,	1890			34·91			15·45	
,,	1900			34·82			14·93	

These have been plotted on the diagram; it will be seen at once that, omitting Piazzi, which is discordant, they do indicate a curvature of about the required amount, as compared with Mr Lewis' diagram of the orbit. Hence they support the adopted mass-ratio as at least approximately correct. Obviously no curvature would be shown if a_2 had much the greater mass.

As Castor is a Greenwich clock star, it would seem to be advisable to follow the method that has already been adopted for Procyon, viz. to apply a double correction, one for the proper motion of the centre of gravity, the other for orbital motion round this point.

For example, in reducing the 1890 place of Castor to 1920 we should have to apply $-^s{\cdot}0135 \times 30 - ^s{\cdot}11$ to the R.A., and $+''{\cdot}120 \times 30 - ''{\cdot}56$ to the N.P.D. (in addition to precession). The second term is the orbital correction deduced from Mr Lewis' diagram on the assumption of mass-ratio 1 to 6.

These amount to
$$-^s{\cdot}405 - ^s{\cdot}11 = -^s{\cdot}515$$
$$+3''{\cdot}60 - ''{\cdot}56 = +3''{\cdot}04$$

whereas if Auwers' proper motion were used they would be

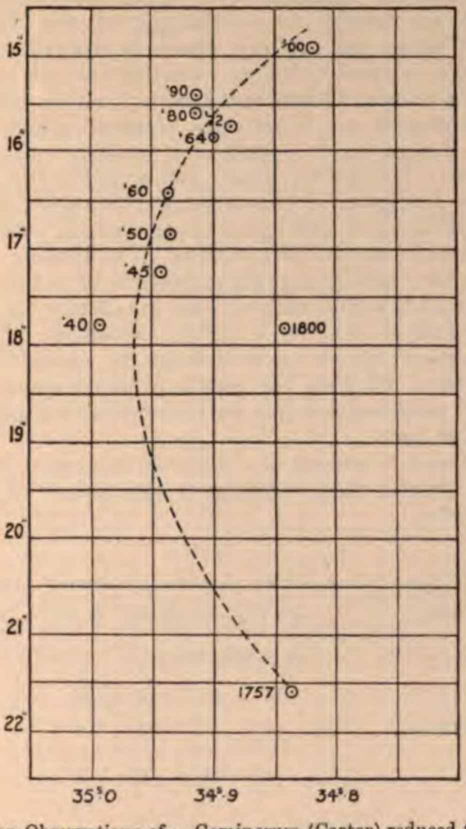
$$-^s{\cdot}0151 \times 30,\ +''{\cdot}079 \times 30 \quad = -^s{\cdot}453,\ +2''{\cdot}37$$

Hence even now the effect of the correction is sensible, and it will rapidly increase as periastron is approached. The direction of orbital motion in N.P.D. will be reversed in a few years, after which Auwers' and Newcomb's values of the motion in N.P.D. will become altogether erroneous.

This is, I believe, the first occasion when spectroscopic obser-

ions have assisted in determining the proper motion of a star. e possibility of mutual assistance of this kind forms another k connecting physical astronomy with astronomy of position.

Perhaps it is as well to call attention to an erratum in Mr wis' Memoir which misled me for a time and may mislead



Meridian Observations of α_2 Geminorum (Castor) reduced to 1890, with Proper Motion $-^{s}\cdot0135$, $+''\cdot120$.

iers. On p. 21 Mr Furner's determination is printed as though fainter star had 20 times the mass of the brighter. His deter- nation on p. 215 of the Memoir made the bright star 20 times e faint, and, although the reverse is probably true, the two pages ould be made consistent.

Estimate of the Number of Stars within Certain Limits of Proper Motion. By W. G. Thackeray.

The following data are derived from discussions of statistics of proper motions, published (1) by Prof. Auwers in the introduction of the Berlin "A" Catalogue of the Astronomische Gesellschaft series, pp. 141–143, where he discusses the Bradley proper motion referred to here as "Bradley," as well as certain of his zone stars referred to here as "Auwers," and though the separation of the stars is not in all cases identical with the grouping adopted in this paper, it is quite near enough for the purposes of this paper; (2) by Prof. Dyson and myself in the introduction to the New Reduction of Groombridge's Catalogue, p. xcii; and (3) by myself on some Carrington proper motions, published in the present volume of the *Monthly Notices.* The Bradley observations extend over the northern sky, Groombridge's lie within 52° of the pole, Carrington's within only 9°, and the Berlin "A" series are within the zone of +15° to +20° of Declination. The material under discussion cannot be considered to extend beyond 9·4 magnitude stars, but from the results it would appear as though the adopted percentages might be used further without much risk of appreciable error.

The agreement between the different catalogues is especially good for the fainter stars, and suggests the absence of any serious systematic error.

Percentage of Stars within Certain Limits of Centennial Proper Motions.

Authority.	0″–5″	5″–10″	10″–20″	>20″	No. of Stars.
	Magnitude 1–4·9.				
Bradley . . .	42	22	18	18	711
Groombridge .	45	21	20	14	234
	43	22	18	17	
	Magnitude 5·0–5·9.				
Bradley . .	49	26	16	9	951
Groombridge .	59	21	14	6	538
	55	23	15	7	
	Magnitude 6·0–6·9.				
Bradley . .	52	29	13	6	1367
Groombridge .	66½	21	8	4½	1149
	60	25	10	5	

Authority.	0″–5″	5″–10″	10″–20″	>20″	No. of Stars.
	Magnitude 7·0–7·9.				
Bradley . .	55	26	13	6	188
Groombridge .	74	18	5½	2½	1369
Auwers . . .	76	14	7	2	1420
Carrington . .	64	22	12	2	114
	75	17	6	2	
	Magnitude 8·0–8·4.				
Groombridge .	78½	17	2½	2	940
Auwers . . .	80	11	7	2	1595
Carrington . .	69	19	9	4	179
	79	14	5	2	
	Magnitude 8·5–9·0.				
Auwers . . .	77	18	4	1	3366
Carrington . .	79	13	7	1	442
	78	15	6	1	
	Magnitude 9·0–9·4.				
Auwers . . .	79	13	7	1	1022
Carrington . .	80	15	5	1	378
	79	14	6	1	

With reference to the estimate of the number of the stars for the adopted groups of magnitude, Seeliger gives for the counts in the B.D. for the northern hemisphere the following figures :—

Mag. 1 –6·5	4,120 stars.
6·6–7·0	3,887
7·1–7·5	6,054
7·6–8·0	11,168
8·1–8·5	22,898
8·6–9·0	52,852
9·1–9·5	213,973
Total	314,952

…r) scale there would be 630,000 in the

…n this estimate, I have counted the …ages in different volumes of the … 100, 125, 81, 124, 143, 131, 93, …e of 106·5 out of every 300

stars, say 35 per cent. Excluding these, the number of stars down to 9·4 magnitude would thus be some 400,000.

The adopted values of the numbers for the different groups of magnitudes, and the percentage values within certain limits of proper motion, are given in the following table :—

Percentage of Stars within Certain Limits of Centennial Proper Motions in Order of Magnitude, with Estimated Number of Stars.

Mag.	$0''-5''$	$5''-10''$	$10''-20''$	$>20''$	Estimated No. of Stars in Thousands.
1 –4·9	43	22	18	17	1
5·0–5·9	55	23	15	7	3
6·0–6·9	60	25	10	5	12
7·0–7·9	75	17	6	2	35
8·0–9·4	79	14	6	1	350

Estimated Number of Stars within Certain Limits of Centennial Proper Motions in Order of Magnitude.

1 –4·9	430	220	180	170	1
5·0–5·9	1,650	690	450	210	3
6·0–6·9	7,200	3,000	1,200	600	12
7·0–7·9	26,250	5,950	2,100	700	35
8·0–9·4	276,500	49,000	21,000	3,500	350
	312,030	58,860	24,930	5,180	401

From a comparison of the Groombridge and Carrington proper motions, for the purpose of estimating the effect of accidental error, the probable error of a centennial proper motion in N.P.D. is $\pm 0''\cdot 8$ and in R.A. $\pm 0''\cdot 9$; thus the probable error of a resultant centennial proper motion is $\pm 1''\cdot 2$. It would, therefore, be reasonable to infer that the numbers corresponding to these limits of proper motion would not be liable to any serious alteration for the effect of accidental error of observation.

If the group $0''-5''$ is further broken up it will be found that for all magnitudes the stars tend to accumulate somewhere round $2''\cdot 5$ as a resultant centennial proper motion, and this seems too large a quantity to be due to systematic or accidental error.

Notes on some Proper Motions derived from a Comparison of Carrington's Catalogue with the Greenwich Places for 1900. By W. G. Thackeray.

In the Greenwich Catalogue for 1900, which is now in course of construction, there are 1185 stars to be found in Carrington's Circumpolar Catalogue for 1855. Of these stars, 94 common to the Groombridge-Greenwich system have been used to discuss the

systematic corrections applicable to Carrington's places to bring them into line with the Groombridge-Greenwich system, and the results have been published in *Monthly Notices*, lxvi. pp. 320–323.

After applying these corrections to the Carrington Catalogue places, proper motions for these 1187 stars have been derived by a simple comparison of the Greenwich observed places for 1900 with the corrected Carrington places brought up to 1900 by Struve-Peters precession constants.

Taking those stars which are common to Groombridge and Carrington, and forming the differences of proper motion derived from the Groombridge and Carrington Catalogue places respectively in the sense of correction to Groombridge for every three hours of right ascension, we get corrections which can be compared with the tables of corrections given by Boss in his paper, "The New Reduction of the Meridian Observations of Groombridge" (*Monthly Notices*, lxvi. p. 513).

The tables are given in the form of corrections to proper motion in arc in both elements. Boss's corrections in N.P.D. are not given, as being practically insensible.

Boss's corrections in R.A. are found by multiplying by $\frac{3}{18}$ the quantities given in Table II., Zone IV., *Monthly Notices*, lxvi. p. 563.

Corrections to Groombridge's Proper Motions in Order of R.A.

R.A.	Right Ascension. Boss.	Right Ascension. Carrrington-Greenwich.	N.P.D. Carrington-Greenwich.
0^h	″·000		
$1\frac{1}{2}$		+″·003	−″·003
2	+·009		
4	−·006		
$4\frac{1}{2}$		−·003	−·010
6	−·010		
$7\frac{1}{2}$		−·010	−·005
8	−·016		
10	−·001		
$10\frac{1}{2}$		+·003	−·005
12	+·017		
$13\frac{1}{2}$		+·020	−·003
14	+·016		
16	+·014		
$16\frac{1}{2}$		+·013	−·002
18	·000		
$19\frac{1}{2}$		−·004	−·004
20	+·002		
22	−·006		
$22\frac{1}{2}$		+·002	−·001

The agreement between Boss and Carrington is notable, and shows the excellence of Carrington's observations. The small

mean difference in N.P.D. represents but a small systemati discordance.

Again arranging these Carrington proper motions in orde of magnitudes and in octants of R.A., in the same manner as wa adopted in the case of the Groombridge Catalogue, we get th following tables of numbers and percentages :—

Numbers of Stars—Centennial Proper Motions in Order of Stars' Magnitude

Magnitude.	Total Number of Stars.	0″–5″	5″–10″	10″–20″	>20″
m m					
5·0–5·9	19	5	8	6	0
6·0–6·9	46	28	12	4	2
7·0–7·9	114	73	25	14	2
8·0–8·4	179	121	35	16	7
8·5–8·9	442	338	82	17	5
9·0–	378	302	56	17	3
Total .	1178	867	218	74	19

Percentage of Stars—Centennial Proper Motions in Order of Stars' Magnitude

m m					
5·0–5·9	19	26	42	32	0
6·0–6·9	46	61	26	9	4
7·0–7·9	114	64	22	12	2
8·0–8·4	179	69	19	9	4
8·5–8·9	442	77	18	4	1
9·0–	378	80	15	5	1

Percentage of Stars—Centennial Proper Motions in Order of R.A.

Limits of R.A.	Total Number of Stars.	0″–5″	5″–10″	10″–20″	>20″
h h					
0– 3	149	67	28	5	1
3– 6	127	77	16	7	0
6– 9	137	80	12	7	2
9–12	170	70	22	5	3
12–15	172	71	20	8	1
15–18	140	75	19	4	2
18–21	147	82	15	2	1
21– 0	136	74	11	13	2

The mean discordance of a determination of proper motio from Groombridge and Carrington is ±″·010 in N.P.D. an ±″·011 in R.A., using those stars only which have little or n systematic error which would give a probable error for a re centennial proper motion of 1±″·2 as the effect of s error.

On the Accidental Production of Temporary Errors of Division on a Graduated Circle. By W. M. Witchell.

(*Communicated by the Astronomer Royal.*)

During an examination, undertaken recently, of the micrometer screws of the Greenwich Meridian Circle reading microscopes, a suggestive discovery was made.

Six observations of "runs" had been taken over each of three consecutive intervals of 5′, and these agreed among themselves quite normally so long as the same interval was under consideration; but when the results from the different intervals were compared a discordance much beyond the probable accidental error of observation appeared between the values obtained at pointer reading 89° 35′–40′, and those at 89° 40′–45′.

As will be seen from the following figures, the discordance amounted to $0^{r}\cdot014$ or $0''\cdot84$, and an attempt to trace its origin was necessary.

Value of 5′ of Circle in "Mean Micrometer."

	Pointer 89° 35′ to 89° 40′.	Pointer 89° 40′ to 89° 45′.
	r	r
Set 1 . .	4·903	4·893
,, 2 . .	·912	·892
,, 3 . .	·908	·897
,, 4 . .	·907	·894
,, 5 . .	·904	·893
,, 6 . .	·908	·891
	4·907	4·893

Difference : $0^{r}\cdot014$

An accidentally large deviation from the mean division error at this part of the circle was at first suspected. A search for others of like magnitude in the neighbourhood, however, produced negative results. But when these observations (which consisted of three sets of runs over each 5′ interval from pointer reading 88° 20′ to 89° 20′) were arranged so as to exhibit the values from the six micrometers individually, and were compared with the former series similarly arranged, it was at once seen that the discordance took its origin in a large apparent error of the particular graduation viewed by microscope D when the pointer reads 89° 40′. The figures follow. They were considered to give strong evidence of an error amounting to $0^{r}\cdot03$ of micrometer, or $1''\cdot8$ in this graduation, inasmuch as the mean screw measurements of the two adjacent intervals appeared to be too large and too small respectively by this quantity.

Measurement of 5′ Interval, 89° 35′–89° 40′ pointer reading.

Micrometer	A	B	C	D	E	F
	r	r	r	r	r	r
Set 1 . .	4·910	4·905	4·887	4·931	4·909	4·874
„ 2 . .	·914	·917	·915	·937	·914	·876
„ 3 . .	·927	·908	·899	·927	·918	·870
„ 4 . .	·911	·920	·897	·923	·910	·883
„ 5 . .	·899	·918	·896	·912	·906	·891
„ 6 . .	·920	·921	·891	·924	·901	·889
Mean .	4·914	4·915	4·898	4·926	4·910	4·880

Corresponding means of 36 other sets, viz. 3 over each 5′ interval from 88° 20′ to 89° 20′:—

4^r·923	4^r·918	4^r·890	4^r·901	4^r·916	4^r·884

Measurement of 5′ Interval, 89° 40′–89° 45′ pointer reading.

Micrometer.	A	B	C	D	E	F
	r	r	r	r	r	r
Set 1 . .	4·916	4·921	4·871	4·868	4·902	4·880
„ 2 . .	·913	·910	·869	·867	·905	·885
„ 3 . .	·917	·892	·881	·873	·916	·902
„ 4 . .	·910	·899	·889	·890	·902	·876
„ 5 . .	·896	·916	·868	·875	·911	·890
„ 6 . .	·925	·908	·891	·854	·905	·863
Mean .	4·913	4·908	4·878	4·871	4·907	4·883

Now in the winter months it has been customary to protect the circle from tarnish by applying to it a thin film of vaseline, which, however, gradually accumulates small particles of dust and is removed from time to time.

Before proceeding to a systematic examination of the circle for possible errors of like nature in other graduations, the part under actual suspicion was wiped clean; when it was found that the supposed error of nearly 2″ in the division in question had absolutely disappeared, presumably with the dusty vaseline.

It should be stated here that the microscopes were in good focus and adjustment.

Other divisions being similarly experimented upon gave no difference, before and after, at all comparable with the foregoing, except in one case (the figures for which are quoted below), so that the liability to error introduced by this method of preserving the circle is probably both slight and casual. At the same time it is distinctly real, and may be the explanation of discordances which cannot be ascribed with certainty to other agencies.

The following are micrometer readings for ten bisections of the division under microscope E (pointer reading 164° 45′) before and after wiping off the vaseline :—

Before.		After.	
	r	r	r
608	·611	·633	·618
·613	·612	·627	·620
·603	·604	·629	·624
·589	·593	·620	·632
·600	·590	·622	·624
Mean r·602		Mean r·625	

Difference : r·023 = 1″·40

As a consequence of these revelations, the cleaning of the circle, which hitherto has naturally been avoided as much as possible for fear of injuring the graduations, will be carried out more frequently.

Observations of Minor Planets from Photographs taken with the 30-inch Reflector of the Thompson Equatorial at the Royal Observatory, Greenwich, during the year 1903.

(*Communicated by the Astronomer Royal.*)

The following positions of minor planets were obtained from photographs taken with the 30-inch Reflector during the year 1903.

The plates were measured with the astrographic micrometer. Four reference stars were, as a rule, measured with the planet, their positions being derived when possible from the Catalogues of the Astronomische Gesellschaft.

The positions given are not corrected for Parallax.

log Parallax Correction = log Parallax Factor − log Δ.

The anonymous planet was found on the same plate as (407) Arachne.

Date and G.M.T. 1903.		Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Log. Parallax Factor. Dec.
d	h m s	h m s	° ′ ″		
		(258) Tyche.			
May 25	10 41 29	14 38 6·48	−7 40 58·1	+8·321	+0·876
26	9 55 41	14 37 25·65	−7 34 56·3	+8·669	+0·876
		(68) Leto.			
May 29	11 25 13	15 13 48·14	−20 43 7·1	+8·723	+0·921
June 2	10 46 9	15 10 18·77	−20 38 24·5	+8·462	+0·921
3	10 29 3	15 9 28·86	−20 37 15·3	+7·931	+0·922
4	10 4 18	15 8 40·18	−20 36 7·5	−8·408	+0·921

Date and G.M.T. 1903.		Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Dec.
d	h m s	h m s	° ′ ″		
		(17) Thetis.			
May 29	11 58 12	16 59 51·84	−14 11 6·9	−8·772	+0·902
June 2	11 18 49	16 56 12·69	−14 11 50·4	−8·961	+0·900
3	11 11 36	16 55 16·89	−14 12 15·7	−8·983	+0·900
4	11 47 10	16 54 18·99	−14 12 45·5	−8·472	+0·902
22	11 5 57	16 38 20·83	−14 38 54·8	+8·664	+0·904
		(511) Davida.			
June 2	11 48 24	16 49 57·68	−9 13 4·5	−8·511	+0·883
3	12 9 25	16 49 10·12	−9 13 46·2	+7·909	+0·883
4	11 27 1	16 48 24·63	−9 14 30·2	−8·726	+0·883
		(304) Olga.			
June 3	11 42 3	15 29 10·14	+8 17 20·4	+8·960	+0·779
4	10 28 42	15 28 23·81	+8 17 39·8	−8·249	+0·778
		(434) Hungaria.			
June 2	12 23 56	16 16 25·39	+22 29 49·8	+8·925	+0·631
4	10 57 43	16 14 36·50	+22 36 36·4	−8·684	+0·626
22	10 35 59	16 0 57·07	+21 55 8·2	+8·779	+0·636
24	10 22 53	15 59 57·33	+21 40 3·6	+8·711	+0·640
		(432) Pythia.			
June 22	11 27 42	16 52 3·91	−19 46 40·1	+8·760	+0·919
27	10 36 45	16 47 36·66	−20 26 1·9	+8·077	+0·921
July 1	10 36 53	16 44 29·33	−20 57 31·9	+8·647	+0·922
		(405) Thia.			
June 22	11 54 50	17 23 0·93	−19 29 50·0	+8·733	+0·918
27	11 43 24	17 18 40·46	−18 53 17·8	+8·878	+0·915
July 1	11 5 2	17 15 36·32	−18 26 19·0	+8·624	+0·915
		(270) Anahita.			
Aug. 4	11 57 52	20 22 25·63	−15 13 57·6	+8·616	+0·906
5	10 38 24	20 21 32·44	−15 16 5·6	−8·915	+0·904
10	10 29 57	20 17 3·00	−15 27 24·4	−8·749	+0·906
13	*9 58 44*	20 14 35·56	−15 34 7·1	−8·924	+0·905

Date and G.M.T. 1903.		Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Dec.
		(57) Mnemosyne.			
d	h m s	h m s	° ′ ″		
Aug. 4	11 39 53	19 39 27·24	+0 10 56·1	+8·899	+0·835
5	9 44 42	19 38 50·55	+0 6 49·2	−8·984	+0·835
6	10 9 25	19 38 10·50	+0 2 8·5	−8·694	+0·836
7	10 6 44	19 37 31·86	−0 2 33·6	−8·665	+0·836
10	10 9 34	19 35 40·30	−0 17 17·0	−8·317	+0·838
13	9 28 56	19 33 56·84	−0 32 43·2	−8·803	+0·839
		(147) Protogenia.			
Aug. 5	11 8 19	21 24 0·62	−12 32 43·0	−9·123	+0·892
13	11 7 46	21 17 52·19	−12 59 26·6	−8·865	+0·897
22	11 14 1	21 11 5·03	−13 29 53·9	+7·693	+0·900
		(407) Arachne.			
Aug. 5	11 25 49	21 44 50·01	−7 53 4·4	−9·127	+0·875
6	10 59 25	21 43 58·25	−7 53 50·5	−9·221	+0·873
31	11 25 11	21 21 21·29	−8 33 47·6	+8·808	+0·880
Sept. 1	9 59 38	21 20 36·37	−8 35 37·0	−8·819	+0·880
		(Anonymous.)			
Aug. 6	10 59 25	21 40 47·82	−7 20 8·6	−9·209	+0·871
31	11 25 11	21 20 30·19	−8 17 4·9	+8·819	+0·879
Sept. 1	9 59 38	21 19 52·08	−8 19 35·8	−8·819	+0·879
		(324) Bamberga.			
Aug. 31	11 51 34	22 3 40·60	−9 43 8·3	+8·589	+0·885
Sept. 1	10 17 35	22 2 42·34	−9 37 21·3	−9·023	+0·883
7	9 40 11	21 56 45·23	−8 59 41·2	−9·071	+0·880
9	10 12 59	21 54 53·28	−8 46 38·3	−8·699	+0·881
11	10 32 9	21 53 7·53	−8 33 31·1	−7·509	+0·881
		(333) Badenia.			
Sept. 11	10 58 46	22 32 49·07	−10 51 22·3	−8·387	+0·890
		(184) Dejopeja.			
Aug. 31	12 54 27	22 41 3·51	−8 13 1·4	+8·895	+0·878
		(514) 1903. M. B.			
Aug. 31	13 20 47	22 54 43·86	−0 54 48·9	+8·991	+0·841
		(513) 1903. L. Y.			
Aug. 31	12 20 27	23 11 34·82	+0 28 50·7	−8·379	+0·833

Enhanced Lines of Iron in the Region F to C. By A. Fowler.

In view of the now generally recognised importance of enhanced lines in the interpretation of solar and stellar spectra, it is thought that the accompanying observations of the enhanced lines of iron which occur in the less refrangible parts of the spectrum may be useful to other workers.

The lines have been observed and photographed under various conditions: in the spark, in the arc at reduced pressure, in the arc in hydrogen, and on the positive pole of an ordinary continuous-current arc in air at atmospheric pressure, metallic electrodes being used throughout. There is a distinct gain in producing the lines without the use of a spark in a few cases, inasmuch as there is no air spectrum to interfere with their detection.

The occurrence of the enhanced lines on the positive pole of the arc affords a particularly convenient mode of identifying them, except towards the red, where the continuous spectrum tends to mask the fainter lines. When the bright spot on the positive pole is carefully adjusted on the slit, the lines in question are observed as very short lines, quite different in appearance from the arc lines, which are also present and provide a convenient reference spectrum; unlike the enhanced lines, the arc lines are either weakened or unchanged on the positive pole. A similar appearance is observed on the negative pole, but the enhanced lines are not so bright. In these experiments the current has ranged from 12 to 0·4 ampères, on 110-volt circuit, with an approximately constant potential difference of 40 volts between the electrodes, and the intensities of the enhanced lines have not been found to be materially changed as compared with the arc lines observed at the same time. Even with 0·4 ampères, the arc burning continuously, all the arc lines remained visible when the proper part of the image was brought on the slit, and the enhanced lines were still very distinct in the immediate neighbourhood of the poles.

This result is somewhat different from that obtained by Hartmann with magnesium poles, in which case the enhanced line 4481 is greatly strengthened as the current is reduced. It differs also from Prof. Hale's recent observations of the iron spectrum,* in which a 2-ampère arc was found to give a spectrum closely approximating to that of the outer flame of an ordinary arc with greater current, though no material change was observed in passing from 30 to 15 ampères. It may be that the difference is partly due to the use of metallic electrodes in my experiments, while Prof. Hale appears to have used the metal on carbon poles; under the latter conditions I have obtained similar results with the 2-ampère arc, but only when the quantity of iron on the poles was small. At all events, when metallic poles are used, reduction of current strength does not appear to be accompanied by a reduction of temperature sufficient to produce any notable differences in the

* *Astrophys. Journal*, vol. xxiv. p. 208.

spectrum, if corresponding parts of the arc be observed in each case. Indeed, this observation accords well with Prof. J. J. Thomson's remark * that in the arc "the temperature of the crater of the positive terminal remains constant even when the current varies."

It should be remarked, however, that the similarity of the phenomena in the 2-ampère iron arc with those observed when the current strength is greater does not in the least invalidate Prof. Hale's conclusion as to the probable low temperature of sun-spots. As in my own discussion of this point,† Prof. Hale's result ultimately depends upon a comparison of spot spectra with the spectrum of the arc-flame.

The wave-lengths of all the lines given in the table have been determined from a new series of photographs in which the linear dispersion from C to F is 24 cm. The results have differed so slightly (rarely more than 0·02) from solar lines of appropriate intensity tabulated by Rowland that there can be no doubt as to their identity, and to avoid any possible confusion, Rowland's wave-lengths have been adopted. This procedure is, in fact, justified by Lockyer and Baxandall's demonstration ‡ of the presence of enhanced lines of iron in the more refrangible parts of the solar spectrum.

The representation of the additional lines with proper intensities in the Fraunhofer spectrum, together with their special behaviour in the chromosphere and spots, § is valuable confirmation of their classification as enhanced lines. A few lines, notably two at wave-lengths 5260·50 and 5100·95 which appear in some of the spectra, have not been included in the table because they failed to satisfy these conditions, although the probable impurity producing them has not yet been traced. That such lines were due to some substance other than iron was further suggested by their variable intensities with respect to undoubted enhanced lines. Prof. Hale has met with a somewhat similar case in a supposed enhanced line of iron at 5218·37, of which he says that "the enhancement of this line may vary." Here, however, there can be no hesitation in attributing the line to copper, which is a very common impurity in iron.

There is, in fact, no reason to suppose that the intensities of the enhanced lines are appreciably different relatively to each other under any of the conditions of experiment which have been mentioned, though their intensities, as compared with the arc lines, are not the same in all cases. The enhancement is most marked in the spark spectrum. Estimates of the relative intensities are given in the table.

For the sake of completeness, the table also shows the be- of the lines in the chromosphere and sunspots, Y, H, M, nectively indicating Young, Hale, Mitchell, and Fowler.

harge of *Electricity through Gases* (1903), p. 417.
Int. Solar Union, vol. i. p. 228 (1906).
vol. lxxiv. p. 225 (1904).
vol. lxvi. p. 361 (1906).

Enhanced Fe.		⊙ Rowland.		Behaviour in Sun-Spots.	Intensity in Chromosphere.			Remarks.
Wave-length.	Int.	Origin.	Int.		Y	M	F	
4924·11*	9	Fe	5	Weakened H, F	10	15	25	
5018·63*	9	Fe	4	,, H, F	15	10	25	
5169·22*	10	Fe	4	,, H, M, F	25	25	40	
5197·74†	4	...	2	,, H, M, F	10	15	25	
5234·79†	4	...	2	,, H, M, F	10	15	25	
5264·98	1	...	0	,, H, F	3	7	15	
5276·17*	6	Fe ?	3	Not clearly affected M, F	10	8	35	Compound line in ⊙, 76·24, 76·17.
5316·79*	9	Fe	4	Weakened Y, H, F	2–20	20	45	
5325·74	2	...	(2)	,, Y, F	2	10	15	Rowland's ⊙ int. too high.
5363·06†	5	...	3	,, F	5–10	8	30	
5535·06†	4	Fe	2	,, H, M, F	12	12	35	Masked by air line in spark.
6238·50	3	...	2	,, M, F	2	...	15	
6247·77	3	...	2	,, M, F	4	...	20	Observed also in spark by Mitchell.
6417·13	2	Fe ?	1	,, M, F	2	...	...	
6456·60†	6	...	3	,, M, F	3–10	5	20	

It will be seen that the representation of the enhanced lines in the solar spectrum, and in the spectrum of the chromosphere and sun-spots, is quite consistent throughout. As in the case of Lockyer and Baxandall's investigation of the more refrangible parts of the spectrum, origins are now provided for several lines unidentified or doubtfully identified by Rowland, the reason being that he did not ordinarily obtain the weaker enhanced lines in his photographs, and probably attributed their occasional appearance to impurities.

There is abundant evidence that all the enhanced lines of iron in the region F to C are weakened in the spectra of sun-spots, and the work at Mount Wilson has already shown that the same is true of many of the lines of this class in the blue and violet.

All the lines in question are also prominent in the spectrum of the chromosphere, and, according to my previous observations, are of the high-level type, with the possible exception of 6417·7, which is not yet included in my list, but is given by Young.

It is sufficiently clear, therefore, that the enhanced lines constitute a special system of lines which vary together both in laboratory experiments and in the various parts of the sun where they are observed.

Note.—It may be mentioned that, while generally confirming Lockyer's list of enhanced lines of iron in the more refrangible parts of the spectrum, the special photographs of the arc reveal an additional line of considerable intensity at 4416·98, which is masked by an air line in the spark. This line also is weakened in the sun-spot spectrum.

* Previously recorded by Lockyer, *Pub. Sol. Phys. Obs.*, 1906, etc.
† ,, ,, Author, *Monthly Notices*, vol. lxvi. p. 364, 1906.

Note on Silicon in the Chromosphere. By A. Fowler.

Pending a more complete investigation of the spectrum of silicon in relation to the chromosphere and Sun-spots, it may be of interest to draw attention to the identification of two strong red lines of this element with well-marked chromospheric lines. The lines in question have been previously observed in the spark spectrum by Salet (λλ 6341, 6366), by the Count de Gramont (λλ 6342, 6370), and more recently by Lunt, who gives the approximate wave-lengths 6346·9 and 6371·2.*

A careful re-determination of the positions of these lines, from photographs giving a linear dispersion of 10 tenth-metres to the millimetre in this part of the spectrum, leaves no doubt as to their coincidence with the previously unidentified high-level chromospheric lines at 6347·31 and 6371·57. The more refrangible of the two lines is the stronger, in the proportion of about 10 to 6, and their intensities in the chromosphere, according to my own observations,† are 25 and 15 respectively. Both lines occur in the Fraunhofer spectrum, with intensities and characters 2N and 1Nd? respectively, and Rowland assigns the latter to iron, while leaving the other unidentified. Kayser and Runge also give a faint line at 6371·60 in the arc spectrum of iron, but it does not appear on my photographs of the iron spectrum, except when the presence of silicon is indicated by the other line at 6347·31. In any case, if there be an iron line at 6371·57, it is not an enhanced line, and is not of sufficient intensity in the arc to account entirely either for the Fraunhofer or chromospheric line at the same wave-length, which must accordingly be attributed chiefly to silicon.

In Sun-spots, according to the observations of Mitchell‡ and myself, the two lines are almost obliterated, so that there is a complete agreement of behaviour and intensities throughout.

The relationships of the different families of silicon lines have not yet been fully worked out, but it is probable that the two red lines, like so many of the other high-level chromospheric lines which are weakened in spots, belong to the enhanced line class. Like the enhanced lines of iron, they appear close to the positive pole when a little silica is introduced into the iron arc, and the wave-lengths have been determined by reference to adjacent iron lines which occur with the silicon lines under these conditions. In his recent discussion of the violet part of the flash spectrum Prof. Dyson§ concluded that there was "a fair degree of probability for the existence of silicon in the chromosphere," and all [illegible] removed by the identification of the red lines.

* [illegible] *Cape Observatory*, vol. x., Part II., p. 18B (1906).
† [illegible], vol. lxvi. p. 365 (1906).
‡ [illegible] vol. xxiv. p. 92 (1906).
§ [illegible] [illegible]vi. A, p. 440 (1906).

Notes on some Spectroscopic Observations of the Sun.
By H. F. Newall, M.A., F.R.S.

Spectroscopic observations of the Sun have been carried on during the past year at Cambridge. In the earlier part of the year the observations were made with the 25-inch equatorial (the Newall telescope), to which a diffraction-grating spectroscope was attached. In the later months a fixed horizontal telescope was improvised, light being directed into it by means of a cœlostat and an auxiliary mirror; and a much more powerful grating spectroscope was used. It will be convenient to refer to these two equipments as the equatorial and the horizontal.

The general programme was to test by preliminary observations, whether the conditions of atmosphere over the flat plains of Cambridgeshire would justify a special outfit which should enable us to take some active part in solar work. It seemed that the study of sun-spot spectra would be a fitting one for making the trials. For, in this work, not only is a fairly clear atmosphere desirable, such as will not prevent the detection of faint characteristic detail in the sun-spot spectra by the overpowering effect of integrated sunlight scattered by intervening haze; but also a steady atmosphere is needed, such as will not give rise to jumping images and the blurring effects due to the spread of circumpenumbral light over the umbra in consequence of atmospheric tremor.

Accordingly a programme was adopted to include spectroscopic observations of sun-spots and other details on the solar surface, but not observations of prominences at the limb.

My idea was that it would be best, in the first instance, to utilise the equatorial equipment to gain practical ideas of the difficulties to be met, and the advantages to be gained, in direct pointing at the Sun with a telescope whose object-glass would in general be about 30 feet above the ground. And then, if the observations proved to be generally encouraging as regards atmospheric conditions, a horizontal equipment was to be tried, with the beam about 6 feet above the ground.

The result of these trials has been completely satisfactory and decisive. The observations with the equatorial equipment showed that good conditions of observing were available in the early morning between 6 and 9 A.M.

The observations with the horizontal equipment showed that these conditions are not seriously impaired by bringing the beam nearer to the ground; any deterioration arising from greater proximity to the ground is probably more than balanced by the vastly freer circulation of air in the path of the optical beam in the open dome as opposed to the closed tube of the equatorial. The conveniences of working with the horizontal equipment—on a fixed floor, with fixed apparatus, with eye-piece in constant and *convenient* position, with all the parts easily accessible, and most of

all, with the power to use very powerful spectroscopes—can hardly be overstated.

The Equatorial Equipment.—For these solar observations the aperture of the 25-inch object-glass was reduced to 12 inches, and the object-glass was usually between 25 and 30 feet above the ground.

A powerful grating spectroscope was attached to the equatorial in the autumn of 1905. A special mounting of a plane grating had been constructed to replace the prism-head of the four-prism spectrograph which has been used in stellar work (*M.N.*, lxv., Plates 18, 19). The beam which issues from the collimator is

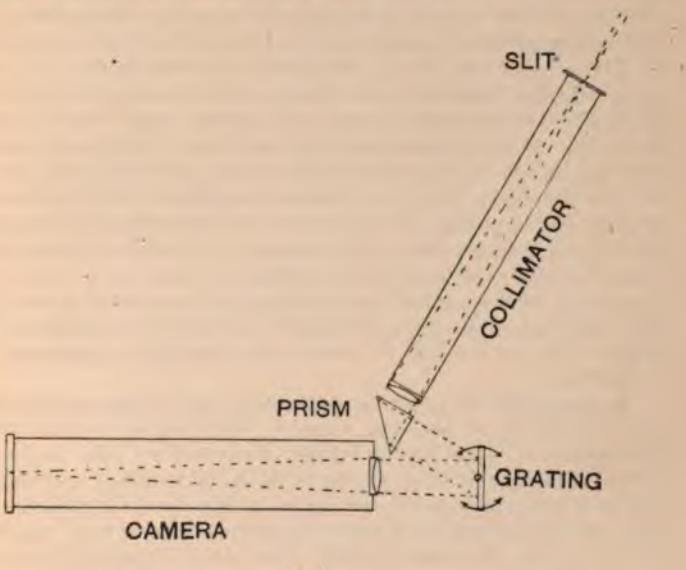


Fig. 1.

deflected through a right angle by a prism fixed in the grating-head; the deflected beam falls on the grating, and the camera is fixed so as to receive a beam emergent at an angle of 30° to the incident beam, and at an angle of 120° to the axis of the equatorial and true collimator. Different orders of spectrum are brought into the field of view by turning the grating, which is geometrically mounted in a cast-iron frame capable of rotation about an axis coincident with the middle ruling of the grating.

The grating is one ruled by Rowland; the ruled surface has an area $3\frac{1}{2}$ inches × 2 inches on speculum metal, and the lines are ruled 14,438 to the inch.

The grating and collimator and camera are of such dimensions to transmit a 2-inch beam. The ratio a/f is $\frac{1}{10}$ for both and camera, the effective aperture of the diaphragmed of the equatorial is only $\frac{1}{21}$, but the slit of the spectro-

scope is adjusted to such narrowness that the diffractional spread of the beam in the collimator fills the lens laterally, and consequently the resolving power of the grating in a 2-inch beam is utilisable.

The spectrum of the second order has generally been used; and of the two spectra of that order, that in which the dispersion and magnification are greater has been chosen. The grating has been adjusted in its frame in such a way that this special spectrum shall be the brightest available.

In the winter months work with the equatorial was sometimes interrupted by condensation of moisture on that surface of the object-glass which is nearest to the observer. Openings were cut through the tube of the equatorial close to the object-glass; and ventilation, controlled by shutters of the "hit-and-miss" type, was promoted. The troubles of condensation did not recur.

The metal diaphragm, used in the first instance to cut down the aperture of the object-glass to 12 inches, was found to cause great disturbance of the image on the slit. When this easily heated diaphragm was shielded by white cardboard and supplemented by a short tube (12 inches in diameter) made of tin-sheet and projecting about 12 inches outwards towards the Sun, the air currents were in part mitigated and in residue diverted from the path of the beam used in observation.

One of the 4-inch finders attached to the equatorial was provided with a projection screen, and was used in setting any desired spot into the field of view of the guiding eye-piece of the spectroscope.

It was found of great convenience to get a quick record of the spots visible on the disc, together with means of orientation. Photographic "printing-out" paper was laid on the projection screen, and half-a-minute's exposure to the Sun's image was enough to give a negative print of the requisite density. Orientation was given by the properly adjusted cross-wires in the focal plane of the finder; they appear as white lines on the negative print of the Sun's disc.

The Horizontal Equipment.—The object-glass of the 25-inch equatorial was dismounted, and fixed in a stout wooden frame, with its optical axis horizontal at a height of about 6 feet above the ground. The aperture was reduced to 12 inches, and sometimes to 6 inches. A 16-inch cœlostat was set up in a small building to the south of the dome of the Newall telescope. By an auxiliary mirror (also 16 inches) the beam from the cœlostat was reflected northwards through an opening made in the dome, and therein the beam passed through the object-glass, which was set with its axis pointing northwards along a line which lay a few feet to the west of the pillar of the temporarily disused eq
The object-glass was placed within a few feet of the
of the dome; and as the dome is 40 feet in diameter
focal length of the object-glass is 29 feet, there was
to set up a large spectroscope in the northern pa
with the slit in the focal plane of the object-glass.

The spectroscope used was the fine instrument built by Messrs Cooke of York, for the late Professor Piazzi Smyth; it has been put at the disposal of the Observatory by the Royal Society. The lenses of the collimator and camera are 4 inches in diameter. The focal length of collimator is 53 inches, and that of the simple camera is 67 inches; a negative enlarging lens can be inserted in the camera, and it has been used in such adjustment as to give an effective focal length of 12 feet 9 inches for the camera.

The plane grating used with this instrument is one ruled by Rowland, and has a ruled surface of 5 inches by $3\frac{1}{2}$ inches on speculum metal, and the lines are ruled 14,438 to the inch. The second order of spectrum was generally used. The slit was usually made narrow enough to give a diffractional indicator less than 1 on the 4-inch lens of the collimator (*M.N.*, lxv. p. 611). Records of the positions and number of spots were printed photographically as in the case of the equatorial equipment; but in this case the printing-out paper was held on the slit-plate in the focal plane of the object-glass, orientation being recorded by the shadow of a wire stretched in front of the photographic paper.

The horizontal equipment was put together in a temporary manner, but in building the cœlostat house and making the windows in the wall of the dome, all arrangements were made in such a way that, if the preliminary observations proved satisfactory, a permanent equipment could be installed, on the same general lines, but with this following modification :—

Permanent Horizontal Equipment in Process of Construction.

The cœlostat house is to contain the cœlostat and auxiliary mirror and an adjustable object-glass of aperture 12 inches and focal length about 60 feet. The focal plane of this horizontal telescope will lie in the North Annex, a convenient laboratory on the north side of the dome of the Newall telescope. The spectroscopic and other apparatus for studying the image of the Sun will be entirely in the annex. The dome itself will be used simply as an optical gallery so far as the solar equipment is concerned; the equatorial will be thus completely available for stellar work, unhampered by any solar apparatus in the dome.

For the solar work, the beam from the cœlostat to the annex will thus pass through shady places well protected from direct solar radiation and from disturbance from detrimental air currents.

The cost of the equipment for solar observations, which represents a new departure at the Cambridge Observatory, will be met by an appropriation from the Frank McClean bequest.

l Conditions between 1905 *Nov.* 24 *and* 1906 *Sept.* 30 (310 days).

…ummarises the state of the sky for
…" means that for at least 2 hours in

		Overcast.	Bright.	Sunshine between Clouds.	Haze.	Observing Days.
1905	Nov. . .	5	2	...	...	1
	Dec. . .	22	7	2	...	6
1906	Jan. . .	14	6	10	1	8
	Feb. . .	18	2	6	2	3
	Mar. . .	22	7	2	...	4
	Apr. . .	13	9	7	1	2
	May . .	23	4	3	1	2
	June . .	14	5	8	3	4
	July . .	16	12	1	...	9
	Aug. . .	13	13	2	1	15
	Sept. . .	13	11	4	2	5
		173	78	45	11	59
			134			

No record on 3 days.

Nearly all observers agree that the early hours of the morning are the most favourable for solar observations. Experience soon showed that Cambridge was no exception to the general rule, and it is probably no exaggeration to say that two minutes' observations between 6.0 and 9.0 A.M. reveal more than an hour's work near mid-day. Nearly all the observations after May were made in the early hours.

Observations on the Nature and Amount of Sky-glare.

Sun-spots are observed under trying conditions. We look at them through a bright sunlit veil in the Earth's atmosphere. If the veil is illumined by the Sun to such an extent that the veil is as bright as the umbra, then it is to be expected that details in the spectrum of the umbra will only be discerned with difficulty.

Evershed's experiments * led him to the view that in the green and yellow part of the spectrum the brightness of the umbra was about $\frac{1}{20}$th of that of the Sun's surface. W. E. Wilson found † that his measures with Boys' radiomicrometer indicated a ratio of about $\frac{1}{5}$.

If we adopt Evershed's value as applicable to observations in the visual part of the spectrum, we should expect that characteristic details in the sun-spot spectrum would be difficult to discern, if the brightness of the sky-glare were $\frac{1}{20}$th of that of the solar surface.

Now this veil or sky-glare is to be attributed to two main causes: (1) scattering by fog or haze or air, (2) scattering by atmospheric tremor. The first of these, acting alone, would give a

* *Observatory* (1898), xxi., 404.
† *Observatory* (1898), xxi., 379.

milky sky in the neighbourhood of the Sun in the sky; and the milkiness would not be very different over the centre of the Sun's disc from what it is at, let us say, 5′ or 10′ outside the Sun's limb. On the other hand, scattering by atmospheric tremor, though nearly uniform over the Sun's disc, would fall away very rapidly outside the Sun's limb, and would probably not be very marked at 20″ or 30″ from the limb; it would be evidenced by the "boiling" of the limb and would be greater with large object-glasses than with small.

Some attempts were made in the early part of the year at Cambridge to get from a series of photographs quantitative information as to the brightness of these components of sky-glare. We may express the brightness of the haze by hs, h being the haze coefficient and s being the brightness of the Sun's surface reduced by absorption from its true value S as it would appear outside the Earth's atmosphere. Similarly we may express the brightness of the veil spread by tremor by ts, where t is the tremor coefficient. By spectrum photographs which show a comparison of the Sun's surface with the sky at 5′ from the limb, and by spectrum photographs which show a comparison of the veil and umbra with the sky, we get two equations which would give h and t, if we knew u/s the ratio of the brightness of the umbra to that of the Sun's surface.

If there were no illumined veil but only an absorbent atmosphere between the observer and the Sun, the ratio $u/s=\mathrm{R}$ would be the same as U/S, the true ratio of umbra and Sun's surface seen from outside our atmosphere. But the veil gives rise to an equation of the form (approximate)

$$u+hs+ts=ks$$

where k is the observed ratio of umbra and Sun's surface, seen through the veil. [Evershed measured k under presumably favourable circumstances, and found $k=\frac{1}{20}$.]

We see that to this approximation

$$\mathrm{R}=\frac{u}{s}=k-(h+t)$$

The observations at Cambridge were directed to getting some idea of the order of magnitude of the quantities R, k, h, and t. The results hardly warrant more than hazarded opinions that (1) R the true ratio U/S is more usually nearly $\frac{1}{40}$, and in certain cases it is probably as small as $\frac{1}{100}$; and (2) with regard to the values of h and t, though we found that h ranged from $\frac{1}{15}$ to $\frac{1}{300}$, and that t might rise to as much as $\frac{1}{2}$, without completely obliterating the most marked characteristics of sun-spot spectra, quite usual values for h and t are $\frac{1}{200}$ and $\frac{1}{20}$.

It is evident that the nature of the case is not such as to justify elaborate measurements. The estimates here given are based on comparisons of density of deposit in photographs taken with varied and relatively adjusted exposures, under conditions

chosen for fair comparison. I should add that the observations were carried out mainly in the winter and early spring, generally on days when the seeing was not good enough to encourage the search for special detail in the spectrum but not bad enough to obliterate it altogether.

One point stands out very clearly, namely, that since the characteristic details of sun-spot spectra are discernible, even when the veil of integrated sunlight is as much as perhaps a quarter of the brightness of the Sun's surface, those details must be very marked.

Another point which I would like to emphasise is this:—

The haze component of the veil is probably hardly to be distinguished spectroscopically from true integrated sunlight, except in the relative brightness of widely separated regions in the spectrum. The tremor component, on the other hand, integrates the sunlight only within, let us say, 30″ of the point considered. Thus over the umbra of a spot the tremor component throws a veil of illumination which comes only from the neighbourhood of the spot—that is, from regions which are comparable with the large flocculi exhibited in Hale's spectroheliograms, and which in other parts of these notes I have ventured to call circumpenumbral regions, where the spectroscopic phenomena certainly indicate special disturbed conditions of solar strata. It is obvious that these points have a very marked bearing upon the interpretation which must be put upon bolometric observations of sun-spots.

Differences in the Ways in which Lines are affected.

These preliminary observations have led me strongly to the view that one of the most promising lines of study must be that of the differences between the ways in which various lines are affected in or near sun-spots.

My observations corroborate in very many points those of many observers as to the existence of long lines, short lines, winged lines, etc., such as are epitomised in the drawings of characteristic types given by W. M. Mitchell (*Astrophys. Jour.*, xxii. p. 6, Pl. II.) and by Fowler (*Int. Sol. Research*, vol. i. p. 228, Plate). Sharp dark lines have been seen in the spectrum of the umbra, and also hazy dark lines: some of the dark lines which are fairly strong over the circumpenumbral regions have been seen nearly, if not quite, obliterated over the umbra. But I incline to agree with Professor Hale in requiring further evidence before I am prepared to say that I have seen lines actually reversed into brightness over the umbra. With respect to the *widening* of lines, I am in some doubt; it would appear indisputable that with certain instrumental conditions lines appear to be widened, but it seems to me that the phenomenon disappears when spectroscopes of high powers are used. I revert to this in the next section.

My observations completely corroborate Professor Fowler's to the effect that there are frequent cases where bright intervals between

faint Fraunhofer lines on circumpenumbral regions seem to be continued without diminution of brightness over the umbra. The meaning of the observation is by no means clear.

In my incomplete knowledge of the literature of the subject, I have not yet found whether it is already generally recognised that many lines can be seen to be darkened over considerable regions outside the penumbra. For instance, the lines attributed to iron at wave-lengths

6280·83
6298·01
6301·72

have on several occasions been seen to be darkened over circumpenumbral regions to distances amounting to two or three diameters of a large spot, *i.e.* to distances comparable with two or three minutes of arc.

The extensive flocculi, which Hale has disclosed with his splendid spectroheliographic records, show that the spot proper is but a small part of the whole area of disturbance. We may reasonably expect lines to be affected over regions comparable with the area of such flocculi; and in attempting to get spectroheliographic records in the light of the iron lines, it would be well to distinguish between iron lines which show circumpenumbral darkening and those which show no such darkening.

Of the reality of the phenomenon I have been able to assure myself by observing the spectrum of the circumpenumbral regions without having the spot upon the slit. The darkening of the lines was detected, for instance, in the case of a spot having a penumbral diameter of 40″, and was noted as being quite marked when the penumbra was still 40″–50″ from the slit.

Mr Evershed, whose attention I called to the effect, told me it was one of those phenomena which he had recognised, but had felt unsuitable for systematic recording because the more one tried to fix it, the more it evaded one's power of recording.

It is difficult to know whether to interpret it as a true darkening of the lines, or rather as due to the brightening of the continuous spectrum.

It will be noticed that one of the lines 6301·7 instanced above as exhibiting this circumpenumbral darkening is one of the lines used by Dunér in his spectroscopic determination of solar rotation.

On several occasions I have been able to see
of the outer and dark edge of the penu
identical with those seen in the umbra
ought to be of importance as bearing on
form and level of sun-spots.

It does not seem necessary at prese
wave-lengths of lines observed as aff
have been recorded by Fowler, Corti
tions support theirs in a vast numb

to stating that, after a good deal of preliminary study, I devoted myself to the careful recording of lines affected in the region C–D, as seen on five good observing days, with the equatorial equipment. I have compared these records with the photographic records obtained with the horizontal equipment, and have found that the photographs contain evidence of more "affected" lines than were seen visually. They also contain evidence (which has not been yet systematised or exhausted) of the difference between the ways in which lines are affected over the umbra, the penumbra, and the circumpenumbral regions.

Out of 120 lines which in the visual observations with the equatorial equipment were recorded as "affected lines," I find that

66 are entered as seen in the umbra only.
20 are seen darkened in the umbra and penumbra.
5 are entered as darkened over circumpenumbral regions.
7 are recorded as obliterated.
3 are regarded as exhibiting true widening.
19 appear to belong either to the umbral or to the penumbral group, but the records are indecisive.
10 are recorded as unidentified with Fraunhofer lines.

Photographic records have been secured of the sun-spot spectrum from 5100 to 6700, thanks to the excellent pinocyanol-bathed plates of Wratten and Wainwright. Plate 3 is a photographic reproduction (untouched by hand) of part of one of the original negatives. It serves as a specimen of the kind of material obtained in the red region of the spectrum, where the selective absorption of vanadium is very marked; no less than 10 of the lines which are intensified over the umbra, are due to that element (Rowland's identification).

Remarks on Widening and Darkening of Affected Lines.

Had the observations of sun-spot spectra been restricted to those made with the horizontal equipment and the more powerful spectroscope, I should have unhesitatingly said that the term "widening" was a misnomer. Most of the lines that attract attention as characteristic of the sunspot spectrum are lines that are faintly represented in the (Fraunhofer) spectrum of the neighbouring solar surface. The observations at Cambridge are on this point in agreement with those of Professor Fowler and of Messrs Hale and Adams. But the "affected" lines seem to me to be *darkened* rather than *widened*; so seldom do they appear to be widened when seen through the powerful spectroscope, that I have been tempted to think that the few outstanding cases of apparent widening, would, with higher power, be resolved.

Both visual observations and photographic records afford strong evidence of this. Plate 3 exhibits many instances of lines that stand out as umbra lines and are quite as sharp as any Fraunhofer lines. I would maintain, not that apparent widening does not

SUNSPOT SPECTRUM, PHOTOGRAPHED AT CAMBRIDGE OBSERVATORY 1906 AUGUST 31

exist, but that widening is not a characteristic peculiar to sun-spot spectra. The widening due to the symmetrical wings of the magnesium lines at *b* is more marked generally over circum-penumbral regions than over the umbra, according to my observations this summer; whilst the reverse is more generally true of the sodium lines at D. The three lines recorded in the last section as exhibiting true widening are all calcium lines.

A good typical instance of what I refer to is afforded by the line at 6306. This line has been described as "generally much widened." It is often entered as due to terrestrial oxygen, Rowland's identification of the line 6306·024 being accepted. Professor Fowler has ascribed it to scandium. My observations of the line with the equatorial equipment supported the idea of true widening (in spite of its unsymmetrical nature) to such an extent that I find in the diary notes the following entry: "1906 July 18. Umbra line at 6306 studied carefully. This is a strong line and sometimes looks like a much widened line, sometimes looks more as if it [Rowland's A(O) line] had a dark line near it. To-day it certainly looks like true widening, the only case of widening that I am quite sure of yet."

It hardly needed more than a glance with the more powerful spectroscope of the horizontal equipment to decide the case against widening. The fact is, a new line appears in the spectrum of the umbra on the violet side of the oxygen line. Even the photographic records show the line as double. Measurements give the wave-length of the umbra line as 6305·86, whilst the A(O) line has wave-length 6306·02. Rowland gives a Fraunhofer line of intensity 0000 at wave-length 6305·878. It would be interesting if this turns out to be the scandium line which Professor Fowler finds in Thalén's tables.

These details have been given because they bear on the general question of "darkening" *versus* "widening" of sun-spot lines; they bear also on the curious coincidence of telluric lines with affected lines in the sun-spot spectrum, to which Father Cortie and other observers have referred.

Bands in the Spectra of Sun-spots.

Suspected Fluting in the Green Region, 5000–5168.—There are many isolated lines confined to the umbra (*e.g.* vanadium lines) which cannot be yet put into the category of elements of a band or fluting; for criterion as to whether a given line is isolated or an element of a band, we require analysis such as Alexander Herschel was the first to suggest and to apply, and such as Deslandres has applied with such success to the banded spectrum of nitrogen, etc. Still, as a first attempt, the observer is led by a sort of instinct to pick out lines as belonging to a band; and in my earlier observations in 1905 December and 1906 January, I devoted some time to the study of umbra lines in the region 5100 to 5168, estimating

wave-lengths with a view to attempting to determine the probable chemical origin of the fluting.

It was at once evident that my observations confirmed Fowler's view that the lines which seemed to be specially characteristic of the sun-spot spectrum were in reality also feebly represented in the solar spectrum.

The lines do not agree in position with Rowland's carbon lines, nor can they be made to agree either by the assumption of some displacement common to all, or by the assumption that the observed dark lines are intervals between bright lines.

I expected, by plotting the estimated positions of the lines carefully on an enlarged reproduction of Rowland's map, drawn (on a scale of 1 cm. to 1 tenth metre) on a paper millimetre scale, to get values of wave-length trustworthy to the tenth of a tenth metre—accuracy sufficient for identification of the fluting. The recorded positions were read off with care to the hundredth of a tenth metre, and I intended to discard the second decimal. But a comparison of my results with the mean values which were published shortly afterwards by Hale and Adams, as determined from their beautiful photographs of sun-spot spectra,* showed such unexpected precision that it seems well to record the experience for the benefit of observers who might use the method in the absence of refined micrometers.

Photographic Measures, Hale and Adams.	Visual Estimates, N.	
5135·83		
	36·50	pair
36.65	36·65	
38·58	38·59	
38·96	38·97	
40·44		
41·40	41·48	
	43·60	pair
43·83	43·81	
44·21		
45·93		
	47·92	
48·93	48·89	
49·67		
49·99		
50·36	50·30	pair
	50·55	

Photographic Measures, Hale and Adams.	Visual Estimates. N.	
5151·37		
51·63		
	52·28	
52·82		
56·53	56·53	pair
	56·74	
57·20		
57·79		
58·81		
59·96	59·92	pair
	60·11	
60·39		
	62·90	pair
63·10	63·11	
	63·59	
63·79		

My attempts to find a known fluting that will correspond to the lines recorded in the table have been hitherto unsuccessful.

* *Astrophys. Jour.* xxiii. 11–44, 1906.

My estimates of the wave-lengths agree more closely with Hale and Adams' measures than with Mitchell's observations.

Suspected Flutings in the Red Region, 6370–6390.—The groups of lines recorded by Young, Mitchell, Cortie, and Fowler, as seen in the red region of the spectrum about wave-length 6381 and 6388, have been studied carefully on several occasions; and the set of observations made with the equatorial equipment on July 30, 1906, have given the following data for 21 lines between wave-lengths 6377–6393. The measures given in the last column were made on a photograph (R.S., 208) taken 1906 August 31.

Visual Estimates.	Intensity.	Photographic Measures.
6377·81	Dark line 3	·76
78·15	Narrow bright streak	
78·48	Dark line 3	·47
78·94	,, 4	·99
79·64	,, 3	·47, ·94
80·31	,, 4	0·94
81·83	,, 3	1·60
82·33	,, 4	·37
82·65	,, 2	·75
82·84	Limits of	·84
....	bright streak	
83·75		·55
84·04	Dark line 2	3·98
84·43	,, 2	·36
84·84	,, 3	·86
85·10	Limits of	4·96
85·45	bright streak	5·26
85·72	Dark line 4	·82
6386·06	Dark line 4	·11
86·76	,, 3	·57, ·79
87·57	,, 3	·27, ·77
88·46	,, 3	·36, ·60
89·02	,, 3	·00
89·56	,, 4	·18, ·48
89·73	Limits of	·68
90·58	bright streak	·39
90·82	Dark line 3	·49, ·81
91·10	Limits of	0·99
91·44	bright streak	
92·57	Dark line 3	·76
93·18	,, 2	·20
93·84	Unaffected Fraunhofer line, intensity 6 on same scale	

These groups often have the appearance of bands degraded towards the violet side and with heads at

	6382·33 [illegible]	[illegible]389·56
Cortie gives the values	6380·96	[illegible]88·63
Fowler gives	6381·6	[illegible]390·0

The banded appearance is naturally least [illegible] than the li[illegible] are seen most distinctly. From the [illegible] [illegible]nsity giv[illegible] in the table above, it would be diffic[illegible] [illegible] the "head[illegible] of the bands.

On two occasions towards the end [illegible] [illegible]ions in la[illegible] August (1906), there were signs of an[illegible] [illegible] degrad[illegible]

towards the violet, and with its head not far from wave-length 5210. Attempts to determine the position of the head on two different days led to the following results:—

1906, Aug. 11	5210·2
" " 21	5211·0

This dark hypothetical fluting is recorded "as quite marked at times and not an illusion due to the proximity of Fraunhofer lines, for the E group is not so affected. The part of the spectrum between 5210 and 5200 is the darkest part of spot spectrum in the region between 5100 and 5300." This fluting is at a region where there are many umbra lines, and had the appearance of a darkening of the unresolved background of the spot spectrum.

Laboratory Experiments on Resolution of the Background of Arc Spectra.

Probably all spectroscopists have realised that, in studying the arc spectrum of, let us say, iron, they concentrate attention upon the bright lines of iron and consent to disregard the "continuous background" on which the lines are generally seen. When, however, powerful spectroscopes are used and attention is paid to the background, it is seen to be resolved into fine lines under conditions which are generally not quite under the control of the observer. In many cases the lines show signs of arrangement in bands and flutings; in other cases the signs of structure of that kind are not detectable.

The resemblance of this varied structure in the background of bright arc spectra to the structure that is visible under what we are tempted to call the finest conditions of seeing in the spot spectrum, must have struck many observers. At any rate it caused me no surprise when, in the spring of this year, I gathered from Mr Fowler that he had been looking for detail in arc spectra similar to that which had led him to identify some flutings in star spectra with the reversal of the flutings discovered by him in the titanium spectrum and attributed by him to oxide of titanium.

I have, both before and after my conversation with Mr Fowler, secured a few photographs of the background of some arc spectra with a very powerful grating spectroscope, and have compared some of the details exhibited on the photographs with the fine structure of the sun-spot spectrum. So far I have not had much success in finding similarity. But in one case I find resemblances which call for further investigation of the sort; for bright flutings degraded towards the red have been found in the spectrum of the arc between iron terminals, which had been tipped with lime,—flutings which coincide in position and direction of degradation with unsymmetrically winged lines darkened in the sun-spot spectrum close to the wave-lengths (*a*) 5504 and (*b*) 5528.

I should add, however, that I have found no flutings recorded by other observers of sun-spot spectra at these wave-lengths.

Mitchell's observations agree with those of Hale and Adams with respect to darkened lines near one position (*a*), but not in the second case (*b*).

	Mitchell.		Hale.	
(*a*)	5504·12 Ti	Darkened	04·12	
	06·10 Ma	Generally much darkened	06·10	
	07·00 Fe	Winged	07·00	Winged
(*b*)			20·73	
	5522·66			
	25·77	Hazy		
			27·80	
			29·38	
	30·99	Darkened		

Search for Evidence of Local Convection Currents on the Sun's Surface.

On many occasions when we were using the equatorial equipment, special attention was given to a search for violent motions in the line of sight in the neighbourhood of sun-spots. In 1895 I spent five weeks at the Riffelberg, making spectroscopic observations of the Sun's surface with a grating spectroscope. Several times I found evidences of violent local downrushes of hydrogen, which were detected by what one may call isolated islands of the C line, showing large displacement towards the ultra-red. These downrushes of hydrogen were indicated by a dot in the spectrum, and the absence of other general darkening of the spectrum proved that they were not connected with any *formed* sun-spot. They persisted on two or three occasions for many hours, and could easily be picked up by allowing the solar image to trail across the slit of the spectroscope; the small black island of C line would suddenly flash out and in another moment be gone, unless the trailing of the image were stopped. Some correspondence that I had with Dr Vogel and Father Sidgreaves at the time, led me to the view that the phenomenon was unusual. I had, without great success, tried to connect the positions of the down[illegible] hydrogen with positions in which spots appeared later.

I made further attempts during the past sum[illegible] similar phenomena, but though I regularly exa[illegible] surface in this way with the equatorial equip[illegible] operation of trailing the whole surface of the Sun [illegible] 30 minutes to complete on each day that it was [illegible] could detect a single instance of the occurre[illegible] downrush. It may be pointed out that it is j[illegible]

the period of the sun-spot cycle—since the previous observations at the Riffelberg disclosed the apparent frequency of the phenomenon.

The observations, however, served to show also how seldom it occurs that large disturbances of the kind that would be detected by distortion of the lines in the spectrum are to be seen on the Sun's surface. Belopolsky has called attention to this point in his note published in vol. i. of the *Transactions of the Union for Solar Research.* If this observation is further corroborated, it would tend to show that the violent convection currents, which we believe are needed to keep up the supply of radiation from the Sun's surface, and which are so well brought to mind by Professor Schuster in his Glasgow lecture on solar evolution (*Astrophys. Jour.*, xvii. 173), must be confined to strata below the reversing layer. This is a result which it seems difficult to reconcile with other considerations.

I have great pleasure in acknowledging my obligations to Mr W. H. Manning for his efficient help in these observations under circumstances that called for much patience.

To Mr J. B. Hubrecht also, whom we are glad to have at the Observatory, working as a research student in astrophysics, I would here render thanks for his help on many occasions when he has been at hand making preliminary observations for a determination of the rotation of the Sun, to be derived from photographic records taken in the spectroscopic method.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXVII. JANUARY 11, 1907. No. 3

W. H. MAW, Esq., PRESIDENT, in the Chair.

Rodney Boyce, Soudan Survey Department, c/o Royal Colonial Institute, Northumberland Avenue, London, S.W.;
Arthur Cleminson, Deputy Commissioner of Lands, Lagos, West Africa;
George Innes, M.P.S., Olive Bank, Liberton Brae, Edinburgh;
Arthur Kent Lucke, Suez Canal Company's Service, Transit Department, Ismaïlia, Egypt; and
George Street, M.A., Merton House, Southwick, Sussex,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Edward George Bloomfield Barlow, Ditton Lodge, Stourwood Avenue, Bournemouth (proposed by Col. E. E. Markwick);
Lieut. F. G. Cooper, R.N.R., H.M.S. *Ocean*, 131 Sutton Court, Chiswick, W. (proposed by E. W. Owens); and
Edward Power, F.S.A., F.G.S., 16 Southwell Gardens, S.W., and Watership, Newbury, Berks (proposed by W. S. Franks).

Fifty-one presents were announced as [illegible] been received since the last meeting, including, amongst [illegible]:—

W. H. Pickering, Lunar and Hawaiian [illegible] features com-pared, presented by the author; 20 char[illegible] Astrographic Chart of the heavens, presented by the Roy[illegible] Greenwich; Engraving from portrait [illegible] W[illegible] for the Royal Society, presented [illegible] the [illegible] Portrait Fund.

The Perturbations of Halley's Comet. By P. H. Cowell and A. C. D. Crommelin.

Shortly after the December meeting we decided to undertake jointly the computation of the perturbations of Halley's Comet. Wishing to ascertain as rapidly as possible whether Pontécoulant's date of the next perihelion passage (1910 May 23) was approximately correct, we made a preliminary computation of the Jupiter perturbations, dividing the comet's orbit into eighty portions, and closely following Pontécoulant's method. We introduced, however, two modifications, which we think are improvements:—

(1) Pontécoulant has made his computation needlessly long by the retention of a number of meaningless and superfluous figures; thus he determines the perturbing forces X, Y parallel to the principal axes of the ellipse to two places of decimals, implying, as a rule, three significant figures; but the product of these, by the factor reducing the perturbation to seconds of arc, is given to six significant figures, of which only three can be trusted. We have restricted ourselves to the reliable figures, which involves no loss of accuracy.

(2) Pontécoulant performs the multiplication by these factors separately for each element of the orbit. We have first taken the sum of the components and multiplied by the reducing factor once for all at the end, thus saving much labour.

We are now undertaking a more accurate investigation of the perturbations, dividing the orbit into 180 portions, and including the perturbing effect of Venus, the Earth, and Neptune, which Pontécoulant did not consider. We therefore deem it unnecessary to do more at present than give the two main results of our preliminary work, which are as follows:—

(1) 1910 May is the correct date within a month for the next perihelion passage. Our actual result is a fortnight earlier than Pontécoulant's, but we lay no stress on the difference.

(2) Our computations confirm the suspicion expressed in December, that Pontécoulant's value of the eccentricity in 1910 is notably in error. In fact, we make the perihelion distance appreciably the same as at the last return (0·59), whereas he increased it to 0·68. This change is of some importance, as it would considerably affect the geocentric path of the comet at the next return, and would also considerably modify the point at which the meteors accompanying the comet would intersect the Earth's orbit.

Result (1) indicates that Ångström's curve fails utterly for the next return, and throws much doubt on the reality of his two inequalities. Possibly many of the earlier returns of the comet have been wrongly identified by Hind and Ångström, and the latter's curves may thus be erroneous.

On the Errors of a Photographed Réseau. By W. H. M. Christie, A. S. Eddington, and C. Davidson.

The investigation which is here described was undertaken in connection with the discussion of the results of the Greenwich photographs of Eros 1900–1901. The determination of the division errors of the réseau as imprinted on the photographic plates to the very high degree of accuracy required, is in some respects a new problem, so that some of the methods and results may be of interest.

An account of the preliminary determination of the errors of the réseau (No. 90), which was used for all the Eros photographs, is printed in the Introduction to the Astrographic Catalogue, Greenwich Section, vol. i. p. xxxvii. These provisional division errors were applied in the reduction of the measures of photographs, but a much more extensive investigation was felt to be necessary in order to deduce a trustworthy value of the solar parallax from the results.

A preliminary comparison made in 1903 between the réseau itself and photographs of it on seven plates had shown that there were sensible differences, which called for further discussion after the heavy work of measurement of the Eros plates had been completed. It may here be explained that the réseau is imprinted on the photographic plate by parallel rays from an electric lamp in the focus of the 13-inch object-glass of the Astrographic telescope, the réseau and the plate being mounted face to face, as nearly as possible in contact just outside the object-glass. It is to be noted that the parallel rays necessarily pass through the glass of the réseau from the back before falling on the silver film.

Under the ordinary conditions the réseau is reversed with the telescope in passing from east to west of the pier, the same edge of the plate being in contact with the bearing studs, on both sides of the pier. In consequence of this reversal the division errors of the central line 14 and the adjacent line 13 or 15, between which Eros is usually placed, affect the R.A. of Eros with opposite signs on photographs taken with the telescope east and west of the pier respectively, and thus directly enter into the value of the parallax deduced from morning and evening photographs, with telescope east and west respectively.

It was therefore necessary to determine the division errors of lines 13, 14, 15 with a degree of accuracy far beyond the ... d for the other lines.

What was required in this new investigation w...
that part of the réseau where Eros falls, ref...
that part of the réseau on which those st...
co-ordinates of the plate centre were ded...
results three series of places of Eros hav...
Astrographic photographs using *refer*...
of the centre; (2) Astrographic pho...

selected within 25′ of the centre; (3) Thompson photographs using *comparison stars*. In series (2) the comparison stars may be considered to be, in the mean, fairly uniformly distributed over the square between the lines 9 and 19 in each co-ordinate; while in the other two series the stars were distributed over the square between lines 4 and 24. Different corrections must be applied to Eros in the two cases.

There were three steps in the determination:—

(1) A more accurate measurement of the errors of lines 13, 14, 15 on the silver réseau.

(2) Measurement of the systematic differences between prints of the réseau and the réseau itself.

(3) Examination of the straightness of the réseau lines.

As the declinations of Eros are not very important for parallax, only the errors in x have so far been investigated.

The numerical values of the errors given in this paper are for the scale of the Astrographic telescope ($1^{mm} = 1'$). They must be halved before being applied to the Thompson photographs. The réseau was printed on both sets of photographs at the same time and in precisely the same manner, the object-glass of the Astrographic telescope being used to give a beam of parallel rays.

The micrometer used throughout was described in *M.N.*, vol. liii. p. 326, but has been somewhat modified since then. Two microscopes, whose distance apart can be adjusted, are connected by a bar which can move longitudinally in a slide. The réseau and scale (or photographic plate) to be compared are carried in a large frame, so as to be viewed one by each microscope. The frame can move in a slide perpendicularly to the line joining the microscopes. A fixed wire in the right-hand microscope is set on the required scale division (or other mark) by moving the bar which carries the two microscopes, by means of a slow-motion screw. A movable wire in the left-hand microscope is then set on the required réseau-line by turning a micrometer-head. In order that the same reading of the micrometer may always correspond to the same distance between the two points viewed, however the slides are moved, it is necessary that either the axes of the microscopes should be parallel, or that each slide should move accurately in its plane. Probably an appreciable error arises from the non-fulfilment of these conditions when the outer parts of the réseau are viewed; the error can be detected and eliminated by turning the réseau through 180° and re-measuring. It seems to be inappreciable within the area between lines 9 and 19.

(1) *Corrections to the adopted division errors of lines* 13, 14, 15 *on the silver réseau.*

In the original determination the division errors of lines 8, 14, and 20, referred to 2 and 26, were first determined (along $y = 14$) by comparing the intervals, 2–8, 8–14, 14–20, 20–26, with the

same interval on a glass scale. The errors of the intermediate lines were then determined by the symmetrical method explained in the Introduction to the Astrographic Catalogue, vol. i. p. xxxvii.

In the re-determination, the object aimed at was to refer, with the greatest accuracy attainable, the central lines on which the place of Eros depends to the means of the lines used for reference or comparison stars, 4 to 24 or 9 to 19 respectively. With this object the interval between line 14 and each pair of lines from 4 to 24 was compared by means of the glass scale at $y = 13{\cdot}5$ and $y = 14{\cdot}5$, so that line 14 was referred in succession to the pairs of lines 4, 24; 5, 23; 6, 22 . . . . 13, 15, giving, after application of the provisional divisional errors, ten independent determinations of the division error of line 14 referred to a pair of lines. Similarly, line 13 was referred to each pair of lines 3, 23; 4, 24 . . . . 12, 14 and line 15 to each pair 5, 25; 6, 24 . . . . 14, 16. Since, in this method, each line is necessarily referred to pairs of lines symmetrically situated about itself, lines 13 and 15 could not be referred exactly to the same lines (4 to 24) as line 14, but it is clear that the accidental residual division error arising from the substitution of line 3 for line 24, and line 25 for line 4, in the two cases respectively, is negligible in the means of 20 lines.

The investigation was carried out at $y = 13{\cdot}5$ and $y = 14{\cdot}5$ independently. The accompanying table gives the result.

Table I.—Corrections to Provisional Division Errors.

$x = 13$.			$x = 14$.			$x = 15$.		
Referred to Lines.	Correction. $y = 13{\cdot}5$.	$y = 14{\cdot}5$	Referred to Lines.	Correction. $y = 13{\cdot}5$	$y = 14{\cdot}5$	Referred to Lines.	Correction. $y = 13{\cdot}5$.	$y = 14{\cdot}5$.
3, 23	$+ {\cdot}\overset{''}{0}03$	$- {\cdot}\overset{''}{0}42$	4, 24	$+ {\cdot}\overset{''}{0}87$	$+ {\cdot}\overset{''}{0}29$	5, 25	$+ {\cdot}\overset{''}{0}29$	$+ {\cdot}\overset{''}{0}32$
4, 22	+ ·026	·000	5, 23	+ ·063	+ ·004	6, 24	+ ·016	+ ·031
5, 21	− ·004	+ ·018	6, 22	+ ·025	+ ·046	7, 23	− ·008	+ ·002
6, 20	− ·015	− ·014	7, 21	+ ·004	+ ·001	8, 22	− ·031	+ ·015
7, 19	+ ·021	+ ·028	8, 20	− ·011	− ·034	9, 21	− ·048	+ ·008
8, 18	− ·027	− ·045	9, 19	+ ·004	− ·001	10, 20	− ·047	+ ·014
9, 17	− ·009	− ·035	10, 18	+ ·006	+ ·016	11, 19	− ·029	+ ·005
10, 16	− ·020	+ ·010	11, 17	− ·046	− ·029	12, [illegible]	− ·042	− ·005
11, 15	− ·024	− ·018	12, 16	− ·011	− ·005	[illegible]	[illegible] ·031	[illegible] ·044
12, 14	− ·033	− ·027	13, 15	+ ·016	− ·001	[illegible]	[illegible]	[illegible]
Means 3 to 23	− ·008	− ·013	4 to 24	+ ·014	+ ·003	[illegible]	[illegible]	[illegible]
Means 8 to 18	− ·023	− ·023	9 to 19	− ·006	− ·004	[illegible]	[illegible]	[illegible]

The two sets of means given are the [illegible] applicable to Eros when referred to refer[illegible] respectively on Astrographic photograph[illegible]

It will be noticed that line 15 has a considerable bend (″·04) in passing from $y = 13\frac{1}{2}$ to $14\frac{1}{2}$, whereas lines 13 and 14 are sensibly straight.

(2) *Systematic differences between the réseau and photographs.*

Fifteen plates were selected so as to be well representative of the different batches of Eros plates. These were one by one compared with the silver réseau; the corresponding lines on plate and réseau being compared at $y = 13{\cdot}5$ and at $y = 14{\cdot}5$. All the x lines from 3 to 25 were measured on each plate; a selection from the results is given in Tables II. and III. The errors tabulated are referred to the means of lines 9 to 19 as zero. They are differences between the prints and the silver réseau, and consequently are additional to the errors of the réseau itself. A linear term (corresponding to an apparent difference in scale between the plate and réseau) has been removed from the results for each plate. This is necessary if the plate constants a and e, found with the originally adopted errors, are not to be modified. This correction for scale is given in the last column of the tables for ten réseau intervals as applied to line 4, and with reversed sign to line 24, and proportionately to the other lines, and has been applied in forming the results given. It was determined by comparing the mean of lines 3 to 13 with the mean of lines 15 to 25, giving equal weight to each line, to correspond with the system adopted in forming the plate constants.

Table II.—Comparison of Photographic Prints with Réseau at $y = 13{\cdot}5$

Plate No.	3	4	12	13	14	15	16	24	25	Corr. for Scale.
	″	″	″	″	″	″	″	″	″	″
5160	−·13	−·08	−·01	−·01	−·02	−·02	−·03	·00	−·21	+·08
5172	−·11	−·06	−·05	·00	−·02	−·03	−·01	+·01	−·20	+·16
5183	+·14	+·14	−·06	−·10	−·06	−·04	·00	+·26	+·04	+·31
5190	·00	·00	−·05	−·04	−·04	·00	−·03	−·01	−·27	+·02
5204	−·10	−·02	+·02	·00	−·01	−·08	−·06	+·21	−·05	+·11
5218	−·19	−·09	+·02	−·07	−·05	−·05	·00	−·07	−·20	+·16
5230	−·09	−·05	+·02	−·03	−·05	−·07	+·04	+·02	−·11	+·11
5239	−·09	−·01	−·01	−·02	−·02	−·07	+·02	+·09	−·15	+·01
5260	−·03	+·01	+·02	−·05	−·04	+·04	+·03	+·03	−·05	+·03
5283	+·11	+·12	−·05	−·05	−·04	−·07	−·01	+·13	−·01	−·04
5307	−·39	−·08	−·03	−·04	−·06	−·02	−·05	+·01	−·20	+·07
5324	−·72	−·39	+·03	−·02	−·07	−·06	−·13	−·08	−·36	+·27
5336	−·08	−·06	+·06	−·05	−·04	−·05	−·03	+·08	−·08	+·04
5352	−·33	−·18	+·03	−·06	−·08	−·11	−·06	−·01	−·22	+·12
5368	+·42	+·36	−·03	−·06	−·04	·00	+·07	+·27	+·05	−·20

TABLE III.—COMPARISON OF PHOTOGRAPHIC PRINTS WITH RÉSEAU AT $y=14{\cdot}5$.

Plate No.	3	4	12	13	14	15	16	24	25	Corr. for Scale.
5160	−″·12	−″·04	+″·09	−″·05	−″·06	−″·02	+″·01	−″·05	−″·21	+″·10
5172	−·13	−·09	+·05	−·10	−·07	−·07	+·05	−·07	−·23	+·19
5183	+·08	+·08	−·08	−·06	−·06	−·04	·00	+·20	+·04	+·32
5190	−·02	+·04	+·02	−·09	−·04	+·01	+·03	−·07	−·33	+·06
5204	−·17	−·07	+·10	−·06	−·03	−·04	−·03	+·12	−·06	+·12
5218	−·12	−·12	+·01	−·03	−·08	−·01	−·03	+·04	−·08	+·12
5230	−·12	−·11	+·08	−·11	−·02	−·01	+·06	·00	−·22	+·07
5239	−·07	−·07	−·01	−·06	−·05	−·03	−·04	+·03	−·11	+·01
5260	−·05	+·05	+·03	+·01	−·06	·00	·00	−·04	−·09	+·01
5283	+·06	+·13	+·01	−·12	−·04	−·05	+·02	+·09	+·01	−·06
5307	−·39	−·15	+·02	−·12	−·09	−·04	+·03	−·08	−·25	+·07
5324	−·64	−·43	+·04	−·07	−·04	−·03	−·02	−·07	−·29	+·28
5336	−·17	−·11	+·05	−·04	−·03	−·04	−·01	+·02	−·05	+·08
5352	−·31	−·19	+·02	−·07	−·09	−·07	+·02	−·03	−·20	+·13
5368	+·39	+·38	−·02	−·15	−·06	−·01	+·06	+·23	+·03	−·16

TABLE IV.—COMPARISON OF PHOTOGRAPHS WITH RÉSEAU.

MEAN ERRORS OF 15 PHOTOGRAPHS.

x	y, 13·5	y, 14·5	x	y, 13·5	y, 14·5	Probable Error.
3	−″·106	−″·119	25	−″·135	−″·136	±″·028
4	−·026	−·047	24	+·063	+·021	±·022
5	+·017	−·004	23	+·043	+·017	±·016
6	−·026	+·029	22	+·007	+·003	±·013
7	−·003	+·011	21	−·029	+·030	±·011
8	+·056	+·040	20	−·001	+·007	±·011
9	+·029	+·027	19	+·015	+·016	±·009
10	+·023	+·030	18	+·035	+·008	±·007
11	+·021	+·038	17	+·020	+·011	±·006
12	−·006	+·027	16	−·017	+·010	±·007
13	−·040	−·075	15	−·042	−·030	±·005
14	−·043	−·055	14	−·043	−·055	

The mean differences between the réseau and [illegible] from the fifteen prints examined, are given in [illegible] with the probable errors of the values foun[illegible] mean discordances of the fifteen prints for e[illegible] will be seen that, far from being accura[illegible] réseau, the prints differ from it very [illegible]

centre the prints agree very closely among themselves. The mean discordance of the measured position of a line *on a single print* from the mean print ranges from $\pm''\cdot016$ for line 14 to $\pm''\cdot04$ for lines 9 and 19, and to $\pm''\cdot13$ for the extreme lines 3 and 25, increasing regularly with the distance from the centre. The great variability of the outside lines is mainly due to the method by which the scale correction was found. That the irregularity is not entirely arbitrary can be very well seen by examining the differences between the errors of lines 24 and 25 in Tables II. and III. Whereas the absolute errors vary greatly from print to print, the differences are very accordant.

The correction to the place of Eros derived from Table IV. is very considerable. If it is not applied there will be an apparent spurious parallactic displacement of Eros amounting to about $0''\cdot1$ between east and west positions, and an error of at least $''\cdot025$ in the final value of the solar parallax, deduced from the Astrographic photographs.

(3) *Examination of the Straightness of the Lines.*

So far, all measurements described have been confined to the strip of the réseau near $y=14$, and the result enables Eros to be accurately referred to stars distributed over this strip. But this strip may in the mean have an error relative to the area covered by the reference and comparison stars. If the ruling machine is not absolutely accurate, a curvature of all the réseau lines may exist. Previous investigation had shown that if such a curvature exists at all it must be very small, but it might not be negligible in the present problem.

The straightness was tested by comparing the lines directly with a spider web mounted parallel to them in the double frame of the micrometer as for the comparisons of the réseau and photographic plate. Sir David Gill has described a somewhat similar comparison with a spider line (*Memoirs R.A.S.*, vol. 51), though in that case the réseau line could not be compared directly with the spider-line, as was practicable with the Greenwich measuring apparatus.

In order to eliminate irregularities due to "knots" in the spider line, measures were repeated with the spider line turned over front to back. A more serious error (which was attributed to the slide which carries the spider-line and réseau not moving perfectly in its own plane) could only be eliminated by turning the réseau through 180° and measuring again.*

Experiments were made both on the prints and the réseau

* When the réseau is turned through 180° the curvature or other irregularities measured against the spider-line will be reversed in direction; instrumental errors will be unaltered. The instrumental error associated with, say, $y=9$ réseau direct will be associated with $y=19$ réseau reversed, so that the instrumental error will not be eliminated from the individual points, but from the means of pairs of points such as $y = 9$ and 19, and from the mean curvature.

itself. For the réseau the investigation was almost entirely confined to line $x = 14$, since it happened to be impracticable (without modifying the micrometer) to measure any other line both in the direct and reversed position. Four sets of measures were made at different times, and the following table gives the results.

TABLE V.—CURVATURE OF RÉSEAU LINE $x = 14$.

(*Sagitta for $y = 5$ to 23.*)

	Series.	1	2	3	4
Réseau D Wire D	... ...	″ ·000	″ − ·030	″ − ·022	″ − ·046
,, D ,, R	... ...	...	− ·034	− ·074	− ·030
,, R ,, D	... ...	− ·068	...	+ ·014	+ ·002
,, R ,, R	... ...	...	...	+ ·006	+ ·006
	Means	− ·034	− ·032	− ·019	− ·017

In this table the "curvature" has been measured by the discordance between the middle of the line (represented by the mean of $y = 12, 13, 14, 15, 16$) and the line joining the two ends (represented by the means of $y = 4, 4\frac{1}{2}, 5, 5\frac{1}{2}, 6$, and $y = 22, 22\frac{1}{2}$ $23, 23\frac{1}{2}, 24$). As the réseau was not reversed in Series No. 2, instrumental error is not eliminated from the corresponding mean.

In examining the *photographed* lines the treatment was somewhat different. The aim of the investigation was to obtain the mean error of the central strip with reference to the areas covered respectively by the reference and comparison stars. Eight plates were selected (from the fifteen previously referred to) and measured according to the following scheme. It must be remembered that lines 1 to 13 could only be measured with the réseau direct, and lines 15 to 27 with the réseau reversed.

Straightness of Lines on Prints, Scheme of Measures.

Plate.	Date. 1900.	Réseau Lines. x.	Prints.	Spider Line.
5172	Oct. 20	4, 6, 8, 10, 12, 14	Direct	Reversed
5190	,, 26	14, 16, 18, 20, 22, 24	Reversed	Direct
5204	,, 28	4, 6, 8, 10, 12, 14	Direct	Direct
5218	Nov. 10	14, 15, 17, 19, 21, 23	Reversed	R[illegible]
5239	,, 15	14, 16, 18, 20, 22, 24	Reversed	[illegible]
5260	,, 22	5, 7, 9, 11, 13, 14	Di[illegible]	[illegible]
5307	Dec. 15	14, 15, 17, 19, 21, 23	[illegible]	[illegible]
5352	Jan. 8	5, 7, 9, 11, 13, 14	[illegible]	[illegible]

Measures were made at $y = 9\frac{1}{2}$, $10\frac{1}{2}$ [illegible]
$16\frac{1}{2}$, $17\frac{1}{2}$, and $18\frac{1}{2}$, and also at either [illegible]
$20\frac{1}{2}$, $21\frac{1}{2}$.

Within the area covered by the comparison stars the irregularities appear to be simply accidental. Table VI. shows the results for this area. The means of the vertical columns are made zero. The smallness of the horizontal means seems to show that within this area there is no appreciable error of the ruling machine. The mean deviation from straightness (including error of measurement and accidental error of the prints examined) is $\pm''{\cdot}024$.

The mean error of the central strip referred to the square 9 to 19* is found (from Table VI.) to be $-''{\cdot}006 \pm ''{\cdot}005$, a quantity which is almost negligible even for parallax.

In the larger area occupied by the *reference stars* curvature is sensible. For line 14 it amounts to $-''{\cdot}048$ (measured by the sagitta from 5 to 23), which may be compared with the results for the silver réseau given in Table V. The results for the other lines cannot be given separately, since instrumental error was only eliminated from their mean. The mean curvature of all the even lines from $x=4$ to $x=24$ (measured in the same way) is $-''{\cdot}020$.

It may legitimately be asked whether, in spite of all precautions, the errors determined in the foregoing investigations and the accuracy claimed for the values found may not be to some extent illusory. Is it quite certain that all instrumental errors have been eliminated? Is it certain that there may not be other sources of division error which have not been considered? Fortunately a discussion of the residuals of the comparison and reference stars supplies an independent check on the results obtained by direct measurement. Before the measurements of the réseau and its prints were made, it was realised that considerable corrections were required in order to reconcile the places of the stars determined with the telescope west and east respectively. The plate centre was the same for all plates taken on any one night (evening and morning); if on the evening photographs the image of a star falls at $x=12$ $y=10$, on the morning photographs (with the réseau reversed) it will fall at $x=16$ $y=18$. It can be shown that the difference in the places of the star determined from the two sets of photographs should be equal to the sum of the residual division errors of these two points of the réseau.

An examination of the places of the stars was made for all nights on which at least two plates were taken with the telescope in each position. There were 14 nights thus available; the number of comparison stars discussed was 73, and of reference stars 146. The comparison stars were referred to the mean of the comparison stars on the plate; accordingly the division errors

* It will be seen that the square 9 to 19 is represented by 110 points evenly distributed over it, each measured on at least two prints.

TABLE VI.—TEST OF STRAIGHTNESS OF RÉSEAU LINES ON PRINTS.

y \ x	Plate Direct.						Plate Reversed.						Mean.
	9	10	11	12	13	14	14	15	16	17	18	19	
18½	+″·01	+″·05	−″·01	″·00	+″·02	+″·02	″·00	−″·01	−″·04	″·00	−″·06	−″·02	−″·005
17½	−·01	+·02	+·02	−·02	+·01	+·01	−·01	−·03	−·03	+·03	+·02	−·06	−·004
16½	−·01	−·01	−·03	−·02	−·01	−·01	−·01	−·05	·00	−·05	+·01	−·04	−·021
15½	+·01	+·02	+·03	−·01	−·01	−·03	−·03	·00	+·05	+·01	+·01	−·01	+·008
14½	−·02	−·07	−·03	+·02	−·01	−·01	−·02	+·01	+·01	−·06	+·01	−·01	−·015
13½	−·05	·00	+·01	−·03	−·01	·00	+·01	·00	+·04	+·04	+·02	·00	+·003
12½	·00	+·01	+·02	−·03	+·01	−·02	+·01	+·06	+·03	+·01	−·01	+·02	+·012
11½	+·03	+·02	−·01	+·03	+·04	·00	−·01	·00	−·02	+·02	·00	+·06	+·016
10½	+·03	−·03	·00	+·03	−·03	+·03	+·02	+·03	−·03	+·02	−·01	+·02	+·007
9½	+·01	−·02	·00	+·02	·00	+·03	+·02	−·01	−·01	−·03	·00	+·03	+·001

The two sets of errors for line 14 are derived from plates measured direct and reversed respectively.

deduced are referred to the square 9 to 19. The reference stars similarly give division errors referred to the square 4 to 24.

Sums of Pairs of Residual Division Errors.

Comparison Stars (from $y=9$ to $y=19$).

$x=$	9, 19	10, 18	11, 17	12, 16	13, 15	14, 14
From stars	$+''{\cdot}04$	$+''{\cdot}01$	$+''{\cdot}06$	$''{\cdot}00$	$-''{\cdot}10$	$-''{\cdot}09$
By direct measurement (Table IV.)	$+{\cdot}04$	$+{\cdot}05$	$+{\cdot}04$	$+{\cdot}01$	$-{\cdot}11$	$-{\cdot}10$

Reference Stars (from $y=4$ to $y=24$).

$x=$	5, 23	7, 21	9, 19	11, 17	13, 15
From stars	$+''{\cdot}08$	$+''{\cdot}05$	$+''{\cdot}04$	$-''{\cdot}04$	$-''{\cdot}11$
By direct measurement (deduced from Table IV.)	$+{\cdot}03$	$+{\cdot}03$	$+{\cdot}06$	$+{\cdot}04$	$-{\cdot}08$

In the latter table by the division error about line 5 is meant the mean division error between 4 and 6; similarly for the other lines. The division error has been regarded as independent of y, so that all stars between the given limits of x were used.

Fifteen comparison stars fall in the strip bounded by $y=13$ and 15 $x=9$ and 19. After correcting their residuals for the revised division errors which depend on x (Table IV.), the remaining differences (between their positions determined from plates with réseau direct and reversed) should be equal to twice the error of the strip referred to the whole square. In this way we find from the mean of the fifteen stars the error $-''{\cdot}02 \pm ''{\cdot}006$, which may be compared with the direct determination (by use of the spider-line) $-''{\cdot}006 \pm ''{\cdot}005$.

The probable error $\pm''{\cdot}006$ of the result given above was inferred from the discordances *inter se* of the residuals for the fifteen stars. The mean discordance of these is $\pm''{\cdot}05$, which represents usually Mean of two plates W. – Mean of two plates E., and the apparent correction W. or E. is half this quantity. It results that the mean error of measurement on a single plate is $\pm''{\cdot}05$ for a comparison star. The error of measurement for a reference star (over the wider field) is apparently much larger.

Royal Observatory, Greenwich:
1907 *January* 11.

Note on the Determination of the Wire-Intervals for a Transit Instrument. By W. H. M. Christie.

In the ordinary use of a transit instrument, the transits of stars are observed over the whole system of wires (say 9 or more), and the instrumental errors (collimation in particular) are referred to the central wire, so that the wire-intervals are required mainly to refer the mean of the wires to the central wire with a high degree of accuracy.

A ready method of doing this independently of star transits is to compare the direct and reflected images of each pair of wires (reckoned from the centre), the small asymmetry of each pair, relatively to the central wire, being measured by the micrometer.

Thus, with a system of 9 wires, the observations would be arranged as follows:—

Wire.	Wire.	Wire.	Wire.
1D 9R	5D 5R	5D 5R	9D 1R
2D 8R	5D 5R	5D 5R	8D 2R
3D 7R	5D 5R	5D 5R	7D 3R
4D 6R	5D 5R	5D 5R	6D 4R

—the reflected image in each case being brought into coincidence (or contact on alternate sides) with the direct image.

Thus the interval between the central wire and the mean of the other 8 wires is determined with the full weight of eight observations; and as the distances between the direct and reflected images of the pairs of wires are very small, the result is practically independent of errors of the micrometer screw. A moderate number of determinations made in this way should suffice to reduce complete transits to the central wire.

For incomplete transits of quick-moving stars, the interval of each wire from the centre wire is required with an accuracy which should be several times greater than that of a single observation, any error in the adopted wire-interval being in this case accidental, and not systematic. A few transits of close circumpolars, or a very moderate number of transits of quick-moving stars, would amply suffice for this.

For close circumpolar stars observed on wires other than the centre wire, it would, of course, be necessary to determine the wire-interval from the centre wire with an accuracy consi
exceeding that of the observation; and for such cases,
occur, special provision would have to be made.

The great bulk of transits observed would be
method explained above.

Royal Observatory, Greenwich:
1907 *January* 10.

The Places of Zodiacal Stars for the Epoch 1900.

By A. M. W. Downing, D.Sc., F.R.S.

Some years ago the project was formed of constructing a catalogue of zodiacal stars, from all available sources, that would serve, for a decade or so, for such purposes as the supply of sufficiently accurate places of a large number of stars liable to occultation by the Moon, and of stars suitable as Moon-culminators. This catalogue (hereinafter called the Zodiacal Catalogue) was published in its completed form in 1905, as vol. viii. part iii. of the *Astronomical Papers of the American Ephemeris and Nautical Almanac*, under the title of "Catalogue of Zodiacal Stars for the Epochs 1900 and 1920, reduced to an absolute system." As it is fairly complete to magnitude 7·0, and contains a good many stars fainter than 7·0, the catalogue, as thus described, may appear to be rather an ambitious undertaking. Accordingly, I have been glad to avail myself of the opportunity offered by the publication of the *Cape General Catalogue for* 1900 (part i. of which gives the places of 1265 of our 1607 zodiacal stars) to utilise it for the purpose of checking the general accuracy of the Zodiacal Catalogue at the initial epoch.

The Zodiacal Catalogue is reduced to the system of Newcomb's Fundamental Catalogue, and contains all the suitably placed stars in the latter (211 in number), with their places unchanged. The Cape Catalogue is also reduced to Newcomb's system, both in right ascension and in declination, and, in addition, the right ascensions are corrected for personality depending on magnitude. As the Cape places are printed without inclusion of the proper motions, I have applied, in all cases, the effects of the proper motions given in the Zodiacal Catalogue to the Cape places. Only some 50 stars in the Zodiacal Catalogue have no proper motions assigned to them.

It was decided to reject differences between the catalogues amounting to, or greater than, $0^{s}\cdot2$ in right ascension and $3''$ in declination. The adoption of this criterion resulted in the exclusion of 8 right ascensions and 3 declinations of the Zodiacal Catalogue from the comparison. No Newcomb star has been excluded. A case of discordance in the right ascension of *β Virginis* was traced to an error in the Cape Catalogue, where the seconds of right ascension should be $29^{s}\cdot274$, in place of $29^{s}\cdot082$.

The means of the residuals without regard to sign for three hourly groups of catalogue differences, taken at random, are then found to be $^{s}\cdot030$ and $''\cdot45$ in right ascension and declination respectively for the complete comparison, including the Newcomb stars; and $^{s}\cdot019$ and $''\cdot31$ in right ascension and declination for the Newcomb stars.

The differences between the catalogue places have been combined in groups extending over one hour of right ascension. As the stars here dealt with are all ecliptic stars, it is not possible to separate discordances depending on right ascension from those depending on

declination. The mean declination corresponding to the middle of each hour of right ascension has accordingly been entered in Table I. This table gives the mean differences in right ascension and declination of the Zodiacal Catalogue (including the Newcomb stars) *minus* Cape in the columns headed I., and the corresponding mean differences Newcomb *minus* Cape in the columns headed II. The subscript figures are the numbers of stars occurring in each group.

As before mentioned, the Cape right ascensions are corrected for personality depending on magnitude. The effect of this is apparent in the relative values of the mean differences of the columns I. and II., under the heading $\Delta\alpha$. The mean magnitude of the stars discussed in column I. is about 6; that of the stars discussed in column II. is 4 or 4·5. As the right ascensions of stars of the fainter magnitudes have a relative correction of the negative sign applied to them in the Cape Catalogue, the difference of the means is thus accounted for. The outstanding differences occurring in column I. under $\Delta\alpha$, as well as those under $\Delta\delta$, must be ascribed to the effect of the presence of the fainter stars with proper motions of inferior accuracy to those of the stars included in the columns headed II. It will be remarked, however, that there is considerable uncertainty in the values of the individual groups in the columns headed II., arising from the comparatively small number of stars in each group.

In Table II. the same groups of differences are combined in pairs and arranged in order of declination, so as to exhibit any possible variation depending on that element. In Table III. the readings of the interpolating curves, drawn through the points representing the mean differences of Table I., are given for the beginning of each hour of right ascension, and the corresponding degree of declination on the ecliptic.

It appears from this discussion that (assuming the errors of the Cape places to be relatively insignificant) the places of the Zodiacal Catalogue for 1900 may be relied upon to within quite narrow limits for the great majority of the stars contained therein. Normally, these limits should not exceed $^{s}\!\cdot\!1$ in right ascension and $1''\!\cdot\!5$ in declination, even in unfavourable cases, as may be inferred from the values of the means of the residuals cited above.

But we now require the places of these stars for 1910, and must allow a slightly wider margin of error for that epoch, and, from it, up to the terminal epoch of the catalogue. The average variations from the Cape observations, with which they have been here compared, are not conspicuously more irregular, r' [illegible] 1900, for the fainter stars than for those occurring [illegible] Fundamental Catalogue.

TABLE I.

α,	δ.	Δα. I.	Δα. II.	Δδ. I.	Δδ. II.
h h	°	s	s	″	″
0– 1,	+ 3	+ ·023$_{50}$	+ ·007$_{12}$	+ ·22$_{50}$	+ ·26$_{12}$
1– 2,	+ 9	+ ·017$_{48}$	+ ·009$_{8}$	+ ·17$_{49}$	+ ·20$_{8}$
2– 3,	+15	+ ·010$_{49}$	+ ·011$_{10}$	+ ·09$_{49}$	− ·05$_{10}$
3– 4,	+19	+ ·026$_{55}$	+ ·009$_{9}$	− ·09$_{56}$	+ ·02$_{9}$
4– 5,	+22	+ ·012$_{72}$	+ ·004$_{12}$	+ ·17$_{72}$	− ·05$_{12}$
5– 6,	+23	+ ·015$_{61}$	+ ·012$_{4}$	+ ·05$_{60}$	− ·07$_{4}$
6– 7,	+23	+ ·013$_{58}$	− ·003$_{6}$	+ ·28$_{59}$	+ ·08$_{6}$
7– 8,	+22	+ ·015$_{51}$	+ ·012$_{11}$	+ ·08$_{51}$	+ ·07$_{11}$
8– 9,	+19	+ ·009$_{43}$	+ ·007$_{10}$	− ·01$_{43}$	− ·29$_{10}$
9–10,	+15	+ ·019$_{40}$	− ·005$_{10}$	+ ·01$_{41}$	+ ·03$_{10}$
10–11,	+ 9	+ ·024$_{44}$	·000$_{9}$	+ ·09$_{45}$	+ ·01$_{9}$
11–12,	+ 3	+ ·035$_{47}$	+ ·008$_{10}$	+ ·08$_{47}$	+ ·29$_{10}$
12–13,	− 3	+ ·043$_{47}$	+ ·010$_{5}$	+ ·11$_{46}$	− ·08$_{5}$
13–14,	− 9	+ ·023$_{48}$	− ·005$_{6}$	+ ·06$_{48}$	+ ·03$_{6}$
14–15,	−15	+ ·032$_{47}$	− ·001$_{11}$	+ ·13$_{47}$	− ·11$_{11}$
15–16,	−19	+ ·032$_{55}$	+ ·006$_{10}$	+ ·21$_{55}$	+ ·11$_{10}$
16–17,	−22	+ ·050$_{61}$	+ ·006$_{6}$	+ ·26$_{61}$	− ·02$_{6}$
17–18,	−23	+ ·037$_{56}$	− ·017$_{7}$	+ ·05$_{57}$	− ·13$_{7}$
18–19,	−23	+ ·029$_{62}$	− ·004$_{7}$	+ ·09$_{62}$	− ·09$_{7}$
19–20,	−22	+ ·033$_{59}$	+ ·006$_{10}$	+ ·10$_{60}$	+ ·07$_{10}$
20–21,	−19	+ ·032$_{48}$	+ ·008$_{8}$	+ ·14$_{47}$	+ ·08$_{8}$
21–22,	−15	+ ·015$_{53}$	− ·010$_{10}$	− ·09$_{53}$	− ·16$_{10}$
22–23,	− 9	+ ·019$_{49}$	− ·002$_{7}$	+ ·09$_{50}$	− ·12$_{7}$
23–24,	− 3	+ ·033$_{54}$	+ ·008$_{13}$	+ ·14$_{54}$	+ ·11$_{13}$
Means		+ ·0250$_{1257}$	+ ·0034$_{211}$	+ ·103$_{1262}$	+ ·017$_{211}$

TABLE II.

δ.	Δα. I.	Δα. II.	Δδ. I.	Δδ. II.
°	s	s	″	″
+23	+ ·014$_{119}$	+ ·005$_{10}$	+ ·17$_{119}$	+ ·01$_{10}$
+22	+ ·014$_{123}$	+ ·008$_{23}$	+ ·13$_{123}$	+ ·02$_{23}$
+19	+ ·018$_{98}$	+ ·008$_{19}$	− ·05$_{99}$	− ·14$_{19}$
+15	+ ·015$_{89}$	+ ·003$_{20}$	+ ·05$_{90}$	− ·01$_{20}$
+ 9	+ ·021$_{92}$	+ ·005$_{17}$	+ ·13$_{94}$	+ ·11$_{17}$
+ 3	+ ·029$_{97}$	+ ·008$_{22}$	+ ·15$_{97}$	+ ·28$_{22}$
− 3	+ ·038$_{101}$	+ ·009$_{18}$	+ ·13$_{100}$	+ ·02$_{18}$
− 9	+ ·022$_{97}$	− ·004$_{13}$	+ ·08$_{98}$	− ·05$_{13}$
−15	+ ·024$_{100}$	− ·006$_{21}$	+ ·02$_{100}$	− ·14$_{21}$
−19	+ ·032$_{103}$	+ ·007$_{18}$	+ ·18$_{102}$	+ ·10$_{18}$
−22	+ ·042$_{120}$	+ ·006$_{16}$	+ ·18$_{121}$	+ ·03$_{16}$
−23	+ ·033$_{118}$	− ·011$_{14}$	+ ·07$_{119}$	− ·11$_{14}$

TABLE III.

α,	δ.	Δα. I.	Δα. II.	Δδ. I.	Δδ. II.
h	°	s	s	"	"
0,	0	+·026	+·008	+·17	+·13
1,	+ 6	+·020	+·007	+·18	+·18
2,	+12	+·016	+·010	+·10	+·08
3,	+17	+·018	+·010	+·03	-·01
4,	+21	+·018	+·007	+·04	-·04
5,	+23	+·016	+·006	+·11	-·05
6,	+23	+·016	+·004	+·16	·00
7,	+23	+·016	+·004	+·14	+·01
8,	+21	+·015	+·006	+·06	-·07
9,	+17	+·013	+·002	·00	-·10
10,	+12	+·019	-·001	+·02	·00
11,	+ 6	+·028	+·004	+·08	+·12
12,	+ 0	+·036	+·009	+·10	+·10
13,	- 6	+·033	+·001	+·10	·00
14,	-12	+·030	·000	+·10	-·03
15,	-17	+·033	·000	+·18	·00
16,	-21	+·039	+·002	+·22	+·02
17,	-23	+·042	-·003	+·17	-·07
18,	-23	+·037	-·008	+·10	-·08
19,	-23	+·032	-·001	+·09	-·02
20,	-21	+·030	+·005	+·09	+·02
21,	-17	+·023	+·001	+·03	-·01
22,	-12	+·020	-·003	+·02	-·10
23,	- 6	+·024	+·002	+·09	-·02

Micrometrical Measures of Double Stars (Fourth Series). By the Rev. T. E. Espin.

The stars measured in 1906 have been mainly those of *h*. In the list of these measures which is here given an asterisk against the number of the star denotes that the object observed is not exactly in the place given by *h*, and the difference is then given in the notes. The following stars I have not so far succeeded in identifying:—

h 5455	*h*'s place	h m s 2 49·14	N.P.D.	° ′ 58·10 (1830)	1906 Nov. 13
h 339		3 49·15		58·10	1906 Nov. 30
h 340		3 56·7		58·0	1906 Nov. 30
h 2593		11 51·54		49·10	1906 April 10, 21
h 2670		13 32·50		56·16	1906 April 9, 10
h 995		23 49·26		62·17	1906 Oct. 13, 16

h.	R.A. 1900. h m	Decl. 1900. ° ′	P. °	D. ″	Mags.	Date. 1900+.	Nights.
5450	0 6·7	+35 35	232·6	4·75	9·1, 13·0	6·87	2
622	0 20·6	+34 14	131·3	18·22	9·0, 9·0	4·78	2
1032	0 26·7	+28 58	246·3	12·29	8·3, 9·0	6·80	2
625	0 30·4	+39 43	282·9	14·38	8·9, 14·0	6·71	2
629	0 50·1	+34 1	69·3	10·38	8·5, 11·5	6·86	2 BC
			74·3	71·26	A= 8·5	6·86	2 AB
631	1 59·5	+27 27	156·9	21·92	8·8, 11·0	6·90	2
636	1 8·8	+30 0	287·5	20·57	7·7, 11·0	6·84	3
1078	1 18·9	+27 3	89·8	15·18	8·5, 11·5	6·87	2 AB
			89·7	28·34	C= 10·0	6·87	2 AC
653	2 27·6	+30 58	40·9	22·99	7·8, 11·7	6·85	2
328	2 35·9	+36 3	245·5	13·07	9·5, 11·0	6·47	2
329	2 47·9	+31 18	106·0	35·41	8·0, 13·5	6·86	1
336	3 36·4	+32 37	316·2	31·68	8·0, 11·0	6·92	2 (Σ 432 rej.)
670	3 56·4	+31 53	226·8	10·50	9·5, 9·5	6·92	2
349	4 41·1	+34 35	85·6	9·87	9·1, 9·5	4·98	2
350	4 44·6	+34 37	308·1	5·50	10·7, 10·7	6·06	2
3265	5 1·3	+36 55	137·0	14·63	9·0, 9·0	6·13	2
3266	5 1·5	+36 52	61·7	8·05	9·0, 10·5	6·13	2
3272	5 13·2	+39 14	342·9	18·86	7·7, 13·0	6·09	2 AB
			296·4	27·31	C= 13·3	6·11	3 AC
			42·5	32·69	D= 11·5	6·13	2 AD
713	5 49·6	+33 15	285·0	10·10	8·9, 9·0	6·92	2
380	6 2·8	+34 29	21·8	18·67	9·0, 9·0	6·09	4
3282	6 20·8	+38 10	313·7	15·89	9·0, 12·2	6·04	2
3284	6 39·8	+36 17	84·8	5·42	10·2, 11·2	6·09	2

h.	R.A. 1900. h m	Decl. 1900. ° ′	P. °	D. ″	Mags.	Date. 1900+.	Nights.
3285	6 46·2	+38 15	252·6	11·11	9·6, 10·7	6·04	2
411	7 1·2	+35 22	43·8	9·40	...	6·11	2
757	7 16·2	+34 27	105·9	5·28	9·8, 10·5	6·09	2
3294	7 27·5	+35 51	178·0	4·81	9·4, 9·5	6·10	2
3295	7 31·3	+39 5	15·5	22·75	8·5, 10·0	6·04	3 (Σ 1118 rej.)
3301	7 43·8	+37 28	61·3	23·47	7·7, 13·7	6·07	3
3303	7 49·3	+35 46	34·3	12·05	9·4, 11·5	6·11	2
3305	7 54·4	+37 9	229·0	4·39	9·0, 9·0	6·06	2
772	7 55·3	+35 43	58·5	12·32	9·7, 13·0	6·14	1
436	7 57·1	+35 16	80·5	12·18	9·5, 11·5	6·11	2
3308	8 3·7	+35 46	265·3	46·01	6·2, 10·0	6·19	2
793	8 34·3	+35 29	254·1	10·25	9·0, 10·0	6·12	2
2483	9 5·6	+36 32	193·9	15·19	9·0, 10·0	6·11	3
2491	9 10·6	+34 56	200·4	14·21	10·2, 10·3	6·10	3
2493	9 13·7	+34 9	157·4	9·97	10·1, 11·7	6·14	2
462	9 17·2	+30 34	7·5	18·01	9·3, 9·6	6·27	2
463	9 17·5	+30 40	347·5	24·41	9·1, 10·5	6·27	2
815	9 24·0	+33 20	144·5	13·98	8·9, 13·0	6·24	3
2509	9 46·8	+37 41	68·5	14·02	...	6·25	2
471	9 49·9	+31 9	311·9	10·45	...	6·28	1
3318	9 57·7	+36 44	339·3	23·97	9·1, 9·5	6·14	2
475	10 3·1	+32 6	172·4	27·63	6·7, 14·0	6·27	2
2531	10 22·5	+40 43	2·6	8·74	8·9, 9·3	6·14	3
2532	10 23·7	+38 29	248·2	12·64	9·0, 9·0	6·11	2
482	10 26·2	+32 54	243·4	40·96	6·0, 12·0	6·27	2
483	10 26·3	+32 42	133·5	15·34	8·8, 10·8	6·27	2
2555	10 57·7	+39 7	45·2	11·22	10·0, 11·5	6·19	2
493*	10 58·2	+33 25	327·7	17·18	9·3, 10·8	6·27	2
499	11 22·8	+36 52	254·0	39·59	8·5, 11·7	6·23	3
500	11 26·5	+36 25	26·7	24·10	9·1, 9·2	6·23	3
502	11 28·3	+37 35	219·0	12·42	9·3, 12·0	6·21	2
506*	11 33·5	+39 45	143·5	28·38	7·0, 14·0	6·32	1
510	11 45·0	+38 16	251·7	24·86	9·6, 9·8	6·28	2
844*	12 3·1	+33 29	331·3	11·55	9·0, 10·0	6·30	1
2600	12 6·2	+33 50	341·5	11·41	9·9, 10·2	6·31	2
523	12 47·1	+35 19	183·3	14·41	9·1, 9·2	6·27	2
524	12 47·7	+32 28	288·9	17·87	9·2, 11·0	6·29	3
528	13 10·4	+40 16	179·9	18·75	9·1, 10·9	6·29	2
2681	13 42·3	+33 37	262·9	13·61	11·5, 12·0	6·28	2
587	14 36·3	+37 42	298·4	16·32	8·5, 11·5	6·57	1
263	14 57·9	+38 2	112·2	17·79	9·2, 11·5	6·60	[illegible]
1340	18 38·9	+32 25	82·2	13·09	9·2, 11·6	6·57	[illegible]
1379	19 9·2	+31 28	307·0	9·30	9·3, 11·2	6·69	[illegible]

h.	R.A. 1900. h m	Decl. 1900. ° ′	P. °	D. ″	Mags.	Date. 1900+.	Nights.
1383	19 15·4	+31 22	111·6	11·32	10·0, 10·0	6·64	3
1390	19 18·3	+30 42	131·9	9·44	9·3, 14·0	6·66	2 BC (new)
			100·9	16·94	A = 9·0	6·66	2 AB
1616	21 5·7	+30 36	292·8	8·58	8·9, 9·7	6·65	3
1686	21 40·1	+31 17	230·2	13·28	9·5, 10·1	6·59	2
1688*	21 43·5	+30 48	358·7	14·90	9·5, 11·0	6·69	2
1695	21 44·2	+30 47	116·1	13·07	9·0, 11·0	6·69	2
1707	21 52·0	+31 28	331·4	8·41	9·3, 11·0	6·67	2
1722	22 1·3	+31 26	45·7	16·72	8·9, 9·9	6·64	2
966	22 30·4	+30 17	268·3	13·27	7·0, 11·0	6·68	3 AB
			276·0	36·84	C = 12·0	6·66	2 AC
972	22 48·1	+31 8	193·9	23·99	8·8, 10·0	6·76	2
1834	22 53·3	+29 50	165·9	25·56	...	6·77	2
			4·2	27·94	C = 13·0	6·78	2 (new)
			270·2	58·21	...	6·78	3
1837	22 54·7	+29 33	349·4	19·06	8·6, 12·2	6·80	2
1839	22 55·8	+40 35	294·9	13·58	8·9, 10·3	5·96	2
1858	23 9·5	+29 11	84·1	24·88	8·9, 11·2	6·74	2
1859	23 9·5	+29 18	121·3	34·57	7·0, 10·5	6·74	2
1862	23 10·9	+26 56	233·9	16·79	8·5, 10·0	6·81	1

Various Stars.

Name.	R.A. 1900. h m	Decl. 1900. ° ′	P. °	D. ″	Mags.	Date. 1900+.	Nights.
S 386 ...	0 26·9	+27 57	196·6	42·02	8·8, 8·9	6·78	3
η Cassiopeiæ...	0 41·3	+57 17	233·8	5·68	...	6·08	3
Washburn 6...	1 30·2	+32 31	105·8	2·33	9·0, 9·0	6·07	4
B.D. +29°·330	1 50·7	+29 58	306·4	53·18	8·2, 9·2	6·86	2
B.D. +30°·303	1 51·3	+30 32	270·7	66·50	7·6, 9·0	6·89	2
Σ 187 rej. ...	1 51·8	+31 5	180·2	13·05	8·6, 11·0	6·81	2
15 Trianguli	2 29·7	+34 15	17·1	138·80	5·5, 7·0	6·91	1
Σ 568 rej. ...	4 31·5	+39 17	213·0	21·50	8·0, 11·0	5·02	1
Σ 594 rej. ...	4 42·5	+39 5	335·0	7·86	8·5, 10·0	6·03	1
S 484 ...	5 22·8	+33 25	170·7	59·20	...	6·91	1
Σ 842 rej. ...	6 2·1	+36 32	18·0	29·08	8·2, 9·6	6·55	2
Σ 1139 rej. ...	7 41·8	+37 21	6·9	15·95	8·6, 9·1	6·06	2
Σ 1294 rej. ...	8 51·1	+33 18	339·7	15·01	8·5, 9·0	6·08	2
Σ 1411 rej. ...	10 3·4	+32 50	306·7	35·11	9·0, 9·4	6·08	1
Σ 1492 rej. ...	10 52·1	+31 12	164·7	21·53	7·8, 9·7	6·29	2
Σ 1610 rej. ...	12 6·6	+39 21	330·5	28·93	7·9, 9·2	5·81	2
Σ 1739 rej. ...	13 16·9	+31 1	130·7	12·37	9·2, 10·0	6·31	2
Σ 1749 rej. ...	13 24·3	+31 35	350·0	20·51	8·5, 10·0	6·30	1
Σ 2295 ...	18 8·6	+31 32	171·7	9·62	8·5, 9·0	6·65	2

Name.	R.A. 1900.	Decl. 1900.	P.	D.	Mags.	Date. 1900+.	Nights.
	h m	° ′	°	″			
A.G.C.	... 18 15·6	+29 52	172·3	17·91	9·0, 9·4	6·71	2
Σ 2359 rej.	... 18 34·7	+30 42	291·5	22·55	8·8, 12·0	6·65	1
A.G.C.	... 19 57·9	+31 23	355·8	12·15	8·3, 8·9	6·61	1
Ho 588	... 20 12·9	+31 12	15·8	8·19	8·8, 13·5	6·64	2 BC
			297·4	51·34	A = 6·5	6·64	2 AB
A 1218	... 21 19·1	+30 50	22·2	3·57	8·5, 9·7	6·65	2
Küstner 66	... 22 50·2	+32 32	1·3	3·52	...	6·88	3
Σ 2975 rej.	... 23 1·6	+32 29	288·5	29·95	9·0, 9·0	6·92	2

Notes.

h 5450 *h* has no description.

h 629 The only other measures are those of HΣ:—

AB	71°·45	10″·92	1885·83
AC	74·15	71·35,	,,

h 636 The only other measures are mine:—

287°·5	20″·08	2n	1904·78

h 3272 The only other measures are by β:—

AB	341°·7	19″·58	2n	1879·86
AC	295·0	29·36	2n	,,
AD	42·7	33·48		

h 493 *h*'s place 1^m too small. There is a third star more distant in the same direction.

h 506 *h*'s dec. should be increased 1°.

h 844 Not found; star here given is BD + 33°·2192, preceding *h*'s place by 3^m 10^s.

h 1390 The faint *comes* C is too difficult to measure satisfactorily.

h 1688 No star in this place. The star here given is BD + 30°·4529, which is 2^m following *h*'s place.

h 966 *h*'s observation was made when it was cloudy, and the primary was consequently underrated and the third star missed. The third star wa- measured by HΣ. The only m- those of HΣ:—

268°·90 13″·1-

Various St-

15 Trianguli A, strong oran-

New Double Stars. By Rev. T. E. Espin.

No.	B.D.	R.A. 1900.	Decl. 1900.	P.	D.	Mags.	Date. 1900+.	Nights.
		h m	° ′	°	″			
312	...	0 12·0	+34 35	237·3	2·15	9·6 10·0	6·95	2
313	+32·58	0 18·2	+32 27	16·3	3·95	8·7 12·7	6·95	2
314	+28·95	0 30·6	+28 41	201·7	8·36	8·5 14·0	6·77	3
315	+28·101	0 32·5	+28 40	77·7	2·04	9·1 9·4	6·78	3
316	+32·154	0 47·3	+32 43	292·3	1·92	9·3 9·7	6·95	2
317	...	0 54·9	+31 56	187·1	6·59	9·2 9·4	6·71	2
318	+30·223	1 21·0	+30 55	71·0	2·72	9·5 11·0	6·71	2
319	+32·256	1 22·9	+33 2	290·7	1·75	9·3 9·8	6·95	1
320	+33·310	1 46·5	+33 25	161·2	1·87	8·5 9·5	6·95	2 AB
				259·8	9·95	C=10·0	6·95	2 AC
321	+29·333	1 51·6	+30 5	181·2	3·63	9·2 10·0	6·82	2
322	+32·374	1 59·6	+32 39	92·2	2·32	9·5 9·6	6·96	2
323	+33·425	2 21·9	+33 39	179·8	6·35	9·1 10·3	6·96	2
324	+28·448	2 33·2	+28 28	20·2	1·80	9·1 11·0	6·97	1 BC
				185·8	32·75	A= 9·0	6·97	1 AB
325	+30·465	2 49·8	+31 10	0·1	12·92	7·9 12·5	6·94	3
326	+31·536	2 59·6	+31 39	36·1	4·79	9·8 10·8	6·91	3 BC
				35·8	102·33	A= 8·0	6·88	2 AB
327	+32·652	3 33·5	+33 9	292·9	14·00	8·3 12·0	6·95	1
328	+34·761	3 47·1	+34 46	288·4	6·82	8·3 14·0	6·99	2
329	+30·601	3 53·7	+30 31	255·9	7·27	9·0 12·5	6·94	3
330	+31·834	4 51·7	+31 7	156·1	3·95	9·2 12·0	6·92	2
331	+35·971	4 59·1	+35 32	324·0	7·60	8·6 11·0	6·95	1
332	+33·1017	5 14·8	+33 17	210·1	14·65	8·3 8·5	6·95	2
333	+31·936	5 15·2	+31 22	36·7	3·37	9·2 9·3	6·92	2
334	S 483	5 21·8	+33 42	347·9	15·09	8·0 14·0	6·11	2 BC
				50·5	95·48	A= 7·0	6·11	2 AB (S. 48
335	+32·1012	5 23·9	+32 34	330·6	2·65	9·1 9·2	6·95	1
336	+31·1027	5 31·0	+31 43	258·6	8·47	8·7 9·0	6·92	2
337	+31·1191	5 49·0	+33 13	296·7	5·45	9·1 12·0	6·92	2
338	+36·1361	6 0·9	+36 37	19·5	8·17	8·5 11·5	6·94	2
339	+32·1460	6 54·8	+32 33	186·7	16·40	6·5 13·0	7·04	2
340	+31·1491	7 0·1	+31 51	139·5	5·62	9·0 9·2	6·94	2
341	+32·1522	7 13·0	+32 37	251·6	3·05	9·0 9·0	6·95	1
342	...	17 52·0	+31 21	235·4	5·82	9·0 10·7	6·67	2
343	+31·3133	17 52·7	+31 12	282·8	8·46	9·0 11·7	6·69	2
344	+33·2994	17 53·6	+33 52	30·7	8·96	8·6 9·1	6·63	2

No.	B.D.	R.A. 1900. h m	Decl. 1900. ° ′	P. °	D. ″	Mags.	Date. 1900+.	Nights.
345	+31·3195	18 7·3	+31 22	19·4	2·45	9·1 9·3	6·64	2
346	+32·3102	18 17·1	+32 10	307·0	6·10	9·5 13·0	6·58	2
347	+32·3103	18 17·2	+32 17	66·6	1·75	9·0 9·2	6·62	4
...	+29·3420	18 54·4	+30 1	172·3	17·91	9·0 9·4	6·71	2
348	+28·3210	19 5·0	+28 15	256·3	5·53	8·7 11·2	6·76	2
349	+31·3482	19 6·8	+31 35	220·8	6·65	9·3 13·0	6·58	2
350	+31·3487	19 7·0	+31 57	235·3	5·41	8·5 9·3	6·58	2
351	+33·3398	19 13·3	+33 21	82·4	6·15	8·8 11·0	6·81	1
352	+34·3504	19 17·1	+34 15	133·0	4·67	8·9 10·0	6·81	1
353	+33·3457	19 23·7	+33 7	296·4	3·36	8·6 10·2	6·75	3
354	+31·3785	19 44·5	+31 29	324·4	9·20	8·6 11·5	6·74	4
355	+31·3814	19 48·1	+31 27	294·8	13·05	7·4 13·0	6·70	3
356	+31·3816	19 48·3	+31 24	343·1	5·74	8·9 9·8	6·70	3
357	OΣ 389	19 48·7	+30 52	306·5	9·42	6·5 12·0	6·66	3 AB
				183·4	12·57	C= 8·5	6·66	3 AC (OΣ 389)
358	+31·3914	19 59·7	+31 33	197·3	7·42	8·6 10·0	6·63	3
359	+31·3915	19 59·8	+31 28	61·1	5·71	11·5 13·0	6·60	2 BC
				131·8	27·91	A= 6·5	6·60	2 AB
...	+30·3900	20 2·9	+30 58	62·3	27·37	8·8 9·5	6·78	3 AB
				251·2	33·40	C= 9·3	6·78	3 AC
360	...	20 3·5	+30 48	78·2	2·74	9·8 9·9	6·66	2
361	+30·3908	20 4·0	+30 23	113·5	4·82	9·0 11·0	6·68	2
362	+30·4008	20 19·0	+30 16	268·2	4·48	11·0 13·2	6·67	2 BC
				229·4	9·18	A= 8·7	6·67	2 AB
363	+30·4018	20 19·4	+30 33	280·4	3·05	9·3 9·3	6·71	3
364	+31·4089	20 23·4	+31 25	277·3	8·47	8·7 12·5	6·66	3
365	+31·4125	20 29·0	+31 25	288·2	2·50	12·0 12·1	6·81	3 CD
				262·1	25·72	7·7 11·5	6·73	2 AB
				318·9	33·68	...	6·81	3 AC
366	+30·4159	20 40·4	+31 2	122·7	3·29	9·1 13·0	6·69	2
367	+29·4161	20 41·0	+29 52	242·5	7·27	9·1 14·0	6·88	1
368	+30·4180	20 43·9	+30 38	357·9	2·88	9·5 9·6	6·64	2
369	+31·4226	20 44·6	+31 37	300·4	6·73	8·9 12·0	6·69	2
370	+31·4227	20 45·0	+31 26	39·2	5·81	9·3 13·5	6·82	3
371	+30·4227	20 49·9	+31 0	132·7	3·67	9·0 9·3	6·66	3
372	+31·4272	20 52·1	+31 35	144·2	2·97	9·1 9·4	6·74	4
373	...	20 54·6	+29 56	147·4	2·49	9·8 11·5	6·69	2
374	+31·4319	20 58·6	+31 22	230·4	4·5±	11·0 13·6	6·81	3
				137·1	24·78	A= 8·2	6·75	2

No.	B.D.	R.A. 1900. h m	Decl. 1900. ° ′	P. °	D. ″	Mags.	Date. 1900+.	Nights.
375	+30.4335	21 4.6	+30 38	221.9	4.40	9.0 9.5	6.64	2
376	+30.4411	21 17.9	+30 20	214.7	10.61	8.6 12.7	6.78	2
377	+31.4430	21 18.1	+31 13	210.0	2.49	9.7 10.0	6.84	2 BC
				290.8	49.38	A= 9.1	6.84	2 AB
378	+31.4470	21 24.7	+31 58	202.7	7.74	8.6 11.0	6.69	2
379	+29.4444	21 28.9	+29 48	300.3	8.77	9.0 10.5	6.72	2
380	+29.4452	21 30.4	+29 50	310.9	2.46	11.2 11.5	6.74	3 CD
				52.2	13.85	8.5 14.0	6.79	1 AB
				106.9	57.85	...	6.71	2 AC
381	+31.4539	21 40.5	+31 17	109.5	4.89	8.7 13.5	6.59	2
382	+31.4560	21 46.6	+32 11	319.1	10.95	7.7 14.0	6.73	6 AB
				320.4	60.22	C= 7.8	6.81	1 AC
383	+34.4586	21 57.0	+34 43	168.1	4.62	9.2 11.6	6.95	2
384	+31.4612	21 58.4	+31 41	66.5	2.99	9.1 9.2	6.63	2
385	+32.4340	22 2.4	+34 6	35.6	5.17	9.0 10.5	6.94	2 AB
				36.2	33.60	C=10.0	6.95	1 AC
386	+33.4427	22 2.5	+33 5	75.7	6.95	8.6 12.7	6.94	2
387	...	22 5.0	+32 53	268.5	1.65	10.0 10.2	6.74	4
388	+31.4653	22 8.8	+31 38	262.6	7.56	8.8 9.3	6.65	2
389	+29.4620	22 11.6	+30 7	261.7	7.38	9.0 10.0	6.72	2
390	+32.4406	22 18.2	+32 57	262.1	13.98	9.0 9.3	6.75	3
391	+29.4687	22 28.3	+29 50	347.1	5.64	9.2 11.2	6.79	3
392	8 Lacertæ	22 31.4	+39 6	224.7	9.36	8.8 13.2	6.90	4 Dd
				185.4	21.66	...	6.86	1 AB (Σ 29
				154.3	28.14	...	6.86	1 BC ,,
				115.9	41.67	...	6.86	1 CD ,,
393	+30.4785	22 38.9	+30 42	259.4	9.43	8.8 12.0	6.64	2 BC
				297.5	78.48	A= 8.5	6.63	1 AB
394	+29.4764	22 42.3	+30 4	338.9	4.57	9.1 11.1	6.80	2
395	+29.4812	22 51.8	+30 6	351.3	4.05	9.2 12.0	6.71	2
396	...	22 58.7	+30 50	27.7	3.81	9.3 9.4	6.67	2
397	+32.4598	23 5.6	+32 36	151.8	5.55	9.2 11.4	6.94	2
398	...	23 18.8	+31 45	264.2	4.09	9.1 11.0	6.67	2
399	+29.4937	23 24.3	+29 17	207.1	8.53	9.3 12.0	6.76	2
400	+29.4938	23 24.6	+29 18	211.3	6.40	9.3 9.9	6.76	2
401	+29.4970	23 31.8	+30 14	71.8	1.60	9.3 11.5	6.92	2
402	+31.4949	23 34.8	+32 5	88.1	4.15	9.2 13.0	6.91	1
403	+30.5001	23 35.7	+30 34	294.7	2.75	9.2 9.5	6.92	2

Notes.

322 Discordant angles.

325 Observed, in the first instance, in mistake for *h* 329.

332 A fine pair, not given in any double star catalogue as far as I am aware.

334 South's measures are :—

P. 30° 53′ Nf D. 87″·602, 1825·11. The change is due to the proper motion of A, which, according to Argelander (Bonn Observations, vol. viii.) is 0″·20 at 202° 49′, according to Porter 0″·19 at 180°. The measures of South and my own give 0″·195 at 174°·1. A. has two faint *comites* Nf and Sf.

337 Closely p *h* 713.

351, 352 The second night's measures were obtained with difficulty, through a fog.

357 There seems to be no notice of the closer *comes* in the measures of previous observers. It is a fairly easy object, even in moonlight.

365 BC Angles discordant.

374 Measures of BC very uncertain.

377, 380 Angles of BC discordant.

387 A little pair N of π Pegasi.

392 The faint *comes*, d, was detected in 1892, but has not hitherto been measured.

Observations of the Occultation of Saturn by the Moon,
1906 October 27. By John Tebbutt.

The sky was beautifully clear and the definition good during the observation of this phenomenon. A magnifying power of 74 diameters was employed on the 8-inch equatorial. The following are the observed local mean times of the different phases :—

	h	m	s
Ring began to disappear . . .	8	17	57·3
Bisection of ball	8	18	17·3
Simultaneous disappearances of ball and following edge of ring . .	8	18	48·2
Disappearance of Titan . . .	8	20	9·9
Ball began to emerge . . .	9	33	39·5
Last contact of ball . . .	9	34	21·4
Last contact of ring . . .	9	34	58·3

The beats of the chronometer were inaudible at the disappearance. The times were therefore noted by turning the eye instantly from the eyepiece to the face of the chronometer. A large fraction of a second was, doubtless, lost in this way, and the times were consequently noted rather late. The first contact of the ball was unfortunately missed, as it followed so quickly after that of the ring. At the reappearance the chronometer beats were audible,

and the observations made by eye and ear. It was difficult to fix the exact times of the contacts of the ring in consequence of its great ellipticity. A star of about the eighth magnitude, which I took to be Titan, disappeared suddenly, but, owing to the overpowering brilliancy of the bright limb, its reappearance could not be observed. It became visible when two or three minutes of arc from the limb. The brilliancy of the planet was, of course, very much less than that of the Moon. This is the third occultation of Saturn seen at this Observatory.

The Peninsula, Windsor, N. S. Wales:
1906 *November* 25.

Mean Areas and Heliographic Latitudes of Sun-spots in the Year 1905, *deduced from Photographs taken at the Royal Observatory, Greenwich; at Dehra Dûn; at Kodaikánal Observatory, India; and in Mauritius.*

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those printed in the *Monthly Notices*, vol. lxvi. p. 85, and are deduced from the measurements of photographs taken at the Royal Observatory, Greenwich; at Dehra Dûn; at the Kodaikánal Observatory, India; and at the Royal Alfred Observatory, Mauritius.

Table I. gives the mean daily area of umbræ, whole spots, and faculæ for each synodic rotation of the Sun in 1905; and Table II. gives the same particulars for the entire year 1905 and the four preceding years, for the sake of comparison. The areas are given in two forms: first, projected areas; that is to say, as seen and measured on the photographs, these being expressed as millionths of the Sun's apparent disc; and next, areas as corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. exhibits for each rotation in 1905 the mean daily area of the whole spots (corrected for foreshortening), and the mean heliographic latitude of the spotted area for spots north and for spots south of the equator, together with the mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots; and Table IV. gives the same information for the year as a whole, similar results for the four preceding years being added, as in the case of Table II.

Tables II. and IV. are thus in continuation of the similar tables for the years 1874 to 1888 on pp. 381 and 382 of vol. xlix. of the *Monthly Notices*, and for the years 1889 to 1902 on pp. 465 and 466 of vol. lxiii., and for the years 1901 to 1904 on pp. 86 and 87 of vol. lxvi.

The rotations in Table I. and Table III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made*

at Redhill, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node at mean noon of 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days. The dates of the commencement of the rotations are given in Greenwich civil time, reckoning from mean midnight.

TABLE I.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Mean of Daily Areas.					
			Projected.			Corrected for Foreshortening.		
			Umbræ.	Whole Spots.	Faculæ.	Umbræ.	Whole Spots.	Faculæ.
	1905. d							
686	Jan. 5·53	28	229	1655	1987	166	1255	2128
687	Feb. 1·87	27	395	2746	2584	270	1938	2703
688	Mar. 1·20	28	229	1992	2234	155	1385	2394
689	28·52	27	99	632	2194	66	421	2333
690	Apr. 24·80	27	170	1145	1865	113	782	2009
691	May 22·02	27	87	567	2348	65	437	2632
692	June 18·22	27	222	1523	2309	156	1123	2533
693	July 15·43	28	309	2174	2633	230	1672	2902
694	Aug. 11·64	27	181	1200	3057	130	882	3238
695	Sept. 7·89	27	136	916	2476	98	692	2642
696	Oct. 5·16	28	399	3096	2401	288	2206	2610
697	Nov. 1·45	27	365	2607	3009	263	1939	3176
698	28·76	27	190	1204	2707	133	850	2829

TABLE II.

Year.	No. of Days on which Photographs were taken.	Mean of Daily Areas.					
		Projected.			Corrected for Foreshorteni[illegible]		
		Umbræ.	Whole Spots.	Faculæ.	Umbræ.	Whole Spots.	[illegible]
1901	359	14	41	23	9	2[illegible]	[illegible]
1902	349	14	86	163	10	6[illegible]	[illegible]
1903	350	67	434	875	51	[illegible]	[illegible]
1904	363	93	653	1639	67	[illegible]	[illegible]
1905	364	230	1637	2433	163	1[illegible]	[illegible]

TABLE III.

No. of Rotation.	Date of Commencement of each Rotation.		No. of Days on which Photographs were taken.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heliographic Latitude of Entire Spotted Area.	Mean Distance from Equator of all Spots.
				Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
	1905.	d							
686	Jan.	5·53	28	594	11°·88	661	15°·29	− 2°·44	13°·68
687	Feb.	1·87	27	682	13·37	1256	16·75	− 6·14	15·56
688	Mar.	1·20	28	875	10·52	510	16·48	+ 0·58	12·71
689		28·52	27	175	16·42	246	18·08	− 3·71	17·39
690	Apr.	24·80	27	439	16·55	343	15·88	+ 2·34	16·26
691	May	22·02	27	224	11·61	213	14·25	− 0·99	12·90
692	June	18·22	27	688	9·52	435	14·47	+ 0·23	11·44
693	July	15·43	28	1142	12·55	530	14·96	+ 3·83	13·31
694	Aug.	11·64	27	638	12·04	245	17·65	+ 3·81	13·60
695	Sept.	7·89	27	423	13·69	268	14·38	+ 2·80	13·96
696	Oct.	5·16	28	2168	11·31	38	13·74	+ 10·88	11·35
697	Nov.	1·45	27	1422	9·59	517	14·71	+ 3·11	10·96
698		28·76	27	429	10·55	421	15·31	− 2·25	12·91

TABLE IV.

Year.	No. of Days on which Photographs were taken.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heliographic Latitude of Entire Spotted Area.	Mean Distance from Equator of all Spots.
		Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
1901	359	22	8°·59	7	16°·27	+2°·82	10°·37
1902	349	42	18·81	21	15·29	+7·48	17·64
1903	350	132	18·11	208	21·15	−5·85	19·94
1904	363	268	16·33	220	16·88	+1·37	16·57
1905	364	750	11·66	440	15·55	+1·60	13·10

The principal features of the record for 1905 are—

1. The great increase in the mean daily spotted area as compared with 1904, both the umbræ and the whole spots showing an advance of 144 per cent. The actual area attained, 1191, surpassed that of the year 1883, the year of maximum in the first complete cycle registered at Greenwich.

2. This increase has been fairly general throughout the year, no fewer than seven rotations in 1905 exceeding in area the most active rotation of 1904. Three periods of remarkable activity were noticed,—the first three months of the year, the month of July, and the months of October and November.

3. The faculæ have, as usual, maintained a more steady rate of advance than the spots, the mean daily area showing no great fluctuation from one rotation to another, and the whole year showing an advance upon 1904 of only 48 per cent.

4. Comparing the whole spots of the two hemispheres, the northern hemisphere has preserved and increased its superiority over the southern, the area for the former being to that of the latter as 63 to 37. In the two preceding cycles this proportion between the two hemispheres was reached about two years before the maximum, the balance being heavily in favour of the southern hemisphere by the time the maximum was reached.

5. Notwithstanding this small relative activity of the southern hemisphere, usually much the more disturbed at maximum, the distribution of spots in latitude appears to point to 1905 having been the year of maximum of the present cycle, since the mean distance from the equator of all spots barely exceeded 13°. This corresponds very closely to the values obtained in 1883 and 1893, the years of maximum in the two preceding cycles.

6. Whilst the Sun was never free from spots in 1904, there were two days on which this occurred in 1905.

7. The distribution of spots in latitude was somewhat wider than in 1904, every latitude from the equator up to 32° being represented, an arrangement usually characteristic of the maximum year of the cycle.

8. The number of separate groups of spots was 355, as compared with 276 in the previous year. The average size of the groups was nearly double that observed in 1904. Indeed, the most striking peculiarity of the Sun-spot record in 1905 was the great number of abnormally large groups; Group No. 5441, seen from 1905 January 29 to February 11, attaining on four consecutive days a greater area than that of any other spot as yet included in the Greenwich measures.

9. Of the 353 separate groups, 209 were in the northern hemisphere and 144 in the southern.

Royal Observatory, Greenwich:
1907 *January* 8.

Observations of Minor Planets from Photographs taken with the 30-inch Reflector of the Thompson Equatorial at the Royal Observatory, Greenwich, during the year 1904.

(*Communicated by the Astronomer Royal.*)

The following positions of minor planets were obtained from photographs taken with the 30-inch Reflector during the year 1904.

The plates were measured with the astrographic micrometer. Four reference stars were, as a rule, measured with the planet, their positions being derived when possible from the Catalogues of the Astronomische Gesellschaft.

The positions given are not corrected for Parallax.

log Parallax Correction = log Parallax Factor − log Δ.

Date and G.M.T. 1904.		Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Dec.
d	h m s	h m s	° ′ ″		
		(388) Charybdis.			
Feb. 15	11 14 15	8 37 23·50	+25 8 58·7	+8·423	+0·588
		(37) Fides.			
Feb. 13	9 16 7	8 38 15·32	+22 51 42·0	−9·271	+0·646
15	10 20 15	8 36 30·84	+22 54 28·3	−8·817	+0·624
17	10 31 32	8 34 53·47	+22 56 38·6	−8·469	+0·621
18	9 0 26	8 34 10·21	+22 57 28·1	−9·241	+0·642
		(106) Dione.			
Feb. 8	10 43 15	8 51 22·22	+24 1 3·2	−9·002	+0·612
13	9 40 59	8 47 21·28	+24 15 35·3	−9·217	+0·622
15	10 45 13	8 45 46·07	+24 20 50·0	−8·605	+0·601
17	11 14 14	8 44 15·15	+24 25 34·7	+8·448	+0·599
18	9 24 5	8 43 34·78	+24 27 36·0	−9·192	+0·617
		(313) Chaldea.			
Feb. 13	10 7 58	9 22 52·37	+1 43 1·5	−9·210	+0·826
18	9 45 8	9 19 6·55	+2 46 31·8	−9·206	+0·820
		(505) Cava.			
Feb. 15	7 7 9	4 59 26·67	+25 19 57·2	−8·423	+0·585

Date and G.M.T. 1904.		Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Dec.
d	h m s	h m s	° ′ ″		
		(454) Mathesis.			
Mar. 10	9 40 24	10 20 17·95	+19 29 59·2	−9·158	+0·677
15	10 19 20	10 16 8·23	+19 35 36·0	−8·609	+0·665
16	10 11 8	10 15 22·35	+19 36 8·9	−8·675	+0·665
		(334) Chicago.			
Mar. 10	10 17 29	10 44 52·75	+10 57 47·2	−9·079	+0·759
16	10 36 16	10 41 22·74	+11 21 57·5	−8·673	+0·752
		(47) Aglaja.			
Mar. 10	10 54 17	10 53 21·84	+ 9 48 15·1	−8·870	+0·766
16	10 56 33	10 48 31·24	+10 9 4·9	−8·386	+0·762
21	11 21 12	10 44 43·14	+10 24 16·4	+8·728	+0·760
		(134) Sophrosyne.			
Mar. 21	10 53 3	11 57 6·16	−4 59 54·4	−9·025	+0·862
Apr. 6	10 14 22	11 41 50·68	−4 23 37·0	−8·635	+0·860
9	11 4 22	11 39 21·15	−4 17 3·0	+8·771	+0·860
12	10 33 53	11 37 5·26	−4 11 1·3	+8·505	+0·859
		(46) Hestia.			
Apr. 6	10 44 2	11 47 34·42	+1 28 10·5	−7·903	+0·827
9	11 35 41	11 45 22·72	+1 44 27·8	+8·984	+0·826
12	11 0 24	11 43 21·85	+1 59 28·1	+8·804	+0·824
		(19) Fortuna.			
Apr. 9	12 0 6	11 54 36·68	−0 30 15·1	+9·082	+0·839
12	11 27 0	11 52 27·80	−0 14 18·3	+8·955	+0·837
		(288) Glauke.			
Apr. 15	10 46 36	12 43 47·46	+3 30 59·7	−8·565	+0·813
18	9 47 51	12 41 50·39	+3 41 49·6	−9·031	+0·813
21	11 18 59	12 39 58·55	+3 51 5·2	+8·771	+0·811
		(403) Cyane.			
Apr. 15	11 20 34	12 55 19·51	−16 56 23·3	−∞	+0·911

Date and G.M.T. 1904.				Apparent R.A.			Apparent Dec.			Log. Parallax Factor. R.A.	Dec.
	d	h	m s	h	m	s	°	′	″		

(317) Roxane.

Apr.	15	11	55 2	12	57	59·69	−3	46	54·9	+8·708	+0·857
	18	10	13 20	12	55	20·35	−3	29	18·2	−8·948	+0·855

(478) Tergeste.

Apr.	15	12	24 4	13	9	2·07	−18	27	7·7	+8·913	+0·914
	18	10	52 17	13	6	55·91	−18	2	50·2	−8·687	+0·914
May	3	9	52 26	12	57	43·24	−15	56	18·0	−8·521	+0·908

(121) Hermione.

May	18	10	28 59	15	39	42·22	−15	49	46·7	−9·144	+0·902
	19	10	25 36	15	38	56·63	−15	48	37·6	−9·140	+0·903
	31	10	26 44	15	30	0·93	−15	36	33·1	−8·650	+0·907
June	3	10	39 31	15	27	54·95	−15	34	16·5	−7·220	+0·907

(90) Antiope.

May	18	10	59 36	15	48	12·92	−19	13	5·0	−9·029	+0·914
	19	11	3 26	15	47	22·69	−19	11	9·4	−8·972	+0·915
	31	11	13 51	15	37	29·78	−18	48	13·6	+8·306	+0·917

(79) Eurynome.

May	18	11	31 28	16	10	8·24	−16	55	13·9	−8·951	+0·909
	19	11	48 16	16	9	10·23	−16	51	14·5	−8·726	+0·911
	31	10	52 42	15	57	40·48	−16	5	26·7	−8·681	+0·908
June	3	11	36 22	15	54	51·46	−15	54	37·3	+8·682	+0·908

(153) Hilda.

May	18	12	5 30	16	9	8·83	−18	19	39·6	−8·527	+0·915

(176) Idunna.

June	3	12	8 10	16	34	24·59	+2	5	38·3	+8·546	+0·823

(322) Phæo.

June	20	11	44 54	17	41	49·22	−20	25	32·9	−7·533	+0·913
July	5	12	1 37	17	27	56·92	−19	37	49·2	+9·163	+0·912
	6	10	55 49	17	27	9·34	−19	34	58·4	+8·660	+0·918
	8	11	2 37	17	25	32·43	−19	29	8·1	+8·860	+0·917

Date and G.M.T. 1904.		Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Dec.
d	h m s	h m s	° ′ ″		

(217) Eudora.

July 6	11 54 38	17 39 55·81	−5 32 37·9	+9·061	+0·865
8	11 48 36	17 38 37·13	−5 41 41·6	+9·078	+0·865
9	12 29 54	17 37 58·32	−5 46 42·3	+9·271	+0·863
11	11 6 18	17 36 48·81	−5 56 45·6	+8·875	+0·868

(419) Aurelia.

June 18	11 29 32	18 34 19·55	−17 39 12·2	−9·108	+0·909
20	12 14 47	18 32 46·38	−17 33 59·8	−8·585	+0·913
July 1	11 3 37	18 23 59·46	−17 11 37·8	−8·844	+0·911
6	12 14 5	18 20 4·20	−17 4 46·8	+8·935	+0·910

(364) Isara.

July 6	13 11 46	20 16 9·54	−21 26 19·2	−8·013	+0·924
8	13 14 17	20 14 19·22	−21 38 27·8	+8·014	+0·924
15	11 38 22	20 7 23·58	−22 21 30·6	−8·974	+0·923
20	11 51 35	20 2 1·29	−22 52 31·6	−8·469	+0·927
21	11 11 35	20 0 57·75	−22 58 21·0	−8·952	+0·924

(483) Seppina.

July 20	13 31 25	20 44 46·44	+2 15 51·3	+8·804	+0·822
Aug. 2	11 20 2	20 36 19·55	+1 10 8·9	−8·707	+0·829
6	11 9 26	20 33 41·31	+0 44 47·7	−8·583	+0·832
8	10 38 36	20 32 24·21	+0 31 29·0	−8·863	+0·833

(115) Thyra.

Aug. 2	11 52 49	20 54 15·72	−16 46 59·1	−8·477	+0·911
8	11 18 4	20 47 17·32	−16 37 37·3	−8·564	+0·910
12	10 18 24	20 42 44·02	−16 30 50·6	−9·009	+0·907
17	10 26 52	20 37 13·10	−16 21 27·3	−8·667	+0·909

(511) Davida.

July 20	13 1 12	20 31 49·86	−24 24 8·5	+8·586	[illegible]

(539) Pamina.

Aug. 8	11 55 29	21 20 9·94	−6 55 32·7	[illegible]·080	[illegible]
12	11 7 38	21 16 36·39	−6 59 [illegible]	[illegible]·567	[illegible]
17	11 7 37	21 12 8·46	−7 6 [illegible]	[illegible]·528	[illegible]

Date and G.M.T. 1904. d	h m s	Apparent R.A. h m s	Apparent Dec. ° ′ ″	Log. Parallax Factor. R.A.	Dec.
			(236) Honoria.		
Aug. 8	12 22 17	21 17 22·40	−5 20 3·0	+8·351	+0·865
12	10 40 55	21 14 22·18	−5 44 14·6	−9·050	+0·866
17	10 48 11	21 10 35·02	−6 17 35·6	−8·784	+0·870
18	11 7 39	21 9 50·11	−6 24 35·4	−8·382	+0·871
			(372) Palma.		
Aug. 18	12 39 15	22 13 16·96	−10 6 21·7	+8·356	+0·887
Sept. 3	9 26 32	21 57 43·84	− 9 56 22·2	−9·200	+0·881
7	9 23 33	21 53 58·20	− 9 53 10·9	−9·125	+0·883
9	9 48 48	21 52 8·02	− 9 51 21·3	−8·897	+0·885
			(389) Industria.		
Sept. 3	9 48 54	22 15 54·24	+0 27 27·5	−9·177	+0·833
7	9 46 18	22 12 29·89	+0 11 8·5	−9·103	+0·835
9	10 24 0	22 10 49·87	+0 2 34·0	−8·721	+0·836
16	9 12 17	22 5 32·92	−0 27 35·0	−9·054	+0·839
			(95) Arethusa.		
Sept. 7	10 14 0	22 23 46·92	+11 33 29·2	−9·006	+0·753
9	10 50 35	22 22 18·94	+11 21 46·8	−8·442	+0·752
12	11 8 59	22 20 12·51	+11 3 11·3	+8·388	+0·754
			(284) Amalia.		
Sept. 3	11 4 12	23 23 0·11	+11 38 25·2	−9·146	+0·755
7	10 47 1	23 19 40·24	+11 9 28·9	−9·136	+0·759
9	11 47 9	23 17 56·45	+10 53 3·4	−8·415	+0·755
16	9 45 9	23 12 11·47	+ 9 50 59·4	−9·216	+0·772
			(331) Etheridgea.		
Sept. 9	12 12 13	23 16 30·46	−10 58 33·5	+8·253	+0·891
Oct. 3	9 14 12	22 58 44·98	−11 43 26·8	−8·957	+0·892
			(405) Thia.		
Sept. 19	9 19 48	23 50 26·78	+16 58 5·5	−9·384	+0·725
20	10 12 45	23 49 33·81	+16 52 9·0	−9·205	+0·708
Oct. 3	7 53 18	23 38 55·46	+15 26 25·8	−9·423	+0·745
8	9 34 25	23 35 8·40	+14 48 35·9	−8·930	+0·721
12	9 46 10	23 32 24·11	+14 17 56·1	−8·559	+0·723

Date and G.M.T. 1904.		Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Dec.
d	h m s	h m s	° ′ ″		
		(373) Melusina.			
Sept. 19	10 36 16	0 1 27·41	−2 53 23·3	−9·152	+0·851
Oct. 3	9 50 34	23 48 18·73	−2 42 33·8	−9·036	+0·851
8	10 7 27	23 44 1·02	−2 36 7·2	−8·650	+0·851
12	10 10 58	23 40 52·74	−2 29 33·8	−7·982	+0·851
		(258) Tyche.			
Sept. 16	10 11 5	23 59 2·79	+14 4 27·0	−9·298	+0·741
19	9 2 31	23 57 15·55	+13 27 36·7	−9·429	+0·761
20	9 57 23	23 56 37·38	+13 14 7·4	−9·281	+0·747
27	10 43 0	23 52 19·98	+11 36 48·9	−8·863	+0·750
		(375) Ursula.			
Sept. 19	9 42 6	0 1 59·20	+11 33 33·8	−9·347	+0·766
20	10 31 20	0 1 2·27	+11 33 17·4	−9·164	+0·756
Oct. 3	8 26 10	23 49 20·87	+11 20 17·2	−9·369	+0·769
8	10 32 5	23 45 5·19	+11 11 29·6	−7·814	+0·753
		(370) Modestia.			
Oct. 3	10 30 57	0 25 11·04	+18 39 30·5	−9·034	+0·682
8	10 57 55	0 20 22·26	+18 11 5·2	−8·372	+0·681
12	11 4 57	0 16 45·25	+17 45 7·6	+8·304	+0·686
13	10 1 31	0 15 54·76	+17 38 37·8	−8·884	+0·690
		(68) Leto.			
Sept. 16	11 45 16	1 7 57·01	−2 13 55·3	−9·190	+0·848
19	10 55 53	1 5 46·97	−2 20 35·5	−9·309	+0·847
20	10 51 3	1 5 0·96	−2 22 49·0	−9·309	+0·847
27	11 5 31	0 59 10·98	−2 37 53·8	−9·138	+0·850
Oct. 3	10 58 42	0 53 47·49	−2 48 47·7	−9·011	+0·852
Nov. 3	9 48 35	0 30 19·77	−2 33 46·7	+8·158	+0·851
		(482) Petrina.			
Oct. 3	11 19 54	0 49 1·97	−0 45 10·5	[illegible]	[illegible]0·831
13	10 28 10	0 42 10·62	−0 38 54·3	[illegible]	[illegible]0·840
		(447) Valentina.			
Oct. 13	11 23 10	0 48 28·96	−1 36 59·[illegible]	[illegible]	[illegible]
14	11 45 59	0 47 41·85	−1 40 [illegible]	[illegible]	[illegible]

Date and G.M.T. 1904. d	h m s	Apparent R.A. h m s	Apparent Dec. ° ′ ″	Log. Parallax Factor. R.A.	Dec.
		(546) Herodias.			
Oct. 14	11 5 10	0 57 14·71	+3 53 53·3	−8·505	+0·811
		(335) Roberta.			
Nov. 12	9 53 56	2 21 38·28	+5 50 34·0	−8·986	+0·798
14	9 48 59	2 19 56·89	+5 43 42·7	−8·957	+0·799
		(17) Thetis.			
Nov. 12	9 29 0	2 21 42·55	+4 54 36·3	−9·129	+0·806
14	9 25 14	2 19 59·71	+4 48 53·5	−9·104	+0·806
		(170) Maria.			
Nov. 12	10 53 51	2 32 29·58	+38 55 55·8	−8·392	+0·275
14	10 54 13	2 30 22·26	+38 41 29·8	−7·612	+0·283
		(178) Belisana.			
Nov. 14	11 22 22	2 27 5·87	+14 9 55·4	+8·693	+0·725
		(298) Baptistina.			
Dec. 5	10 26 8	3 13 40·46	+27 17 9·6	+8·255	+0·553
7	8 55 47	3 11 47·93	+27 10 44·6	−9·099	+0·569
		(443) Photographica.			
Dec. 5	10 58 53	3 19 56·19	+11 30 35·4	+8·767	+0·751
7	9 26 48	3 18 19·53	+11 25 26·4	−8·881	+0·752
		(154) Bertha.			
Nov. 14	11 49 48	4 36 54·73	+37 17 59·6	−9·158	+0·360

Observations of Occultations of Stars by the Moon, made at the Royal Observatory, Greenwich, in the Year 1906.

(*Communicated by the Astronomer Royal.*)

Day, 1906.		Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
January	4	Disapp. ξ^2 Ceti	Great Equatorial	670	Dark	4 12 3·57	W. B.
	4	,, ,, ...	Astrographic Equatorial ...	225	,,	4 12 2·34	H. F.
	4	,, ,, ...	Sheepshanks Equatorial ...	100	,,	4 12 3·03	J. S.
	4	,, ,, ...	Great Equatorial (Corbett) ...	120	,,	4 12 3·57	B. E.
	10	,, *g* Geminorum ...	Sheepshanks Equatorial ...	100	Bright	12 3 58·84	V.
February	3 (*c*)	,, Aldebaran	Astrographic Equatorial	225	Dark	5 22 30·36	H.
	3 (*g*)	,, ,, ...	Sheepshanks Equatorial ...	100	,,	5 22 29·70	H. F.
	3 (*g*)	,, ,, ...	Sheepshanks Equatorial (finder)	100	,,	5 22 31·10	J. S.
	3 (*a*)	Reapp. ,, ...	Astrographic Equatorial ...	225	Bright	6 28 34·15	H.
	3	,, ,, ...	Old Altazimuth	100	,,	6 28 34·23	A. C.
	3	,, ,, ...	Thompson Equatorial (Hodgson)	100	,,	6 28 (38·02)	D. E.
	3	,, ,, ...	Sheepshanks Equatorial	100	,,	6 28 34·28	W. S.
	3	Disapp. W. B. (2) IV. 724 ...	Astrographic Equatorial ...	225	Dark	8 54 46·89	J. S.
	4	,, 115 Tauri	,, ,, ...	225	,,	5 24 48·29	W. S.
	4 (*f*)	Reapp. ,, ...	,, ,, ...	225	Bright	6 33 10·60	W. S.
	7 (*c*)	Disapp. ζ Cancri	,, ,, ...	400	Dark	7 6 59·59	H.
	7 (*e*)	,, ,, ...	Great Equatorial	670	,,	7 6 58·24	B.
	7 (*c*)	,, ,, ...	Sheepshanks Equatorial	100	,,	7 6 58·01	J. S.
	7	,, ,, ...	Merz Refractor	250	,,	7 6 57·44	P. M.
	7	,, ,, ...	Thompson Equatorial	100	,,	7 6 58·24	R. F.

Day, 1906.		Phenomenon.		Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation.	Observer.
							h m s	
February	7 (c)	Disapp.	ζ Cancri (Comes) ...	Great Equatorial	670	Dark	7 6 58·24	B.
	7	,,	Piazzi VIII. 6 ...	,, ,, ...	670	,,	7 7 7·12	B.
	7 (c)	,,	Piazzi VIII. 6 ...	Sheepshanks Equatorial	100	,,	7 7 5·99	J. S.
	7	,,	Piazzi VIII. 6 ...	Merz Refractor	250	,,	7 7 6·32	P. M.
March	2	,,	θ^2 Tauri	Astrographic Equatorial ...	225	,,	12 2 16·10	W.
	6	,,	B.D. + 18°, 1752	,, ,, ...	225	,,	7 38 36·28	W. S.
	29 (a)	,,	B.D. + 14°, 657 ...	Sheepshanks Equatorial	100	,,	9 0 12·91	R. C.
April	3	,,	d^2 Cancri	Great Equatorial	670	,,	9 37 28·24	W. S.
	3	,,	,, ...	Great Equatorial (Corbett) ...	120	,,	9 37 28·24	W.
	3	,,	,, ...	Sheepshanks Equatorial	100	,,	9 37 28·66	V.
	3	,,	,, ...	Sheepshanks Equatorial (Finder)	100	,,	9 37 28·16	A. W.
	4	,,	π^2 Cancri	Astrographic Equatorial ...	225	,,	6 36 22·99	W. S.
	5	Reapp.	Regulus	Sheepshanks Equatorial	100	Bright	6 42 5·34	A. C.
	6	Disapp.	χ Leonis	,, ,, ...	100	Dark	7 3 7·95	A. C.
	6	,,	,, ...	Great Equatorial	670	,,	7 3 7·80	H. F.
	6	,,	,, ...	Great Equatorial (Corbett) ...	120	,,	7 3 8·30	G. C.
	6	,,	,, ...	Astrographic Equatorial ...	225	,,	7 3 7·50	W. S.
	27 (a)	,,	119 Tauri	,, ,, ...	225	,,	8 49 38·62	H. F.
	27	,,	,, ...	Sheepshanks Equatorial	100	,,	8 49 38·82	J. S.
	27	,,	,, ...	Astrographic Equatorial ...	225	,,	8 49 39·02	W. S.
	27	,,	,, ...	Old Altazimuth	100	,,	8 49 39·02	B. E.
	27 (a)	,,	120 Tauri	Astrographic Equatorial ...	225	,,	9 27 22·31	H. F.

						h m s	
April	27	Disapp. 120 Tauri	Sheepshanks Equatorial	100	Dark	9 27 22·11	J. S.
	27	,, ,, ...	Old Altazimuth	100	,,	9 27 23·06	A. W.
September	9 (*c*) (*d*)	Reapp. B.A.C. 1391 ...	Astrographic Equatorial	225	,,	11 58 13·14	W.
	9 (*a*)	Disapp. Aldebaran	,, ,, ...	225	Bright	14 26 41·78	W.
	9	Reapp. ,, ...	,, ,, ...	225	Dark	14 43 31·21	W.
November	5	,, ν Geminorum ...	Sheepshanks Equatorial	100	,,	12 34 43·49	S. E.
	19	Disapp. ο Sagittarii	Astrographic Equatorial ...	225	,,	5 29 0·96	H. F.
	22	,, W.B. XXI. 1281 ...	Sheepshanks Equatorial	100	,,	6 48 53·21	J. S.
	22	,, ,, ...	Astrographic Equatorial ...	225	,,	6 48 52·16	W. S.
December	5	Reapp. B.A.C. 3029 ...	Sheepshanks Equatorial	60	,,	17 8 1·35	A. C.

(*a*) Instantaneous.
(*c*) Very faint, cloudy.
(*d*) Not a very good observation.
(*e*) Disappeared simultaneously.
(*f*) The observer was looking at the exact point of reappearance.
(*g*) Very windy—clock-beats inaudible.

-f the telescopes used are as follows:—

-ial	28 inches.	Sheepshanks Equatorial	$6\frac{3}{4}$ inches.
-atorial	26 ,,	Great Equatorial (Corbett Telescope) ...	$6\frac{1}{2}$,,
... ...	$12\frac{3}{4}$,,	Thompson Equatorial (Hodgson)	6 ,,
-orial (Guiding Telescope)	10 ,,	Old Altazimuth	4 ,,

B., D. E., W. B., H. F., W., J. S., P. M., W. S., S. E., R. C., V., B. E., R. F., G. C., A. W., are those of -r Bryant, Mr Edney, Mr Bowyer, Mr Furner, Mr Witchell, Mr Storey, Mr Melotte, Mr Stevens, Mr Eddington, Evans, Mr Fowler, Mr Cody, and Mr Witney, respectively.

-y, *Greenwich*.

The Latitude of the Royal Observatory, Edinburgh.

(*Communicated by Professor F. W. Dyson.*)

The latitude of the Royal Observatory, Edinburgh, is given in the *Nautical Almanac* as 55° 55′ 28″·0. This provisional value was derived from an observation of *α Cassiopeiæ* in the prime vertical by Dr Becker on 1889 October 8.

From 1898 April to the present time observations have been made of Sir David Gill's list of Zodiacal Stars. The observations as far as 1906 February have been reduced in the first instance—

(i) With an assumed latitude, 55° 55′ 30″·0.
(ii) Bessel's Refractions.
(iii) No correction for flexure.

Comparison of the resulting declinations has been made with Newcomb's Fundamental Catalogue, and with the Zodiacal Catalogue published in the Papers of the American Ephemeris, vol. vi. pt. 3, which is also reduced to Newcomb's system.

The results of this comparison are shown in the first four columns of the following table. The observations are very scanty from the pole to Z.D. 30° S., but are plentiful from Z.D. 30° to 70°.

The differences, when the observations are corrected to the Pulkowa system of refractions, are given in column 5.

The observations of flexure made between 1898 April 13 and 1906 February are as follows:—

Date.	Hor. Flexure.	Date.	Hor. Flexure.
	″		″
1898 April 13	+1·20	1905 February 23	+0·88
	+1·12	April 15	+1·03
	+0·98		+1·07
	+0·80	April 19	+0·89
1902 September 16	+1·53		+0·80
1903 August 24	+1·62	November 7	+0·84
	+1·52		
1903 August 27	+1·12	Mean	+1″·10
	+1·14		

The observations during the year 1906 have given a somewhat smaller value. I have taken the correction for flexure, in round figures, as −1″·00 sin Z.D. When this correction is applied the differences from Newcomb are those of the last column of the following table:—

Comparison of the Declinations observed at the Royal Observatory, Edinburgh, 1898–1905, with Newcomb's Fundamental System.

Dec.	Z.D.	Number of Obs.	Newc.— Obs. Decns.	N–O (Pulkowa Refractions).	1''·00 sin Z.D.	N–O (Pulkowa Refn. Flexure Correction).
°	°		"	"	"	"
$+91\frac{1}{2}$	−36	23	−0·53	−0·42	−·59	+·17
$88\frac{1}{2}$	−33	19	−0·06	+0·04	−·54	+·58
59	$-3\frac{1}{2}$	17	−0·13	−0·12	−·06	−·06
47	$+8\frac{1}{2}$	9	−0·24	−0·27	+·15	−·42
$37\frac{1}{2}$	+18	8	+0·17	+0·10	+·31	−·21
$32\frac{1}{2}$	+23	34	+0·27	+0·20	+·39	−·19
$27\frac{1}{2}$	+28	211	+0·36	+0·28	+·47	−·19
$22\frac{1}{2}$	+33	685	+0·57	+0·47	+·54	−·07
$17\frac{1}{2}$	+38	566	+0·77	+0·65	+·62	+·03
$12\frac{1}{2}$	+43	360	+1·12	+0·97	+·68	+·29
$7\frac{1}{2}$	+48	376	+0·86	+0·78	+·74	+·04
$2\frac{1}{2}$	+53	260	+1·09	+0·87	+·80	+·07
$-2\frac{1}{2}$	+58	325	+1·14	+0·89	+·85	+·04
$-7\frac{1}{2}$	+63	418	+1·21	+0·90	+·89	+·01
$-12\frac{1}{2}$	+68	297	+1·44	+1·05	+·93	+·12
$-17\frac{1}{2}$	+73	495	+1·43	+0·96	+·96	00
$-22\frac{1}{2}$	+78	196	+1·28	+0·68	+·98	−·30
$-26\frac{1}{2}$	+82	81	+0·93	+0·22	+·99	−·77

Consideration of these residuals shows that the latitude may be taken as 55° 55′ 30″·0. The observations from $-34\frac{1}{2}$° to $+34\frac{1}{2}$°, though few, give this result whatever correction be applied for flexure. The observations from Z.D. + $34\frac{1}{2}$° to Z.D. 73° support the adopted correction for flexure. At the zenith distances 78° to 82° there is a marked discordance, but the observations at higher zenith distances support the change from Bessel's to the Pulkowa refractions.

The reduction of the observations which form the material on which this determination of the latitude is based have been accelerated, thanks to a grant from the Government Grant Committee of the Royal Society.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXVII. FEBRUARY 8, 1907. No. 4

ANNUAL GENERAL MEETING.

Mr W. H. MAW, PRESIDENT, in the Chair.

The Report of the Auditors of the Treasurer's accounts for the year 1906 was read, and is given on p. 220.

The Annual Report of the Council was partly read; see pp. 217 to 299.

The Address was delivered by the President, after which the Gold Medal was handed to the Secretary for transmission to Professor Ernest William Brown, to whom the Medal had been awarded for his researches in the Lunar Theory (see pp. 300 to 313).

The President having appointed the Scrutineers, the Society proceeded to the ballot for Officers and Council for the ensuing year. The names of those elected are given on p. 314.

The thanks of the Meeting were given to the retiring Officers, and also to the Auditors of the Treasurer's accounts and to the Scrutineers of the ballot.

Edgar T. Adams, 5 Warkworth Street, Cambridge, and the Cottage, Halstead, Essex,
Robert Jonckheere, Observatoire Stella, Roubaix (Nord), France,
John Stewart, Chief Officer, R.M.S. "Empress of Ch
The Willows, Wallasey, Birkenhead, and
Samuel Veevers, Normanton, Kimberley Drive, G
near Liverpool,

were balloted for and duly elected Fellows of the Soc

The following Candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Arthur Neville Brown, M.A., Schoolmaster, Ludgrove, New Barnet, Herts (proposed by Col. E. E. Markwick);

Harry Cooper, 19 Cromer Road, Eastville, Bristol (proposed by W. F. Denning);

Phanindralal Gangooly, M.A., Premchand Roychand Student, University of Calcutta (proposed by Asutosh Mukhopadhyay); and

William Newsam McClean, 42 Durdham Park, Bristol (proposed by Sir David Gill).

Report of the Council to the Eighty-seventh Annual General Meeting of the Society.

The following table shows the progress and present state of the Society :—

	Patron	Honorary Members	Fellows: Compounders	Fellows: Annual Subscribers	Associates	Total
1905 December 30	1	2	259	398	49	709
Since elected	...	+ 1	+ 2	+ 33	+ 1	...
Deceased	...	...	− 5	− 9	− 2	...
Resigned	...	...	...	− 6	...	...
Removals	...	...	+ 4	− 4	...	...
Expelled	...	...	...	− 3	...	...
1906 December 31	1	3	260	409	48	721

Major Hills' Account as Treasurer of the Royal

RECEIVED.

	£	s.	d.	£	s.	d.
Balances, 1905 December 31 :—						
At Bankers', as per Pass-book	299	0	2			
Cheques not credited till 1906	7	18	5			
In hand of Assistant Secretary on Account of Turnor and Horrox Fund	4	13	11			
In hand of Assistant Secretary on Petty Cash Account	8	19	10			
				320	12	4
Dividends :—						
£1,250 Metropolitan 3-per-cent. Stock ...	35	12	8			
£932 19 0 Metropolitan 2½-per-cent. Stock ...	22	3	4			
Half-year's Dividend on £1,484 17 5 Swansea Corporation 3½-per-cent. Stock	24	13	9			
Half-year's Dividend on £1,584 17 5 Swansea Corporation 3½-per-cent. Stock	26	7	0			
£3,400 East Indian Railway 3-per-cent. Debenture Stock	96	18	0			
£3,200 London and North-Western Railway 3-per-cent. Debenture Stock	91	4	0			
£4,000 Midland Railway 2½-per-cent. Debenture Stock	95	0	0			
£500 Lancashire and Yorkshire Railway 3-per-cent. Consolidated Preference Stock ...	14	5	0			
£1,860 Gas Light and Coke Co. 3-per-cent. Debenture Stock	53	0	2			
£1,650 Commercial Gas Co. 3-per-cent. Debenture Stock	47	0	6			
				506	4	5
Received on account of Subscriptions :—						
Arrears	128	2	0			
Annual Contributions for 1906	596	8	0			
" " in advance	12	12	0			
Admission Fees	73	10	0			
First Contributions	52	10	0			
				863	2	0
Composition Fees...				87	3	0
Sales of Publications, &c. :—						
At Williams & Norgate's, 1905	11	16	0			
At Society's Rooms, 1906	94	0	8			
Sales of Photographs, 1906	35	12	6			
				141	9	2
Income Tax refunded by Commissioners of Inland Revenue				26	10	6
Bequest of the late Rev. A. S. Farrar				100	0	0
Sale of £932 19 0 Metropolitan 2½-per-cent. Stock				719	9	7
				2,764	11	

Audited and found correct, 1907 Jan. 8 :

G. J. NEWBEGIN.
A. E. CONRADY.
W. W. BRYANT.

Astronomical Society, from 1906 *January* 1 *to December* 31.

PAID.

	£	s.	d.	£	s.	d.
Assistant Secretary: Salary	250	0	0			
" " Editing Society's Publications	50	0	0			
Clerk's Salary	75	0	0			
				375	0	0
House Duty	2	12	6			
Fire Insurance	9	9	6			
				12	2	0
Printing, &c.:—						
Memoirs, vol. lvi. (Spottiswoode & Co.)	649	19	3			
Monthly Notices, " "	376	16	0			
Eclipse Report, 1905 (Harrison & Sons)	17	3	0			
List of Fellows and Miscellaneous (Spottiswoode & Co.)	25	4	0			
Photo-blocks in *Annual Report*	0	14	9			
				1069	17	0
Computation of Ephemerides in *Monthly Notices* ...				15	0	0
Turnor and Horrox Fund, purchase of books for Library				12	7	11
Reproduction of Photographs, Hinton & Co. ...				35	4	2
Cataloguing astronomical literature for the International Catalogue of Scientific Literature ...				30	0	0
Expenses of Meetings	16	19	0			
Lantern expenses	8	12	0			
Time Signal: rental of wire	5	0	0			
				30	11	0
Postage and Telegrams	79	9	6			
Carriage of Parcels	14	13	4			
Stationery (Spottiswoode & Co.)	4	17	6			
Sundry Stationery and Office expenses	5	10	7			
				104	10	11
Illuminating Address (Benjamin Franklin Centenary)				3	8	0
House expenses: Allowance and sundry expenses	59	0	7			
Coal and Gas	42	8	3			
Electric Lighting	18	14	10			
Fittings and Repairs	12	8	7			
Sundries	4	10	9			
				137	3	0
Purchase of £100 Swansea Corporation $3\frac{1}{2}$-per-cent. Stock (Investment of Bequest of the late Rev. A. S. Farrar)				105	16	0
Purchase of £379, 15 2 Swansea Corporation $3\frac{1}{2}$-per-cent. Stock				395	0	0
Power of Attorney for sale of [illegible]politan $2\frac{1}{2}$-per-cent. Stock				0	11	6
Bankers' deductions on cheq[illegible] ...				0	3	0
Repayment of £400 loan fro[illegible]nterest				401	12	7
Cheques outstanding from [illegible] ...				16	19	6
Balances, 1906 December 31 [illegible]						
At Bankers', as per P[illegible]	14	1	0			
In hand of Assistant [illegible] of Turnor and Horro[illegible]	4	6	0			
In hand of Assista[illegible] Account ...	0	17	5			
				19	4	5
				£2,764	11	0

Report of the Auditors.

We have examined the Treasurer's accounts of receipts and expenditure for the year 1906, and have found and certified the same to be correct. The cash in hand on December 31, 1906, including the balance at the bankers', etc., amounted to £19, 4s. 5d.

The overdraft of £400 at the bank has been repaid, but it has been found necessary to sell £932, 19s. Metropolitan $2\frac{1}{2}$-per-cent. Stock, which realised £719, 9s. 7d.

£379, 15s. 2d. Swansea Corporation $3\frac{1}{2}$-per-cent. Stock has been purchased, in addition to the investment of a bequest of £100 from the late Rev. A. S. Farrar, received during the year. The result has been that the expenditure for the year has exceeded the income by about £200, chiefly in consequence of the large amount of the bills for printing. In this connection we are glad to find that a change of printers has taken place, which we trust will result in a considerable reduction of this item.

The books, instruments, and other effects in the possession of the Society have been examined, and they appear to be in a satisfactory condition, with the exception of some of the instruments in the basement, with regard to which we regret to find that so far no practical result has followed from the recommendation made by the Auditors last year to eliminate those of no practical value or historic interest. We desire to repeat this recommendation with greater emphasis, as it is a matter which obviously grows more pressing each year.

We have laid on the table a list of the names of those Fellows who are in arrear for sums due at the last Annual General Meeting of the Society, with the amount due against each Fellow's name.

(*Signed*) WALTER W. BRYANT.
G. J. NEWBEGIN.
A. E. CONRADY.

1907 *January* 11.

Bequests to the General Funds of the Society.

The Carrington Bequest (1876): A sum of £2,000 [illegible]
Sold in 1899, and stands now in £1,881 14s. [illegible]
North-Western Railway 3-per-cent. Debenture [illegible]

The McClean Bequest (1905): A sum of £2,000. [illegible]
sum invested in the purchase of £1,484 17s. [illegible]
Corporation $3\frac{1}{2}$-per-cent. Stock.

The Farrar Bequest (1906): A sum of £100. Invest[illegible]
Corporation $3\frac{1}{2}$-per-cent. Stock.

Trust Funds.

The Turnor Fund: A sum of £464 18*s.* East Indian Railway 3-per-cent. Debenture Stock; the interest to be used in the purchase of books for the Library.

The Horrox Memorial Fund: A sum of £103 6*s.* East Indian Railway 3-per-cent. Debenture Stock; the interest to be used in the purchase of books for the Library.

The Lee and Janson Fund: A sum of £334 10*s.* 9*d.* East Indian Railway 3-per-cent. Debenture Stock; the interest to be given by the Council to the widow or orphan of any deceased Fellow of the Society who may stand in need of it.

The Hannah Jackson (née Gwilt) Fund: A sum of £309 18*s.* 6*d.* East Indian Railway 3-per-cent. Debenture Stock; the interest to be given in Medals or other awards, in accordance with the terms of the Trust.

Assets and Present Property of the Society, 1907 *January* 1.

	£	*s.*	*d.*	£	*s.*	*d.*
Balances, 1905 December 31 :—						
At Bankers'	14	1	0			
In hand of Assistant Secretary on Account of Turnor and Horrox Fund	4	6	0			
In hand of Assistant Secretary on Petty Cash Account	0	17	5			
				19	4	5
Due on account of Subscriptions :—						
4 Contributions of 5 years' standing	42	0	0			
8 „ 4 „	67	4	0			
9 „ 3 „	56	14	0			
33 „ 2 „	138	12	0			
60 „ 1 „	126	0	0			
	430	10	0			
Less Contributions paid in advance	12	12	0			
				417	18	0
Due for Photo[illegible] sold				0	15	0
Due from [illegible] Williams & Norgate for sales of Publications, 1906				44	18	1
£3,400 East [illegible] Railway 3-per-cent. Debenture Stock, including the [illegible] Fund, the Horrox Memorial Fund, the Lee and [illegible] and the Hannah Jackson (*née* Gwilt) Fund.						
£3,[illegible] Western Railway 3-per-cent De-[illegible] Carrington Bequest).						
£4,[illegible] Debenture Stock.						
£500 [illegible] 3-per-cent. Consolid[illegible] [illegible]						
£1,860 [illegible] Debenture Stock.						
£1,650 [illegible] Debenture Stock.						

£1,250 Metropolitan 3-per-cent. Stock.

£1,864 12*s*. 7*d*. Swansea Corporation 3½-per-cent. Stock (including £1,800 of the McClean Bequest of £2,000).

£100 Swansea Corporation 3½-per-cent. Stock (being Investment of Canon Farrar's Bequest).

Astronomical and other Manuscripts, Books, Prints, Photographs, and Instruments.

Furniture, &c.

Stock of Publications of the Society.

Celestial Photographs.

The following is a list of reproductions of Celestial Photographs published by the Royal Astronomical Society for sale to the Fellows :—

R.A.S. Ref. No.	Subject.	Photographed by
1	Total Solar Eclipse, 1889 January 1	W. H. Pickering
2	Total Solar Eclipse, 1893 April 16	J. M. Schaeberle
3	Total Solar Eclipse, 1886 August 29	A. Schuster
4	Nebulæ in the *Pleiades*	Isaac Roberts
5	Nebula M 74 *Piscium* (N.G.C. 628)	Isaac Roberts
6	Great Nebula in *Orion*	Isaac Roberts
7	Milky Way near M 11	E. E. Barnard
8	Milky Way near Cluster in *Perseus*	E. E. Barnard
9	Comet *c* 1893 IV. (Brooks), 1893 October 21	E. E. Barnard
10	Comet *a* 1892 I. (Swift), 1892 April 7	E. E. Barnard
11	Nebula about η *Argus*	David Gill
12	Portion of Moon (Hyginus-Albategnius)	Loewy and Puiseux
13	Comet *c* 1893 IV. (Brooks), 1893 October 22	E. E. Barnard
14	Comet *c* 1893 IV. (Brooks), 1893 October 20	E. E. Barnard
15	Comet *c* 1893 IV. (Brooks), 1893 November 10	E. E. Barnard
16	Comet *a* 1892 I. (Swift), 1892 April 26	E. E. Barnard
17	Comet *f* 1892 III. (Holmes), 1892 November 10	E. E. Barnard
18	Comet *a* 1892 I. (Swift), 1892 April 18	E. E. Barnard
19	Portion of Moon (Alps, Apennines, &c.)	Loewy and Puiseux
20	Nebula in *Andromeda*	Isaac Roberts
21	*Jupiter*, 1892 September 26	Lick Observatory
22	Cluster M 13 *Herculis* (N.G.C. 6205)	W. E. Wilson
23	Total Solar Eclipse, 1893 April 16 (5 sec.)	J. Kearney
24	Total Solar Eclipse, 1893 April 16 (20 sec.)	J. Kearney
25	The Moon (Age $7^d\ 3^h$)	Lick Observatory
26	The Moon (Age $12^d\ 6\frac{1}{2}^h$)	Lick Observatory

R.A.S. Ref. No.	Subject.	Photographed by
27	The Moon (Age 16^d 18^h)	Lick Observatory
28	The Moon (Age 23^d 8^h)	Lick Observatory
29	The Sun, 1892 February 13	Roy. Obs. Greenwich
30	The Sun, 1892 July 8	Roy. Obs. Greenwich
31	Portion of Moon (Region of Maginus)	Loewy and Puiseux
32	The Moon (Age 14^d 1^h)	Lick Observatory
33	Portion of Moon (Ptolemæus, &c.)	Lick Observatory
34	Portion of Moon (Mare Serenitatis)	Lick Observatory
35	Portion of Moon (Clavius, Licetus, &c.)	Lick Observatory
36	Portion of Moon (Regiomontanus, &c.)	Lick Observatory
37	Portion of Moon (Tycho, Thebit, &c.)	Lick Observatory
38	Portion of Moon (Theophilus, &c.)	Lick Observatory
39	Total Solar Eclipse, 1896 August 9 (3 sec.)	S. Kostinsky
40	Total Solar Eclipse, 1896 August 9 (26 sec.)	A. Hansky
41	Cluster M 56 *Lyræ* (N.G.C. 6779)	
42	Nebulæ M 81, 82 *Ursæ Majoris* (N.G.C. 3031, 3034)	
43	Cluster M 56 *Lyræ* (enlarged) (N.G.C. 6779)	
44	Solar Corona, 1871 December 12, Baikul	H. Davis
45	Solar Corona, 1875 April 6, Siam	Lockyer and Schuster
46	Solar Corona, 1878 July 29, Wyoming	W. Harkness
47	Solar Corona, 1882 May 17, Egypt	Abney and Schuster
48	Solar Corona, 1883 May 6, Caroline Island	Lawrance and Woods
49	Solar Corona, 1885 September 9, Wellington, N.Z.	Radford
50	Solar Corona, 1886 August 29, Grenada, W.I.	A. Schuster
51	Solar Corona, 1887 August 19, Japan	M. Sugiyama
52	Solar Corona, 1889 January 1, California	W. H. Pickering
53	Solar Corona, 1889 December 22, Cayenne	J. M. Schaeberle
54	Solar Corona, 1893 April 16, Fundium	J. Kearney
55	Solar Corona, 1893 April 16, Brazil	A. Taylor
56	Great Nebula in *Orion*	W. E. Wilson
57	Dumb-bell Nebula, *Vulpecula* (N.G.C. 6853)	W. E. Wilson
58	Spiral Nebula, *Canes Venatici* (N.G.C. 5194–5)	W. E. Wilson
59	Ditto (enlarged) (N.G.C. 5194)	W. E. Wilson
60	Annular Nebula, *Lyra* (N.G.C. 6720)	W. E. Wilson
61	Meteor Trail and Comet Brooks, 1893 November 13	E. E. Barnard
62	Total Solar Eclipse, 1898 January 22 (5 sec.)	W. H. M. Christie
63	Total Solar Eclipse, 1898 January 22 (20 sec.)	W. H. M. Christie
64	Solar Corona, 1896 August 9, Novaya Zemlya	G. Baden-Powell
65	Solar Corona, 1898 January 22, Pulgaon, India	E. H. Hills
66	Nebula in *Andromeda*	Roy. Obs., Greenwi

R.A.S. Ref. No.	Subject.	Photographed by
67	Spectrum of Sun's Limb, 1898 January 22	E. H. Hills
68	Annual Nebula, *Lyra* (N.G.C. 6720)	Lick Observatory
69	Dumb-bell Nebula, *Vulpecula* (N.G.C. 6853)	Lick Observatory
70	Spiral Nebula, *Canes Venatici* (N.G.C. 5194–5)	Lick Observatory
71	Spiral Nebula, *Ursa Major* (N.G.C. 5457)	Lick Observatory
72	Trifid Nebula, *Sagittarius* (N.G.C. 6514)	Lick Observatory
73	Great Nebula in *Orion*	Lick Observatory
74	Cluster M 13 *Herculis* (N.G.C. 6205)	Lick Observatory
75	Solar Surface with Faculæ, 1893 August 7	G. E. Hale
76	Faculæ and Prominences, 1892 June 25	G. E. Hale
77	Total Solar Eclipse, 1898 Jan. 22 ($\frac{3}{8}$ sec.)	W. H. M. Christie
78	Nebula H V, 14 *Cygni* (N.G.C. 6992)	W. E. Wilson
79	Portion of Moon (Theophilus, &c.)	Yerkes Observatory
80	Total Solar Eclipse, 1900 May 28 (30 sec.)	E. E. Barnard
81	Comet 1901 I., 1901 May 4	Roy. Obs., Cape of G. H
82	Comet 1901 I., 1901 May 6	Roy. Obs., Cape of G. H.
83	Comet 1901 I., 1901 May 9	Perth Obs., W. Australia
84	Solar Surface with Faculæ, 1895 August 18	H. Deslandres
85	Solar Prominences, 1894 April 11	H. Deslandres
86	Nebula about Nova *Persei*, 1901 September 20	G. W. Ritchey
87	Nebula about Nova *Persei*, 1901 November 13	G. W. Ritchey
88	Total Solar Eclipse, 1901 May 18 (10 sec.)	F. W. Dyson
89	Total Solar Eclipse, 1901 May 18 (40 sec.)	F. W. Dyson
90	Comet *b* 1902 III. (Perrine), 1902 Sept. 29	Roy. Obs., Greenwich
91	Portion of Moon (Mare Serenitatis, &c.)	Yerkes Observatory
92	Portion of Moon (Rough Crater Region, Mare Nubium)	Yerkes Observatory
93	Portion of Moon (Tycho, Theophilus, &c.)	Yerkes Observatory
94	Portion of Moon (Bullialdus to Copernicus)	Yerkes Observatory
95	Portion of Moon (Copernicus, enlarged)	Yerkes Observatory
96	Great Nebula in *Orion*	Yerkes Observatory
97	Great Nebula in *Orion* (Central portion)	Yerkes Observatory
98	Nebula in *Andromeda*	Yerkes Observatory
99	Nebula in *Cygnus* (N.G.C. 6960)	Yerkes Observatory
100	Nebula in *Cygnus* (N.G.C. 6992)	Yerkes Observatory
101	Cluster M 13 *Herculis* (N.G.C. 6205)	Yerkes Observatory
102	Cluster M 15 *Pegasi* (N.G.C. 7078)	Yerkes Observatory
103	Solar Surface with Faculæ	Yerkes Observatory
104	The Moon, 1900 April 5	P. Puiseux
105	The Moon, 1902 November 13	P. Puiseux

R.A.S. Ref. No.	Subject.	Photographed by
106	The Moon, 1903 February 6	P. Puiseux
107	The Moon, 1903 September 12	P. Puiseux
108	Nebulosity about 15 *Monocerotis*	E. E. Barnard
109	Milky Way about β *Cygni*	E. E. Barnard
110	Nebulosity near α *Cygni*	E. E. Barnard
111	Milky Way near χ *Cygni*	E. E. Barnard
112	Star Cloud in *Sagittarius*	E. E. Barnard
113	Milky Way in *Cepheus*	E. E. Barnard
114	Milky Way about M 8	E. E. Barnard
115	Milky Way about θ *Ophiuchi*	E. E. Barnard
116	Milky Way near N.G.C. 6475	E. E. Barnard
117	Great Nebula near ρ *Ophiuchi*	E. E. Barnard
118	Milky Way about 58 *Ophiuchi*	E. E. Barnard
119	Milky Way near *Omega* nebula	E. E. Barnard
120	Star Cloud in *Sagittarius*	E. E. Barnard
121	Nebula about ν *Scorpii*	E. E. Barnard
122	Sun, 1905 January 30	Roy. Obs., Greenwich
123	Sun-spot, 1905 January 30	Roy. Obs., Greenwich
124	Sun, 1905 January 31	Roy. Obs., Greenwich
125	Sun-spot, 1905 January 31	Roy. Obs., Greenwich
126	Sun, 1905 February 2	Roy. Obs., Greenwich
127	Sun-spot, 1905 February 2	Roy. Obs., Greenwich
128	Sun, 1905 February 3	Roy. Obs., Greenwich
129	Sun-spot, 1905 February 3	Roy. Obs., Greenwich
130	Sun, 1905 February 5	Roy. Obs., Greenwich
131	Sun-spot, 1905 February 5	Roy. Obs., Greenwich
132	Sun, 1905 February 8	Roy. Obs., Greenwich
133	Sun-spot, 1905 February 8	Roy. Obs., Greenwich
134	Nebula near ψ *Eridani*, 1905 January 8	Max Wolf
135	Nebula M 33 *Trianguli* (N.G.C. 598)	Isaac Roberts
136	Nebula in *Perseus* (N.G.C. 1499)	Isaac Roberts
137	Nebula in *Monoceros* (N.G.C. 2237–9)	Isaac Roberts
138	Nebula ♅ V. 24 *Comæ* (N.G.C. 4565)	Isaac Roberts
139	Nebulæ ♅ V. 42, &c., *Comæ* (N.G.C. 4631)	Isaac Roberts
140	Nebulæ ♅ V. 37 *Cygni* (N.G.C. 7000 ?)	Isaac Roberts
141	Nebula Index Cat. 405 *Persei*	Isaac Roberts
142	Clusters in *Perseus* (N.G.C. 869, 884)	Isaac Roberts
143	Cluster ♅ VI. 30 *Cassiopeiæ* (N.G.C. 7789)	Isaac Roberts
144	Eclipse, 1905 August 30 (5 sec.)	W. H. M. Ch
145	Eclipse, 1905 August 30 (20 sec.)	W. H. M. Cl

R.A.S. Ref. No.	Subject.	Photographed by
146	Eclipse, 1905 August 30 (7 sec.)	W. H. M. Christie
147	Eclipse, 1905 August 30 (20 sec.)	W. H. M. Christie
148	Eclipse, 1905 August 30 (Portion)	W. H. M. Christie
149	Region of Nebula ρ *Ophiuchi*	E. E. Barnard
150	Nebula ρ *Ophiuchi* (enlarged)	E. E. Barnard
151	Region of θ *Ophiuchi*	E. E. Barnard
152	Great Rift near θ *Ophiuchi*	E. E. Barnard
153	Great Star Cloud in *Sagittarius*	E. E. Barnard
154	Small Star Cloud in *Sagittarius*	E. E. Barnard
155	Region of Cluster M 11	E. E. Barnard

Nos. 44–55, Nos. 64, 65, and No. 147 form a series of corona photographs, oriented and reduced to the same scale.

The above photographs are now on sale to Fellows as prints, either platinotype or aristotype, mounted on sunk cut-out mounts, measuring 12 inches by 10 inches; also unmounted, and as lantern slides. Nos. 44–55 and Nos. 64 and 65 are also supplied as transparencies, $6\frac{1}{4}$ inches square.

Price of prints, mounted 1*s*. 6*d*. each, unmounted 1*s*. each; lantern slides, 1*s*. each; packing and postage extra.

Transparencies, $6\frac{1}{4}$ inches square (Nos. 44–55 and Nos. 64 and 65), 3*s*. 6*d*. each.

Orders to be addressed to W. H. Wesley, Burlington House, London, W. In ordering prints or slides the R.A.S. Reference No. only need be quoted, but in the case of prints it should be stated whether platinotypes or aristotypes are required, and whether mounted or unmounted.

The Gold Medal.

The Council have awarded the Society's Gold Medal to Professor Ernest W. Brown, F.R.S., for his researches in the Lunar Theory. The President, in his Address to the Society, gives the grounds upon which the award has been founded.

Publications of the Society.

During the past year vol. lxvi. of the *Monthly Notices* has been issued.

Vol. lvi. of *Memoirs* has been published :—

Thomas Lewis, Measures of the Double Stars contained in the *Mensuræ Micrometricæ* of F. G. W. Struve, collected and discussed; with an Introduction containing general deductions, a list of proper motions of fifty faint stars, and various other information in respect to double stars.

The Royal Society have decided to terminate the arrangement under which the Royal Astronomical Society has been permitted to have reprints of astronomical papers from the *Proceedings* and *Philosophical Transactions*. This decision, however, does not relate to Eclipse reports; and the Reports to the Joint Meeting of the two Societies on 1905 October 19 (reprinted from the *Proceedings*) has been issued to Fellows as a separate publication. The following paper, reprinted from the *Philosophical Transactions*, will shortly be issued as an Appendix to the *Memoirs*:—

F. W. Dyson, Determination of wave-length from spectra obtained at the total solar eclipses of 1900, 1901, and 1905.

Obituary.

The Council regret that they have to record the loss by death of the following Fellows and Associates during the past year:—

Fellows:—Raphael Louis Bischoffsheim.
Rev. John Bone.
Everard Home Roberts Coleman.*
Thomas R. Dallmeyer.
Robert Isaac Finnemore.
Joseph H. Freeman.
Joseph Gledhill.
Charles Jasper Joly.*
John Joynson.
Alfred Edward Nicholls.
Robert Rawson.
William John Reynolds.
Philip E. Sewell.
Rev. George Venables.
Associates:—Samuel Pierpont Langley.
Jean Abraham Chrétien Oudemans.

Obituary notices are also given of the following, who died in January 1907:—

Agnes Mary Clerke (Honorary Member).
William Johnston.
William Simms.

Raphael Louis Bischoffsheim, son of Louis Bischoffsheim, founder of the Bischoffsheim Bank, was born at Amsterdam, 22nd July 1823. At an early age he was sent to Paris, there to study for entry to the Central School of Arts and Manufactures, into which he was admitted in 1839. On leaving the school he was attached to the staff of the Railway de la Haute-Italie as Inspecting Engineer. Later, he took over the direction of his father's bank, finally settling down in Paris, where, on 24th April 1880, he obtained full letters of naturalisation for "services rendered to the country." For many years he was well known for his liberality towards anything tending to promote the welfare of literature, science, and art, and the value of his donations was finally acknowledged by his admission, as life member, to the Académie des Sciences on 16th June 1890.

Especially fond of astronomy, he presented a costly equipment

* Obituary in Annual Report, 1905.

to the Paris Observatory, the Equatorial Coudé and a Meridian Circle being due to his munificence; he subsidised the Observatory on Mont Blanc, and contributed towards the expense of redetermining the length of an arc of the meridian at Quito.

But he will, above all, be remembered as the founder of the Observatory of Nice, in the organisation of which he was assisted by the experience and authority of the Bureau des Longitudes. Having brought into existence this great institution, he entrusted its direction to the able and energetic astronomer Perrotin. He generously endowed the observatory, and finally assured its future by presenting it to the University of Paris. This foundation has rendered the name of Bischoffsheim familiar to astronomers in every land, and the *Annales* of his Observatory will form the imperishable memorial of its founder.

M. Bischoffsheim was elected a Fellow of the Royal Astronomical Society on the 14th January 1881. G. B.

The Rev. John Bone was the second son of John and Mary Bone, of Melton Lodge, Surrey. He was born on 20th October 1834. He graduated at King's College, London, in theology, with first-class honours, in 1861, and in the same year he was ordained Deacon by the Bishop of Rochester. He was first licensed to the curacy of Radwell, Herts, and in 1862 was ordained priest. In 1863 he became curate of Poulton-le-Fylde. Thence he went to Melksham, Wilts, and from 1865 to 1873 he occupied the position of curate-in-charge of North Meols, Lancashire. He also served on the committee of the Southport Infirmary. In 1863 he married Eliza, youngest daughter of Samuel and Mary Mayhew, of Camberwell Park, Surrey.

As incumbent of St Thomas' Church, Lancaster, Mr Bone "read himself in" on the last Sunday in May 1873, so that he had held the living exactly thirty-three years. He was the oldest beneficed clergyman in Lancaster in point of length of time he had held the living.

For some years he conducted a class in astronomy at the Storey Institute, and was never happier than when introducing others into the mysteries and delights of solar science. Whenever any astronomical phenomenon was observable in Lancaster his was the brain that guided local observers, and by his personal observation of the heavens he has at [illegible] times been able to render good service to the cause [illegible] mical research. In the work of the Lancaster Astrono[illegible] tific Association he took a deep interest, contrib[illegible] pers on various phases of the planetary, solar, [illegible] ms. There are many in Lancaster who entertain [illegible] de to Mr Bone for the lead he gave them i[illegible] n astronomy. The establishment of the Greg [illegible] h he was honorary Director, was mainly due t[illegible]

The sudden death of his wife in July [illegible] to him, and for a time he was prostrate[illegible]

and was able to resume and discharge his ministerial duties up to within a few days of his death, which occurred on Sunday, 27th May, at the age of seventy-one. He leaves three sons and two daughters.

Mr Bone was elected a Fellow of the Society on 6th April 1887.

AGNES MARY CLERKE was born, 10th February 1842, at Skibbereen, a small country town in a remote part of the County Cork. Her father was John William Clerke; her mother was a sister of the late Lord-Justice Deasy.

Very early in life she was attracted by the wonders of the heavens, and before the age of fifteen had definitely formed the intention of writing a history of astronomy,—had even actually begun it. Always delicate constitutionally, she found her chief pleasures in study and in music. In 1861 the family moved to Dublin; in 1863 to Queenstown; and the years 1867–77 were spent in Italy, chiefly at Florence, where Miss Clerke studied assiduously in the public library, and wrote her first important article, "Copernicus in Italy," which was accepted by the *Edinburgh Review* (October 1877).

The family then returned to England and settled in London. In 1885 appeared Miss Clerke's *History of Astronomy in the Nineteenth Century*, a work now in its fourth edition, and regarded as the standard work, continuing the *History* of Grant.

Miss Clerke's other works, published at intervals, are as follows:—*The System of the Stars*; *Familiar Studies in Homer* (in part only astronomical); *The Herschels: A Concise History of Astronomy*; *Problems in Astrophysics*; *Modern Cosmogonies*.

Besides these works she contributed fifty-five articles to the *Edinburgh Review*, mainly on subjects connected with Astrophysics; the articles on astronomers to the *Dictionary of National Biography*; some articles on astronomers and on astronomical subjects to the *Encyclopædia Britannica*; and innumerable articles to *Knowledge*, to *The Observatory*, and other periodicals.

In later years Miss Clerke was a frequent attendant at the meetings of the Royal Astronomical Society, and in 1903 received the great honour of being elected an Honorary Member of the Society.

She was also a member of the British Astronomical Association, and constantly attended its meetings.

Miss Clerke was not a practical astronomer; but the three months' visit paid by her in 1888 to the Cape Observatory, as the guest of Sir David and Lady Gill, enabled her to write with increased clearness and confidence. In 1892 she was awarded for her astronomical works the Actonian Prize of 100 guineas by the Royal Institution.

Miss Clerke's ideals of life were lofty; and, loving and lovable, her character was in complete harmony with them. In all her writings, Truth was ever her goal.

Accomplished in many directions, astronomy to the last was her chief intellectual interest.

She died, after a comparatively short illness—in perfect peace, and fully conscious to almost her last moment—on 20th January 1907.

M. L. H.

THOMAS RUDOLPHUS DALLMEYER was born in May 1859. He was the second son of the late Mr J. H. Dallmeyer, photographic optician, his mother being the daughter of another famous optician —Andrew Ross.

He was educated at Mill Hill School and at King's College, London, and it had been intended that he should go through the full university course in mathematics and science. Unfortunately, however, his elder brother died suddenly in 1878, and his father's health also becoming impaired, it was considered desirable that young Dallmeyer should relinquish his studies after passing the first examination for B.Sc., in order to take a share in the management of the firm founded and made famous by his father. His father died in 1885, having handed over the business to his son the year before.

He was thoroughly familiar with photographic optics in all its branches. Perhaps his best work was that done in introducing and perfecting the telephotographic lens, and in working out its theory and applications in his excellent book on "Telephotography."

In 1886 he became a member of the Royal Photographic Society, and the interest which he took in the work of that society eventually led to his election as President for the years 1900, 1901, and 1902.

Although he never contributed any papers to the Society, he rendered many services to astronomy by way of supplying efficient tools. In addition to ordinary telescopes and photographic lenses, of which many must have been used for serious astronomical work, he supplied the six-inch Rapid Rectilinear lens with which the Cape Photographic Durchmusterung was carried out.

He died on Christmas Day 1906, after an illness of only a few days.

He was elected a Fellow of the Royal Astronomical Society in May 1888.

. . C.

ROBERT ISAAC FINNEMORE was the son of
to the Archbishop of Canterbury. When
eight years old the family emigrated to Nat
the ill-fated *Minerva*, which was wrecked off
family being saved with great difficulty.
of his parents that he should become a mi
as a pupil at Bishopstowe under Bishop Co
at Bishopstowe being destroyed by fire,
Maritzburg, where, in 1858, he entered th
as pupil-assistant in the Surveyor-General's
year, while not yet seventeen, he was p

clerk. In 1864 he rose to be chief clerk and draughtsman, and was also admitted a Government land-surveyor. Being of an ambitious disposition, and the Surveyor-General's department offering no prospect of further advancement, he succeeded in being transferred to the Attorney-General's department, where he worked as law-clerk under Sir Michael Galwey for eleven years. In 1876 he accepted the appointment of Postmaster-General. During his tenure of office he introduced separate delivery windows for Europeans and natives, but his attempt to establish a postal delivery was unsuccessful. In 1877 he was appointed Master of the Supreme Court, which post he held till March 1881. He was Chairman of the Zulu War Commission appointed to decide upon the compensation to be paid to relatives of those who had fallen during the war; and for his "admirable report," drafted with the assistance of Sir Henry Bale and Dean Green, he received the thanks of the Government and an expression of the Governor's high appreciation. In 1894 a new office of Crown Solicitor and Parliamentary Draughtsman was created, and the then Prime Minister, Sir John Robinson, offered the post to Mr Finnemore. It was in this capacity that he, in 1896, accompanied Sir Walter Hely-Hutchinson to Volksrust as legal adviser in connection with the Jameson Raid. In November 1896 he succeeded to the Supreme Court Bench, and during part of 1903 and 1904 he acted as Chief-Justice, and towards the end of the latter year he retired.

Apart from these services as a public official, Mr Finnemore did much to advance the cause of philanthropic, religious, and social organisations. He was an enthusiastic Freemason, and held the office of District Grand Master from 1882. To the cause of temperance he devoted much time; and as a member of the Wesleyan Methodist Church he did much for the advancement of that body in Natal. He was, besides, a Fellow of each of the following Societies:—Royal Meteorological, Royal Historical, Zoological, Royal Geographical, Statistical, Anthropological Institution, Royal Colonial Institute, and the Imperial Institute.

Mr Finnemore died 27th July 1906.

He was elected a Fellow of the Society in November 1890.

Joseph H. Freeman was born, 18th April 1845, at Stratford, Essex, where he resided all his life. A schoolmaster by profession, he devoted much time to popular lectures on astronomy. His preference for astronomy was due to his grandfather, Mr John Freeman, who was an intimate friend of Mr Epps, the first Assistant Secretary of the Royal Astronomical Society. Mr Freeman was for many years a most regular attendant at the Society's meetings. He died on 5th February 1906. He was elected a Fellow of the Society on the 10th November 1871.

Joseph Gledhill was born, 17th November 1837, at Bradford, Yorks. His early training was directed to the profession of a

schoolmaster, which he followed for some years. His own tastes, however, gradually won him over to the close prosecution of scientific study, and in 1868, when Mr Edward Crossley established his observatory in Halifax, he asked Mr Gledhill to undertake the work of observer. The measurement of double stars was a distinguishing feature of his work there, and the results were embodied in *A Handbook of Double Stars*, which was written with the co-operation of the Rev. J. Wilson, M.A. (now Canon of Worcester) and Mr Crossley. He also contributed several papers on planetary observations to the *Monthly Notices of the R.A.S.*

The erection of the 3-foot reflector in 1885 (built by Dr Common, and subsequently presented to the Lick Observatory by Mr Crossley) promised a more extended sphere of work, but it was found that, for purposes of exact observation, the climate of Halifax was hopelessly unsuitable, and the serious use of the telescope was soon discontinued.

After about two years of indifferent health, Mr Gledhill (soon after Mr Crossley's death in 1905) removed to Hoddesdon, Herts, where he died on 20th March 1906.

His scientific attainments were by no means a complete indication of his mental activity and varied tastes. He was an omnivorous reader, a very fair amateur violinist and organist, and, later in life, became an enthusiastic fisherman. Apart from his more serious work in astronomy, there are probably not a few amateurs living who have at some time or other felt their indebtedness to his ready and willing assistance in their little difficulties.

Mr Gledhill was elected a Fellow of the Society in May 1874.

F. H. C.

William Johnston was born, 10th August 1819, at "Stockholm Farm," in the parish of Dumfries, Scotland. He was for many years a furniture manufacturer in Gloucester, but later dealt in curios, becoming an authority both in England and America. In 1843 he married Miss Avery, of Gloucester, who died in 1898. He was interested in a variety of subjects, as may be judged from the titles of his pamphlets,—*The Machinery of the Heavens*; *The Immortality of all Living Creatures*; *Are Intoxicating Beverages Necessaries of Life?* He died on 10th January 1907, leavin
sons and four daughters.

Mr Johnston was elected a Fellow of the Socie
January 1897.

Alfred Edward Nicholls was born in
Gloucestershire. Immediately on leaving scho
and after serving in all grades as a mariner, id
a small nautical school in Limehouse, of wh
in 1894. Under his management it becar
cessful schools of navigation in the countr
the Passmore Edwards Sailors' Palace at L

the school was transferred to that building, where it has since been carried on under the name of the King Edward VII. Nautical School, and has had many among its students who have risen high in the profession. Captain Nicholls was the author of several works on navigation and astronomy, including *Seamanship and Guide*, and *Concise Rules to Board of Trade Examinations.* He was a member of the British Astronomical Association, and had taken up work in connection with the Variable Star Section.

He died suddenly, from the result of an operation, on 4th March 1906, leaving a widow and six young children.

He was elected a Fellow of the Royal Astronomical Society in March 1905.

Robert Rawson was born at Brinsley, a small colliery village nine miles from Nottingham, on the 22nd July 1814. His father, John Rawson, was a man of very limited means, whose life seems to have been a long struggle against adverse circumstances.

Robert Rawson was a child of seven when he began work in the coal-mines of Messrs Barter & Walker at Eastwood, two or three miles from his home at Brinsley, and he never forgot the long walks in all weathers to and from the mines. It was during this sixteen years, when working as a collier, that he laid the foundation of his future success. One day he came across a periodical containing a variety of mathematical questions, to some of which he could see the answer; but not being able to master all, he made inquiries and was told that he wanted an "arithmetic." Accordingly, one Saturday afternoon, on leaving the pit, he walked into Nottingham and bought a second-hand arithmetic for twopence. This he used to find the answers to the particular questions; he never worked through it. In the same way he acquired an algebra, then an old *Euclid*, and more advanced works. In 1837, when Robert Stevenson commenced building the Manchester and Leeds Railway, Rawson obtained employment as clerk and draughtsman in a constructor's office at Rochdale. He did not seek this situation; it was offered under the following circumstances. A controversy arose in a local paper as to how the level of the road should be altered at a curve in the railway, so as to counteract the effect of centrifugal force. Rawson gave his solution, and a few days later the railway engineer sent for him. As he presented himself in collier's dress and spoke the colliers' dialect, there was at first some misunderstanding. In the end it was arranged that he should leave the mines and go to the office. When the railway was completed in 1842 he removed to Manchester and became a teacher of mathematics, and kept up a correspondence with the *York Courant*, the *Mathematician*, the *Lady's and Gentleman's Diary*. He also wrote papers on the "Summation of Series," "Definite Integration," etc. etc., which were published in the *Memoirs* of the Manchester Literary and Philosophical Society.

In 1845 he joined the Manchester Society, and two years later had a seat on its council, where he came into contact with leading

practical engineers and chemists. At this time he was employed by Stevenson to calculate the stress on the girders of the Menai Bridge, and by Eaton Hodgkinson to determine the strength of cast-iron pillars and other materials.

In 1843 the Admiralty established a school in each of the dockyards for the education of engineers and shipwright apprentices. The instruction was at first given by the dockyard officers, but in 1847 it was considered that properly qualified masters should be appointed. Hodgkinson, to whom the First Lord applied, recommended Rawson, who was rather dubious, as he had never had experience of school life, in fact had never set his foot inside a school. However, for a period of twenty-eight years he held the post at Portsmouth with credit to himself and advantage to his pupils. Many of those passing through his school are now filling places of high distinction in the service of the Admiralty and of the great shipbuilding yards. It will suffice to mention Sir Philip Watts, K.C.B., the present Director of Naval Construction; Sir John Durston, K.C.B., Engineer-in-chief R.N.; Sir James Williamson; and Professor Francis Elgar, F.R.S.

While at the dockyard school he was appointed by the Lords Commissioners of the Admiralty to make experiments, in conjunction with the master shipwright, John Fincham, to test the validity of Moseley's formulæ for the dynamical stability of ships. Moseley gives an account of these experiments in a paper entitled "On the Dynamical Stability of Ships and on the Oscillations of Floating Bodies," in the *Philosophical Transactions*, 1851, and speaks in high terms of Rawson's ability.

Rawson devised the screw compass, which determines at sight the pitch of the screw: for this invention he received the thanks of the Admiralty. In 1851 he published a treatise on *The Screw-Propeller: an Investigation of its Geometrical Properties*, and some elementary works on arithmethic, mensuration, and trigonometry.

The great bulk of Rawson's papers are in the *Memoirs* of the Manchester Literary and Philosophical Society, but some will be found in the British Association *Reports*, the Naval Architects' *Transactions*, and the *Messenger of Mathematics*. Th
deal with differential equations; prime numbers; fric
oscillation of floating bodies; stability of ships; scr
etc. etc., his last paper being on "Dr Ferrers' The
Numbers" in the *Messenger of Mathematics*, 1895

In April 1894 he was placed on the commiss
for the county of Hampshire, and, notwithstandi
he discharged his duties as a magistrate with gre
up to within a month of his death. He was twic
1851, to Mrs Baldock, daughter of his friend
again, in 1870, to Miss Aylword, daughter
shipwright in Plymouth Dockyard. There
either marriage. Mr Rawson was a man o
temperate living, rising at 5 in the morning

in the evening. His days were spent working in his garden, walking and cycling. In the spring of 1906, while out walking at 6 o'clock on a frosty morning, he slipped and bruised himself. An attack of influenza supervened, which developed into pneumonia, and on 11th March he died at the advanced age of ninety-two. Mr Rawson was one of those men who inspire affection and command the homage of heart as well as intellect. At his funeral, which took place 16th March 1906 at the Havant cemetery, were, besides the representatives of the leading county families and a number of his fellow-magistrates, many of his old pupils, some of whom had travelled great distances to pay their last tribute of respect.

Mr Rawson was an Associate of the Institute of Naval Architects, and a member of the London Mathematical Society. He was elected a Fellow of the Royal Astronomical Society on the 9th November 1889.

William John Reynolds was the eldest son of William Reynolds, a carmine manufacturer of Hackney. He was born at Clapton 2nd September 1845, and attended the local schools, and afterwards a boarding-school at Greenwich, and finally Homerton College, at that time under Dr Unwin. Being of a mechanical turn of mind, he was bound apprentice to a firm of pianoforte makers in Clerkenwell. This firm coming to an abrupt end, Reynolds became clerk in the office of the Metropolitan Drinking Fountain Association in Clement's Lane. At his father's death, he, in conjunction with his brother, carried on the manufacture of carmine at Hackney for many years. Finally he joined with Mr Fred Baddeley, forming the firm of Baddeley & Reynolds, engravers and die-sinkers, in the Old Bailey.

Mr Reynolds married in 1880 and had one daughter. He seems to have interested himself in many subjects, and especially in Horology and Astronomy, and often answered the questions of inquirers in the *English Mechanic*. He also interested himself in social matters, being at one time Secretary to the South Place Ethical Society, and was a man much esteemed amongst his acquaintances. His death occurred on 15th July 1906, at "Varna," Palmer's Green.

He was elected a Fellow of the Society on 11th June 1898.

Philip Edward Sewell was born in London on the 14th January 1822. His father, Isaac Sewell, was a member of the Society of Friends, and his mother (*née* Mary Wright) was a lady of some literary repute. His education commenced at Hackney Grammar School, then in charge of Canon Eden, and finished at the Friends' School at Stoke Newington. On leaving school he joined the London and County Bank at Brighton, of which his father was manager. Shortly afterwards he was entered for Cambridge University, but a breakdown in his health necessitating an outdoor life, he took up civil-engineering, under Mr C. E. Vignoles. In 1849 he married Sarah, youngest daughter of

Samuel Woods, of Tottenham. He was engaged in the construction of the Settle and Carlisle Railway, and subsequently in engineering work upon Seaham harbour. In 1853 he went to Spain as railway engineer, and while there he entertained the members of the Himalaya Eclipse Expedition in June 1860 at Santander, for which he received the thanks of the Society. In 1864 he accepted an appointment in Gurney's Bank; and when this bank, in 1896, assumed the style of Barclay & Co., Mr Sewell was elected one of the Directors for the Norwich District.

Mr Sewell was a man of broad sympathies. In the parish of Buxton, where he had property, he was greatly interested in the Reformatory for juvenile offenders, which he practically kept alive, both morally and financially. He was also an active worker on behalf of the Discharged Prisoners' Aid Society.

Mr Sewell was a justice of the peace, and was one of the original Aldermen appointed on the formation of the Norfolk County Council. Mr Sewell was twice married, but he died a widower, leaving one son, Mr Philip Edward Sewell.

Mr Sewell was elected a Fellow 11th January 1861, and died 6th February 1906.

William Simms, who died on 2nd January last at the age of eighty-nine, was the oldest Fellow of the Society, having been elected in 1851, three years before any existing Fellow. The names of Troughton and of Simms have been so long honourably associated with the history of practical astronomy that the opportunity may be taken to recall a few facts about the different individuals who have borne them.

Edward Troughton (1753–1835) was admitted as a young man to partnership with his uncle of the same name and his eldest brother John, who were settled in London as mathematical instrument makers. About 1782 the Troughtons established themselves in Fleet Street, where they commenced an independent business, called "Ye Orrery," as successors to a series of well-known artists (Wright, and subsequently Cole) who had previously occupied the same premises. After the death of his uncle and his brother John, Edward Troughton alone continued the business until 1826, when he took Mr William Simms (1793–1860) as his partner and successor. After Troughton's retirement in 1831, William Simms continued the business alone till his death in 1[illegible] when he was succeeded by his son James Simms, the present [illegible]prietor.

William Simms had five brothers; on[illegible]rick Walter Simms (1803–1865), was for a shor[illegible] assistant at the Royal Observatory unde[illegible] of our Society; and another brother, Ja[illegible] of ships' and other compasses in Grevill[illegible] Our recently deceased Fellow, William Sim[illegible]the son of this brother James, and was [illegible]his father's business. But about 1836 [illegible]ays had commenced, and his uncle's occup[illegible]ical

and surveying instruments) seemed so much more promising than that of a compass-maker, that he was led to apply to the well-known firm for a situation, which was easily found for him. He became superintendent of the business, and afterwards a partner with his cousin; but in 1871 (after being partner for some ten years) he retired, during a loss of health which he regarded as permanent, but which fortunately proved only temporary.

The *Monthly Notices* contain several papers by "William Simms jun.," * especially on the improvement of instruments. The hole in the central cube of the Greenwich transit-circle telescope was first suggested by him, according to the recollection of Mr James Simms; and (on the same authority) he greatly improved the cutting apparatus of the dividing machines, while as a hand-divider he could scarcely be excelled. For a long time he was employed upon the construction of the National Standard of Length, and greatly assisted the Rev. R. Sheepshanks in that work, a large number of the observations, as well as much in the way of construction, being due to him. He also helped the elder Lord Rosse, put up Mr Carrington's telescope, and did work at Bidston Observatory. His astronomical interests were wide in range; and his daughter (Mrs M'Lachlan) well remembers an occasion when letters announcing some interesting and novel observation made independently by Sir George Airy and himself crossed in the post. We learn from her also that observations on the Sun caused the loss of her father's eyesight.

While in business, and after his marriage in 1844 to Charlotte, daughter of Francis Needham, of Wymondham, Leicestershire, William Simms lived at Granville Square (in London), at Putney, and at Charlton. After his retirement he lived at Burnham, in Somerset; and afterwards in the Isle of Wight, at Ryde and Shanklin. He had one son, who is a surveyor in New Zealand, and the daughter above mentioned. His widow survives him, aged ninety-six, and also quite blind. The late Queen was much interested in the aged couple, and shortly before her death Mr William Simms was granted a small Civil List pension, on account of his blindness, caused by devotion to science.

He was born in London, 22nd June 1817; and died at Albert Lodge, Shanklin, I.W., 2nd January 1907. He was elected a Fellow on 10th January 1851 (the Report of Jurors of the great Exhibition of 1851 mentions his management of the Astronomical Exhibits), and served on the Council 1867–69. H. H. T.

The Rev. George Venables was born at Hamptongay, Oxon, 24th April 1821. He was educated at St Edmund Hall, Oxford, where he took the degree of Student of Civil Law. In 1843 he married Miss Davis of Loudwater, Berks, by whom he had six children. The wife and one daughter survive him. In 1852 he

* In one case there is some confusion with another "William Simms junr.," afterwards described as W. H. Simms. He was the *son* of William Simms the elder, and brother of the present sole representative of the firm.

was ordained priest by the Bishop of Oxford, and finally appointed as first Vicar of St Paul's, Chatham, 1855. In 1858 he accepted the private living of Friezland, in the West Riding of Yorkshire, which he left in 1870 to become Vicar of St Matthew's, Leicester. In 1874 he was transferred to the important vicarage of Great Yarmouth, and in 1881 he was made a Canon of Norwich Cathedral. Finally, on the nomination of the Lord Chancellor, he became Rector of Burgh Castle, near Great Yarmouth. While at Great Yarmouth he did much to restore the historic parish church, to which he presented a magnificent oak pulpit. Canon Venables was a prolific writer on Church matters. He was a member of the Royal Commission on Patronage, and was select Preacher at Cambridge in 1883. His death took place 30th December 1906, at the age of eighty-five. He was elected a Fellow of the Society on the 11th of January 1856.

Samuel Pierpont Langley was born at Roxbury, Massachusetts, on 22nd August 1834. After he had graduated at Boston High School in 1851, he took up civil-engineering and architecture as his profession. From childhood he had shown great devotion to scientific pursuits, especially to astronomy. He practised his profession for some thirteen years, but after spending part of the years 1864 and 1865 in travelling in Europe, and visiting foreign observatories and learned institutions, he decided on his return to devote his life to the pursuit of science.

His first scientific appointment was that of Assistant at the Harvard College Observatory in 1865. In the following year he was made Assistant Professor of Mathematics in the Naval Academy at Annapolis, a post he almost immediately relinquished to become Director of the Allegheny Observatory at Pittsburg, where he remained for twenty years.

In 1867–68 he was busy with the equipment of the observatory, and a little latter he arranged and cárried out a plan for distributing standard observatory time to the existing railway system of the country. In 1869 he went to Kentucky, in charge of a party of the United States Coast Survey, to observe the total eclipse of the Sun, and in 1870 he joined a Government eclipse ex[illegible] and went to Jerez in Spain. The only other eclipse exp[illegible] took part in was one to observe the eclipse of 1878 fr[illegible] of Pike's Peak.

His first interest at Allegheny was the Sun, an[illegible] he was there that he made his well-known drawi[illegible] and other details of the Sun's surface. An emin[illegible] said of these drawings that the better the d[illegible] surface is seen, the more closely does it r[illegible] drawings.

It was about 1875 that he began to devo[illegible] the measurement of the heat spectra of the [illegible] bodies. Experience convinced him of the [illegible] thermopile as a measuring instrument, an[illegible]

years he was successful in the invention of the Bolometer. This instrument, as is well known, is an electrical thermometer on the principle of Wheatstone's bridge. With it he at once set to work to explore the infra-red spectrum of the Sun; he extended it to regions of ten times the wave-length of the visible spectrum and mapped the lines in it; other researches with the bolometer were on the action of the Earth's atmosphere in scattering and absorbing selectively the Sun's rays of all wave-lengths; on the infra-red spectrum of the Moon, and a determination of its temperature, which he found to be a little above 0° C.; an estimate of the constant of solar radiation by a new method; and on the connection between temperature and distribution of radiation in the spectrum of heated terrestrial sources. All this was done between 1880 and 1888.

In 1888 he was appointed Secretary of the Smithsonian Institution. The Smithsonian Institution officially represents the interest of the United States in science, and the chairman of its board of directors is the President of the United States. Its immediate affairs are administered by the secretary, who has charge of many and various scientific interests, such as the National Museum, the International Exchanges, the Bureau of American Ethnology, the National Zoological Park, and others. These administrative duties necessarily occupied much of Langley's time, and it was now impossible for him to devote the same personal attention to his scientific researches as before.

A few years after his appointment he founded the Smithsonian Astrophysical Observatory, the object of which was to increase our knowledge of the natural agencies which control climate and life. One of the most important of these he believed to be the amount of heat radiated to the Earth by the Sun, and at the time of his death he was engaged in the problem of finding whether the amount was constant, or varied sufficiently to affect the climate of the Earth. About the same time he improved his bolometer by adding to it a photographic arrangement to record its readings automatically and continuously: by its means it became possible to map the whole of the energy spectrum of the Sun in a few minutes.

Perhaps Langley was more widely known for his studies in the problem of flight than for his astrophysical work, or his able direction of a great institution. He had always been interested in the question of flight, and first took it up seriously in 1889. In 1891 he published his "Experiments in Aerodynamics," and in 1893 "The Internal Work of the Wind." In these papers he demonstrated the theoretical possibility of flight by means of large sustaining surfaces and mechanical motors, and for the remaining years of his life he gave much time and thought to its practical realisation. Large models of his design for a flying machine were successfully flown between 1896 and 1903, and he was encouraged by his success to construct an aerodrome large enough to carry a man. Two trials of this machine were made on the Potomac river in 1903, but were foiled by accidents in the launching, fortunately

without loss of life. No further trials were made owing to a combination of unfavourable circumstances, including failing health.

Langley's published papers are over a hundred in number. It was his aim in all his publications to write them in the simplest and clearest language, so that they might be understood by educated persons not specially read in the subject, and he took immense pains to try to achieve his aim. He was a Foreign Member of the Royal Society of London, a Correspondent of the Institute of France, Member of the Accademia dei Lincei of Rome, and of many others. He received the degree of D.C.L. from Oxford, and Sc.D. from Cambridge, besides many other honorary degrees; and he was a medallist of the Royal Society of London, the American National Academy of Sciences, and the Institute of France, and received the Diploma of the Royal Institution.

He had many interests outside his scientific work. One was psychical research, and for many years he was associated with both the American and the British Societies for Psychical Research. He showed a keen appreciation of the best in English and French literature, and had a special interest in George Borrow, of whose manuscripts he had a large collection. He was also much interested in the fine arts, and had a considerable knowledge of pictures.

He died on the 27th February 1906, at Aiken, South Carolina.

He was elected Associate of the Royal Astronomical Society in 1883.

B. C.

Jean Abraham Chrétien Oudemans was born at Amsterdam on the 16th of December 1827. At the age of sixteen he went to the University of Leiden to study astronomy under Professor Kaiser, and took his degree in 1852. In 1853 he was appointed as Astronomer at the Observatory which was then established on the roof of the University building, and occupied himself mostly with observations and computations of planets, comets and variable stars, which are published chiefly in the *Astronomische Nachrichten*. His first astronomical publication, however, dates from 1846.

In 1856 Oudemans was named Professor of Astronomy at the University of Utrecht, but the following year he resigned his professorship to take up his appointment of Chief of the Geographical Service in the East Indian Colonies. There he determined, in different parts of the archipelago, the l[illegible]nd longitude of a great number of stations, and executed [illegible] of engineers the triangulation of the whole island [illegible] found time for more strictly astronomical work, [illegible] total solar eclipses, and taking part in the [illegible] transit of Venus in 1874 as Chief of the [illegible] island of Réunion, his colleagues being [illegible] Professor Kaiser.

In 1875 he returned to Europe, an[illegible] time, Professor of Astronomy and Dire[illegible] Utrecht. This position he held till 18[illegible] age of seventy years.

When in Batavia he published the first part of the measurement of the base line near Sunplak in Java, the second part being published shortly after his return to Holland. When all the observations for the triangulation of Java were completed, he was charged by the Government with their reduction. During the years 1891–1900 he published the results in four large volumes.

As a member of the Dutch Geodetic Committee, Oudemans had the direction of the geodetic astronomical observations, and in 1905 he published a volume containing the results of the observations of latitude and azimuth made at thirteen stations, together with a study of the instruments employed and of their different errors. Notwithstanding his occupation with his geodetic work, Oudemans published a great number of papers on different astronomical subjects, *e.g.* "On the theory of instruments"; "On the ring-system of Saturn"; "On the position of the equator of Mars"; "On the Moon's diameter"; "On the parallax of fixed stars."

He published also a popular work on astronomy for the schools in Java, and revised Kaiser's Popular Astronomy, *The Starry Heavens*.

Oudemans was indefatigably busy with astronomical work to the end of his life: only some few days before his death he corrected the proofs of his last paper, "On the mutual occultations and eclipses of the Satellites of Jupiter," published by the Academy of Sciences at Amsterdam. He was a painstaking astronomer, with a vast knowledge, who in all his work strove his utmost to attain the highest accuracy and completeness. He had a noble and open character, and was much esteemed and beloved by all who knew him.

His death at the age of seventy-nine took place, after a short illness, on 14th December 1906.

He was elected an Associate of the Royal Astronomical Society in 1883, and a Correspondent of the French Academy in 1901.

H. G. VAN D. S. B.

Proceedings of Observatories.

Royal Observatory, Greenwich.
(*Director, Sir William Christie, K.C.B., Astronomer Royal.*)

Transit Circle.—During the year 5877 observations of transits and 5605 of meridian zenith distances have been obtained. The Sun has been observed 123 times and the Moon 94 times. The lunar crater Mösting A has also been observed 47 times. Reflexion observations of stars have been obtained on 69 nights.

The object-glass of the transit-circle was repolished at the beginning of the year, but in the process the figure was affected owing to the thinness of the lenses, and it was not till the end of April that the refiguring was satisfactorily completed. There was consequently much interruption in the observations during the first four months of the year.

Some observations with the reversion prism, designed to ascertain whether the direction of motion had a systematic effect, revealed the fact that the adopted wire intervals were sensibly in error. It had been assumed that when one or two wires were renewed the intervals of the remainder were unaffected, but it now appears probable that the process of inserting new wires is liable to displace the existing wires. A redetermination of the intervals of the wires for the periods covered by the forthcoming Second Nine Year Catalogue, 1897–1905, was undertaken, 1200 transits being worked up in each year. The result of this extensive discussion disclosed sensible errors in the adopted wire intervals, but as these are of the nature of accidental errors, no corrections have been applied to the transits, the systematic effect on the catalogue places being presumably negligible.

The reductions for the Second Nine Year Catalogue for 1900 are progressing. The zones from the Pole to 15° N.P.D., Part ii. containing the astrographic reference stars, are completed, and the copy for press ready. Some 1200 of these stars which are common to the Carrington Catalogue for 1855 have been compared with it, and from the comparison, after corrections for systematic discordances have been applied, proper motions have been determined. These proper motions and those derived from the new reduction of Groombridge's Catalogue have been used by Mr Eddington as the data for a paper on the "Systematic Motions of the Stars," in which Kapteyn's two-star drift hypothesis has been examined [illegible] independently confirmed (*M. N.*, lxvii. p. 34).

The new working catalogue, as stated last year, consi[illegible] reference stars down to $9^{m}\cdot0$ in the Oxford Astrographic [illegible] between the limits of 24° and 32° north declination. T[illegible] fundamental stars will also be observed.

Altazimuth.—This instrument has been used as a meridian transit instrument during the second and third quarters of the Moon for the observation of the Sun, Moon, planets, and the stars in Newcomb's Fundamental Catalogue. The instrument is reversed every two months, alternately by reversing in its Y*s*, and by turning the instrument through 180° of azimuth.

The total number of meridian transits obtained during the year is 1636, including 77 of the Moon's limbs, and 44 of the lunar crater Mösting A. The number of extra-meridian observations of the Moon obtained during her first and last quarters is 40.

A new set of wires was inserted in the instrument in October, and in connection with the determination of their intervals a re-determination has been made of the errors of the three screws of the Right Ascension, Polar Distance, and Position Micrometers. The third of these micrometers serves to readily connect the two former with each other, and thus simplifies the determination of their errors.

Reflex Zenith Tube.—During the year, 1170 double observations and 37 single observations have been obtained, the brighter stars having been observed over as long periods as possible. γ *Draconis* was observed on 82 days, β *Draconis* on 56 days, ι^2 *Cygni* on 41 days, and θ *Ursæ Majoris* on 27 days.

Occultations.—During the year 17 disappearances and 6 re-appearances of stars occulted by the Moon have been observed by one or more observers. The results have already been communicated to the Society.

28-*inch Refractor.*—A new working catalogue of double stars was brought into use in the course of 1906. This catalogue includes, as its main feature, the double stars discovered by Hough, the remaining pairs being selected from Struve, Otto Struve, Burnham, Hussey, and Aitken. The weather has not been at all favourable for double-star work, the long spells of non-observational weather causing serious interruptions in the programme of observation. However, the 414 stars observed have been measured, on the average, on two nights.

An analysis of the observations gives—

Hough Stars.

18 under $0''.5$ separation
18 between 0.5 and $1''.0$
36 between 1.0 and 2.0
165 over 2.0

Miscellaneous Stars.

37 under $0''.5$ separation
45 between 0.5 and $1''.0$
37 between 1.0 and 2.0
58 over 2.0

κ *Pegasi* was observed on 14 nights, δ *Equulei* on 9 nights, and 70 *Ophiuchi* on 11 nights. The measures of δ *Equulei* confirm the short period (5·7 years).

The equatorial and polar diameters of *Jupiter* were measured on 17 nights with the bi-filar micrometer, and also with the double-image. The satellites were measured on 2 nights. Owing to the necessity for renewing a portion of the wheel-work for opening the shutters of the dome, the 28-inch was out of use from June 27 to July 31.

Thompson Equatorial.—With the 26-inch refractor 63 photographs of *Neptune* and its satellite have been obtained on 24 nights.

With the 30-inch reflector 24 photographs of *J vi.* were obtained on 11 nights and 4 of *J vii.* on 3 nights from January 1 to February 15. During the whole of the 1905–6 opposition (from 1905 August 23 to 1906 February 15) 86 photographs of *J vi.* were obtained on 36 nights and 19 of *J vii.* on 15 nights. During the present opposition, up to December 31, 31 photographs of *J vi.* have been obtained on 16 nights, the series commencing on August 28, and 6 photographs of *J vii.* on 4 nights (November 17, 22, December 10 and 12).

Photographs of comets have been taken in the year 1906 as follows:—

Comet *c* 1905,	4	photographs on	4	nights.
a 1906,	19	"	14	"
b 1906,	20	"	15	"
d 1906,	21	"	19	"
e 1906,	7	"	6	"
g 1906,	5	"	5	"

78 photographs of 31 minor planets have also been obtained.

These photographs were all taken for determination of position.

Photographs of the nebulæ, M. 51 Can. Ven. (100^m and 60^m), M. 57 Lyræ (5^m to 20^m), and M. 31 Andromedæ (100^m), have also been obtained.

In addition, a few photographs have been taken for instrumental adjustments.

As regards the measurement of photographs and reductions—

The printing of the *Eros* measures and reductions is complete, with the exception of the discussion of results. In the cour[illegible] this discussion the errors of the réseau as photographed o[illegible] plates have been discussed with special reference to their eff[illegible] the determination of the solar parallax (*M. N.*, January 190[illegible]

The photographs of *Neptune* and its satellite are measur[illegible] discussed as far as the end of the last opp[illegible] and th[illegible] published.

The photographs of *J vi.* and *J vii.* fo[illegible]sition [illegible] and of Comets 1903 *c*, 1904 *a*, 1904 *e*, 19[illegible]5 *c*, 19[illegible] 1906 *b*, have been measured, and the resul[illegible]hed.

The measures of photographs of Comets 1906 *d*, 1906 *e*, and 1906 *g* are in hand.

The minor planets photographed in the years 1903 and 1904 have been measured, and the results published. Those photographed in 1905 are ready for publication, and those observed in 1906 are in hand.

Astrographic Equatorial.—Work with this instrument has still mainly been confined to replacing chart plates which, though satisfactory in other respects, are, owing to slight photographic defects, unsuitable for production of enlarged prints. During the year 133 chart plates, 4 catalogue plates, 22 plates of the field round *Jupiter*, one of *Mars*, and one of Comet 1905 *c* were taken. Of the chart plates 30 were rejected, principally because they did not come up to the standard in showing faint stars or for photographic defects.

The measurement of the catalogue plates has been completed by remeasuring the images showing discordances in the Zones 85° to the Pole. The means of measures for these zones have been taken and copy for press prepared. The printing of the measures of Zones 80° to the Pole is practically finished, and the second volume of measures complete, with the exception of the Introduction. In view of the large overlap of the plates immediately surrounding the Pole, it was thought advisable for the plates from declination 87° to the Pole to measure all the stars within the limits of the réseau, and these measures have been printed on a different system to those of the lower zones, with reference numbers showing all the plates on which each star occurs. The assigning and printing of these reference numbers has taken more time than was anticipated.

The computation of the constants of the plates in the second volume has been begun during the year. Standard co-ordinates deduced from the places of the reference stars in the Greenwich Second Nine Year Catalogue, 1900, now in course of preparation, have been computed for the plates of the Zones declination 76° to 80° and 87° to the Pole, and the scale value and orientation constants computed for these plates, with the exception of those of Zone 76°.

The counting of the chart plates (declination 64° to the Pole) is now complete with the exception of a few fields for which better photographs will be obtained, 285 plates having been counted during the year.

Enlarged reproductions of the chart plates have been made for 213 plates during the year. The total reproduced to December 31 is 538, the six Zones 65° to 70° being complete, and the Zones 71° to 74° nearly half finished.

Photoheliograph.—Photographs of the Sun have been taken on 209 days with the Thompson (9-inch) Photoheliograph, and on 6 days with the Dallmeyer (4-inch) during the time that the 30-inch mirror was being resilvered, November 3–12, when the Thompson instrument was not available. Of the photographs taken, 576 have been selected for preservation. Photographs have

been received through the Solar Physics Committee from Dehra Dûn, India, up to 1906 November 27, and these have been further supplemented by photographs from the Observatory at Kodaikánal, India, and from the Royal Alfred Observatory, Mauritius. The daily record thus made up is complete, without break, from 1905 January 2 to 1906 November 27, the date of the last Indian photograph received. The last photograph received from Kodaikánal is dated 1906 September 13, and the last from Mauritius 1906 March 26. The photographs from these two observatories have all been measured; those from Dehra Dûn have been measured up to 1906 July 21, and those taken at Greenwich to 1906 May 28. The solar activity during the year has not been quite so great as in the previous year, but the work of measurement and reduction still continues to be very heavy.

Concurrently with this, the preparation of the Supplementary Photoheliographic Results, 1874 to 1885, referred to in the last Annual Report, has been carried on during the year, and is now approaching completion. Mr Maunder, after an inspection of the photographs stored at South Kensington, found there 35 photographs taken in Mauritius which filled 23 gaps previously existing in the record for 1878 and 1879, all of these days being occasions on which the Sun showed spots or faculæ, or both. These have been lent to the observatory, and have been measured and reduced, and the results incorporated with those previously published in the Greenwich volumes for those years, or by the Solar Physics Committee. The ledgers of spot groups for the twelve years 1874–1885 have been passed for press; the daily projected areas have been computed for the entire period; and the computation of the mean daily areas and mean latitudes of the spots for each rotation of the Sun and for each year alone requires completion, and is already in hand.

The Photoheliographic Results for 1905 have been passed for press.

Printing.—The copies of the volume of Greenwich observations for 1903 were distributed in April 1906, together with the New Reduction of Groombridge's Catalogue of Circumpolar Stars and the Determinations of Longitude 1888–1902. The volume for 1904 was distributed in November and December, with an Appendix—Meridian Zenith Distances of γ *Draconis* with the Reflex Zenith Tube 1886–1899; also the Meteorological Reduc tions, Part IV., Temperature 1891–1905.

The printing of the 1905 volume is nearly completed, and second volume of the *Astrographic Catalogue* will shortly be for distribution. Other publications now in the press, and completion, are the Heliographic Results, 1874 to 18[illegible] Measures of Photographs of *Eros* for determination [illegible] parallax.

Royal Observatory, Cape of Good Hope.
(Director, Sir David Gill, K.C.B., H.M. Astronomer.)

The new transit circle has been in regular use throughout the year in observations for the new Fundamental Catalogue, and in certain researches on the magnitude correction in R.A. which are subsequently mentioned. The Repsold-Struve method of observation has been continued without use of clockwork. Experiments have been continued with the clock-driven wire, and so long as the self-regulating motor preserves a nearly uniform rate the cone-apparatus and differential gear work to perfection, and the star can be maintained in perfect bisection on the travelling wire throughout the transit by pressing one or other of two keys, of which the action of one is to accelerate, the other to retard the rate of motion of the wire by 4 per cent. But unfortunately the rate of the motor varies by quantities fully greater than 4 per cent., so that, until this variation of rate is corrected, the perfection of bisection during the transit of a star cannot be maintained. It is proposed to substitute a weight-driven clockwork, similar to that on the astrographic telescope.

The underground azimuth marks have retained their constancy of relative azimuth in a remarkable manner.

The old non-reversible transit circle was employed in completing the observations of the list of stars of which observations were requested by Professor Boss, and of the miscellaneous stars used in the determination of latitude in connection with the Geodetic Survey, and stars of which occultations have been observed. This work was completed by the beginning of August. Since that time a series of observations of selected stars whose relative positions have been accurately determined by heliometer measurement has been made in order to determine the amount of personality of the observers depending on magnitude. That is to say, a bright star having two much fainter stars, the one preceding, the other following it, is selected—all three stars having nearly the same declination, and the fainter stars situated nearly symmetrically with respect to the brighter star. Heliometer observations of the position-angle and distance of the bright star with respect to the fainter stars obviously determine the R.A. of the bright star relative to the fainter stars, free from personality depending on the magnitude of the stars or of the adopted scale-value of the heliometer. This relative R.A. so found is compared with the relative R.A. derived from the meridian observations, with the satisfactory result that the magnitude corrections derived for Mr Power and Mr Pead with the old transit circle by means of screen observations are confirmed for stars from Mag. 4 to 8, whilst for the observers with the moving wire the magnitude correction is nearly zero.

The following observations have been secured with the meridian instruments :—

With the old Non-reversible Transit Circle:—

Number of transits	. . . . .	4665
Determinations of Z.D.	. . . .	4032
"	collimation . . .	93
"	level	239
"	azimuth	249
"	run	249
"	nadir	230
"	flexure	11

With the new Reversible Transit Circle:—

Number of transits	. . . . .	3590
Determinations of Z.D.	. . . .	3627
"	collimation by collimators	48
"	collimation by reflection and reversal . .	41
"	level	422
"	azimuth	389
Observations of N. azimuth mark	. .	370
" S. "	. .	369
Determinations of run	. . . .	249
"	nadir	301
"	flexure	64

The following oppositions of the major planets have been observed with the heliometer during the year:—

	No. of Obs.	No. of Nights
Opposition of Neptune (1905–6) .	38 *	6
" Uranus 1906 . .	64	7
" Saturn 1906 . .	33	4
" Jupiter 1906 . .	48	8

The following heliometer observations have been made during the year in connection with the triangulation of the comparison stars:—

Uranus 1903–5, Saturn 1900 .	101	observations.
Neptune 1905–8	86	"
Mars 1905	61	"
Jupiter 1905	17	"
Miscellaneous	87	"

The triangulation of comparison stars for Mars 1905, with 135 observations, is now complete. 212 observations in distance and 130 in position angle have been made in connection with the above-mentioned determinations of magnitude-errors in R.A. as observed with the transit circle.

Thirty-one observations each of distance and position angle of the components of α Centauri have been made by Mr J. M. Baldwin.

Comet 1905 *c* has been observed on 9 nights and Comet Finlay 1906 *d* on 11 nights with the heliometer.

* Including 20 observations in 1905.

At the request of Dr T. Banachiewicz, observations of the relative positions of Saturn and h_1 Aquarii were made on 4 nights.

Thirty-three separate phenomena of occultations have been observed, viz.—

Disappearances of stars at the dark limb . .	14
Reappearances " " . .	14
Disappearances " bright limb .	1
Disappearance of Saturn – 4 phenomena . .	4

With the astrographic telescope the following work has been accomplished :—

Description of Plate.	No. of Plates.	No. of Exposures.	Duration of Exposure.
Triple exposure Chart plates .	15	45	20^m 20^m 20^m
Single exposure " .	10	10	30^m
Catalogue plates, second series .	23	69	6^m 3^m 20^s
Kapteyn areas	14	28	30^m 30^m
Adjustment plates . . .	11	...	various
Experiments with Cooke lens .	10	...	"
Miscellaneous	5	...	"

The "Kapteyn-areas" plates were taken at the request of Professor J. C. Kapteyn in connection with his scheme for the determination of the parallaxes and proper motions of faint stars.

During the year 185 Catalogue plates, containing 103,296 stars, including 2167 standard stars, have been measured, both in direct and reversed positions of each plate; the standard stars being measured in like manner by each of the two observers by whom the alternate zones of the plate are measured. 18 plates measured in former years have since been rejected.

The total number of plates now measured is 1086, containing about 600,000 star-images, corresponding to over 250,000 different stars.

The actual state of the measurement on December 31 is as follows :—

	No. of Plates measured.			Copied for Press.		
Zone.	Before 1906.	During 1906.	Outstanding.	Before 1906.	During 1906.	Outstanding.
–41°	140	4	...	140	...	...
–42	144	...	...	144	...	...
–43	142	...	2	115	27	...
–44	131	1	12	66	58½	1
–45	138	2	4	19	108	3
–46	113	29	2	3	82½	38
–47	65	73	6	2	1	67½
–48	25	69	50	5	...	14
–49	1	6	113	1	...	...
–50	2	1	117	2	1	...
–51	0	0	120	0	0	...
	901	185	426	497	278	123½

The Victoria telescope has been employed principally for the determination of the radial velocities of stars by means of spectra photographed with the 4-prism spectrograph attached to the 24-inch telescope. The instrument was used on 113 nights for spectroscopic work, and photographs of 245 spectra were secured by Messrs Lunt and Simpson. 123 separate spectra have been measured, and the radial velocity computed by Messrs Lunt, Goatcher, and Baldwin; Mr Simpson has given much assistance in the reductions.

Mr Lunt has prepared a paper on "The presence of Europium in Stars," for communication to the Royal Society.

The 18-inch visual telescope has been employed on 17 nights for observations of Comet Finlay and Comet Metcalf, by Messrs Hough, Lunt, Baldwin, and Simpson.

Daylight observations of the position-angle and distance of the components of α Centauri were made on 8 days by Messrs Baldwin and Innes.

The printing of the Catalogue of 8560 Astrographic Standard Stars has been completed, and copies forwarded to Greenwich for distribution.

The Cape General Catalogue for 1900, based on observations with the 8-inch transit circle during the years 1900–04, and containing 2798 zodiacal stars and all C.P.D. stars not fainter than 8·5 mag. (except those of zone -42° to -50°) which are not previously contained in any catalogue of precision, has been passed through press, and copies have been sent to Greenwich for distribution.

Meridian observations 1900–04. The outstanding sheets of this volume, containing the results of meridian observations 1900–04, all reduced to the equinox 1900, have been passed through press and sent to Greenwich for distribution. Vol. x. part 2 of the Annals of the Cape Observatory, containing researches by Mr Lunt on the spectra of Silicon and Fluorine, has been issued.

Vol. xii. part 2, containing Mr Cookson's observations with the heliometer of the relative distances and position angles of Jupiter's Satellites, and his derivation from them of the mass of Jupiter and the inequalities of the Satellites, and vol. xvi. part 3, containing Mr de Sitter's discussion of the inclination and nodes of the orbits of Jupiter's Satellites from photographs taken at the Cape in the years 1891, 1903, and 1904, have been delivered at Greenwich, and will be circulated at the next annual distribution. Vol. ii. part 5, containing meridian observations of the Sun, Mercury, and Venus during the years 1884–92; vol. ii. part 6, containing the results of occultations of stars by the Moon during the years 1896–1906 and vol. vii. part 1, containing heliometer observations of t major planets during the years 1897–1904, have been forwarded press.

The Cape day-numbers for 1908–9 have been printed distributed.

The reductions of the meridian observations with the

transit circle are complete to the end of the year. The results have been forwarded to Professor Boss. The demand for early completion of the observations of the Boss-stars has delayed the reductions of observations made with the new transit circle. The right ascensions of the latter have been completely reduced and examined to 1906 October 25. The corrected circle readings have been derived to 1896 November 25, and the reductions to apparent place applied to 1905 November 23. Owing, however, to the absence of Mr Cox in England, the examination of the N.P.D. reductions has fallen considerably in arrear.

The constants for reduction of the Astrographic Catalogue plates for the zones -41°, -42°, and -43° have been derived, as also those for 64 plates of zone -44° and for 19 plates of zone -45°.

The heliometer observations of comets and of the conjunction of Saturn and *h* Aquarii have been completed and sent for publication to the *Astronomische Nachrichten*. The heliometer observations of sets of stars required for the determination of magnitude personal equation in meridian observing have been completely reduced, and a comparison made with the transit circle results.

The records of the seismograph have been regularly forwarded to Professor Milne during the year. A duplicate copy, at the request of the German Government, is supplied through the Consul-General for Germany, Cape Town, for transmission to the central station for the investigation of earthquakes at Strassburg.

The meteorological observations have, as usual, been communicated to the Meteorological Commission, Cape Town.

Colonel Morris has completed the reductions of the Geodetic Survey of the Transvaal and Orange River Colony, and he will return to England early in 1907.

The chain of triangulation connecting the northern end of the 30th meridian arc in the Transvaal with the existing triangulation in Rhodesia has been completed by Captain Gordon, R.E., in course of the year.

No reports have been received from Dr Rubin since May 1905 in spite of the most urgent requests made to him both directly and through the Administrator.

From reports obtained from Mr M'Caw, Dr Rubin's assistant, it appeared that the triangulation is completed as far as latitude $-10^\circ\ 30'$. It has been found necessary, for various reasons, to suspend operations for the present; the assistants have been recalled, but Dr Rubin has not yet returned. The observations, so far as received, are almost completely reduced.

Mr J. M. Baldwin, a research student in Astronomy from Melbourne University, arrived at the Observatory on July 18 for a six months' course of study in practical Astronomy. Since his arrival he has devoted himself to the study of the system of α Centauri, both visually and spectroscopically, and has assisted in the general work of the Heliometer and the Victoria Telescope.

On the 20th February 1907 Sir David Gill will retire after

completing his twenty-eighth year of service as His Majesty's Astronomer at the Cape. Mr S. S. Hough, Chief Assistant, has been nominated by the Lords Commissioners of the Admiralty as his successor.

Royal Observatory, Edinburgh.
(*Director, Prof. F. W. Dyson, Astronomer Royal for Scotland.*)

Meridian Circle.—During the year several changes have been made in the instrument. The most important of these are the substitution of a spring suspension near the pivots (as in the Greenwich altazimuth) for the old method of counterpoising, and the introduction of a recording micrometer for the transits. The new counterpoise, introduced early in the year, is quite satisfactory. The recording micrometer is only just completed, and it is too soon to speak of its performance. These and some other minor alterations, combined with unfavourable weather, have made the number of observations of zodiacal stars during the year somewhat small. The number of transits and zenith distances observed during the year is 900.

15-*inch Equatorial.*—The Office of Works in July built a platform round the lower part of the pillar of the instrument, to make the handling of the instrument more easy and comfortable. The illuminations of the micrometer and circles have been rearranged, and the instrument got into adjustment for observations of double stars. Only a few observations have as yet been secured.

Spectroscopic Observation of the Rotation of the Sun.—All favourable occasions have been utilised, and 677 sets of observations made on 67 days. A few measurements have also been made of the distances of the solar lines from the telluric standards near the Sun's centre, for comparison with the means obtained from similar measurements at opposite ends of a diameter.

24-*inch Reflector.*—This instrument has been arranged for photographic determinations of the position of nebulæ. Only a few photographs have as yet been obtained; and although it will probably be impossible to obtain many photographs, it is hoped to put the instrument to this work.

Catalogue of Zodiacal Stars.—The observations have been reduced to 1900·0 as far as 1906 February, and a manuscript catalogue forwarded to Sir D. Gill. In the task of overtaking the arrears of reduction, assistance received from the Government Grant Committee of the Royal Society is gratefully acknowledged. The observations for 1906 are reduced to mean place 1906·0.

Henderson's Catalogue for 1840, re-reduced by Dr Halm, has been published during the year.

Seismographic observations have been made continuously by Mr Heath, and the results communicated to the Committee of the British Association.

Meteorological observations have been regularly made.

Cambridge Observatory. (*Director, Sir R. S. Ball.*)

1. *Meridian Circle.*—The revision and completion for press of the large volume of meridian circle results, 1872–1900, mentioned in last year's report, was finished in September, and application for printing has been made to the Syndics of the University Press. In the course of revision at the meridian circle of all the single observation stars published in the Cambridge A.G. Zone Catalogue, it happened in a considerable number of cases that the star, of which a single observation appeared in the Zone Catalogue, could not be found in the place assigned to it. The question then arose, Was the original observation erroneous, or did it refer to a very faint star only visible in the instrument on exceptional nights? With great kindness, Professor Turner undertook to examine all these doubtful cases on the Astrographic Catalogue plates taken at Oxford which cover the Cambridge zone. Very nearly all the difficulties have thus been cleared up, and the Cambridge Catalogue has derived great advantage from this final revision by the aid of the Oxford photographs.

Towards the end of the year the observation of Gill's Zodiacal Stars was resumed, and 280 stars were observed on 9 nights, with the necessary observations of fundamental stars and instrumental errors. The reductions to mean place are nearly complete to date.

Mr Hartley has been in charge of the meridian circle throughout the year.

2. *Sheepshanks Equatorial.*—One hundred and twenty-seven plates, each containing four or more exposures, have been taken by Mr Hinks during the year, the greater part of them in continuation of the series for the determination of stellar parallax begun in 1904. The observation of the fainter stars of the working list then adopted is practically complete, and the results for 10 stars are reduced and published by Mr Russell (*M. N.*, 1905 June, 1906 December). The observation of the brighter stars was interrupted by the failure of the special colour screen after a considerable number of plates had been taken. A new form of screen was constructed in experimental form in April, and the observation of bright stars has now been resumed. The finished apparatus will be mounted early in 1907.

An extensive new working list of stars has been prepared, and is incorporated with the old as vacancies occur. Observations of these stars were begun in April.

Mr F. J. M. Stratton (Isaac Newton Student) has been occupied with an investigation of the proper motions of faint stars in the Pleiades by comparison of early plates taken at Greenwich and Oxford (kindly lent by the Astronomer Royal and the Director of the Oxford University Observatory) with recent plates taken with the Sheepshanks telescope.

Mr Stratton has also undertaken a new reduction of Schlüter's observations of the Moon for comparison with Hayn's determination

of the constants of the Moon's libration and of the selenographical co-ordinates of Mösting A.

Mr H. N. Russell has continued, at Princeton, U.S.A., the measurement and reduction of plates of the stellar parallax series.

3. *Floating Zenith Telescope.* — Mr Cookson has continued throughout the year his observations with his floating zenith telescope. The climate is very unfavourable to the class of observation required for a determination of the aberration constant, and it has been found impossible to gather enough material in one year. The observations have therefore been extended over another year, and it is expected that the two years together will provide the necessary observations.

4. *Reduction of Eros Photographs.*—The formation of a homogeneous system of comparison star places has made good progress and is approaching completion.

An account of the results of intercomparison of published series from different observatories has been published during the year (*M. N.*, 1906 June, November). This showed the necessity of supplementing the published material with more complete detail, in which the results from individual plates are kept separate. By the courtesy of the Directors concerned, a great part of the required information has been obtained.

The individual observations have been collected in a card catalogue of about 18,000 cards and 50,000 entries. Systematic corrections have been determined and applied (1) to reduce all the observations to the same system of fundamental stars, and (2) to eliminate the very serious errors in the form of magnitude equation which affect the results of several observatories.

Charts of all available stars have been furnished to the Lick Observatory, and the comparison stars for the reduction of the Crossley Reflector photographs have been selected from them. Closely approximate places of the selected stars have been communicated to Professor C. D. Perrine, in advance of the definitive formation of the system.

One of the principal objects in the formation of the close standard system of stars—to provide for the reduction of the very numerous Crossley reflector plates with small measurable field—is thus in a fair way to be accomplished. The same system will be available for all other series in which it is not possible or desirable to use the *étoiles de repère* as standards—*e.g.* the Oxford and Cambridge series—and will also supply the places of a great part of the stars used as comparison stars by the visual observers. A considerable number, however, of the visual comparison stars used at Lick, Yerkes, and Washington were too faint to be found o the photographs hitherto measured and published. The places all these stars were marked on the charts supplied to Lick, most of them have been identified and measured on the Cr plates. This makes good a serious deficiency in the prepar for a general reduction of all Eros observations.

Mr Hinks has been aided in this work throughout the

Miss Julia Bell, and since April Mr S. E. Bowd has been engaged upon the card catalogue. The expense of this assistance has been defrayed by a grant from the Government Grant Fund of the Royal Society.

5. *Instruction.*—Courses of instruction in practical astronomy, and in field astronomy and surveying, have been given by Mr Hinks during the year.

The Newall Telescope, Cambridge Observatory.
(*Mr H. F. Newall.*)

The past year, 1906, was spent, so far as new observational work is concerned, in making spectroscopic observations of the Sun. The work was mainly directed to deciding, by observations of sun-spot spectra and other solar features, whether the conditions of atmosphere over the flat plains of Cambridgeshire would justify a special outfit for solar work.

In the first half of the year the 25-inch equatorial was used with a diffraction spectrograph attached to it. In the latter half a fixed horizontal telescope was improvised by dismounting the 25-inch object-glass and fixing it in a stout wooden frame, with its axis horizontal, at a height of about 6 feet above the ground. Sunlight was directed into it by means of a cœlostat and an auxiliary mirror. The Sun's image was examined with a very powerful grating spectroscope.

The result of the trials is completely decisive. Observations with the equatorial equipment prove that good conditions of observing are available between 6 and 9 a.m. Observations with the horizontal equipment show that these conditions are not seriously impaired by bringing the beam nearer to the ground.

A short account of the observations and of some of the more readily derivable results has been communicated to the Society (*M. N.*, current volume, p. 158).

A permanent horizontal equipment for solar work is now in process of construction; the expense of it is to be met by an appropriation from the McClean bequest. The new grating spectroscope of Littrow form, with collimator-camera 14 feet long, was set up in position in the north annex on December 21.

The measurement and reduction of stellar spectra has been carried on during the year.

Bryan Cookson, M.A., Trinity College, has been appointed Assistant in Astrophysics, a new post with this title having been recently created.

Mr J. B. Hubrecht, research student in astrophysics, is investigating the rotation of the Sun by a spectroscopic method.

Mr W. H. Manning assists in the stellar work, and Mr L. J. Stanley in the solar work.

Dunsink Observatory.
(Director, Prof. E. T. Whittaker, Royal Astronomer of Ireland.)

Professor C. J. Joly, who had held the office of Royal Astronomer of Ireland since 1897, died in January. Mr E. T. Whittaker was appointed his successor, the appointment dating from July 1; during the first half of the year the observatory was in the charge of Mr C. Martin, Assistant Astronomer.

Mr. J. W. Vice entered into residence adjoining the observatory in September as a student of Practical Astronomy, under a scheme sanctioned by the Board of the University of Dublin.

The chief work of the year has consisted in the observation and reduction of the stars in the list of red stars—a work which was commenced by the late Director shortly before his death. The observations, which were made with the Pistor and Martins meridian circle, are now practically completed, and the work of reduction is in a forward state.

The time service to Dublin has been continued as in previous years.

The mirror of the Roberts photographic reflector was resilvered at the observatory at the beginning of December, but weather conditions prevented the use of this instrument during the rest of the month.

The professorial courses of lectures delivered in Dublin by the Director have been entitled "Theory of Optical Instruments" and "Theory of Energy and Radiation." Demonstrations in Practical Astronomy are given at the observatory by the Assistant Astronomer, and the South Equatorial has been employed on the first Saturday in each month in showing objects of interest to visitors.

Glasgow Observatory. (*Director, Prof. L. Becker.*)

The Director and his assistant were principally occupied during the year with the reductions of observations obtained in former years. A paper entitled "The distribution of blue and violet light in the Corona on 1905 August 30, as derived from photographs taken at Kalaa-es-Senam, Tunisia," has been accepted for publication by the Joint Permanent Eclipse Committee (of the Royal Society and Royal Astronomical Society).

The spectroscope attached to the 20-inch reflector was use
19 nights in the first quarter of the year. The observation
then discontinued, as some necessary alterations had to be
The efficiency of the apparatus has been enhanced by a n
holder, which, being curved to the focal surface of the
makes it possible to photograph with perfect definition the
between wave-lengths 6000 and 3200. The film-holder i
on a micrometer slide, and its position can be adjusted or
line.

The observations of stars close to the pole were taken up again after a year's interruption.

The scheme for the removal of the observatory has not yet taken definite shape.

The time service and the meteorological observations have been carried on as in former years.

Liverpool Observatory. (Director, Mr W. E. Plummer.)

The equatorial has been used mainly for the observation of comets, a class of observation with which this observatory has become identified. The observations, fully reduced up to the middle of the year, have been communicated to the Society and printed in their Journals. The observations of double stars have also been continued. During the last year some systematic inquiries have been made with the view of detecting, if possible, the effect of aperture and focal length on the measures. For this purpose attempts have been made to determine the orbits of some of the longer known binaries from observations, limited as closely as possible to the same aperture. It was thought that some of the variations from theory might be explained as due to the employment of larger telescopes than formerly. This work is still in progress.

In other departments there has been no alteration during the year. The meteorological and seismological observations are continuously maintained. Seismograms of the more remarkable earthquakes have been forwarded to the authorities who are discussing particular effects. The testing and rating of chronometers and the examination of other apparatus forms part of the routine work of the observatory. Lectures, in connection with the University of Liverpool, are regularly given in the observatory.

Radcliffe Observatory, Oxford.
(Director, Dr A. A. Rambaut, Radcliffe Observer.)

The principal astronomical work of this observatory during the year 1906 has consisted of investigations of stellar parallax carried on with the 24-inch photographic equatorial according to the method advocated by Professor J. C. Kapteyn (*Bulletin de la Carte du Ciel*, tome i. p. 262). It will be remembered that his method requires that a plate should be exposed at least three times to the same group of stars at about the epochs of maximum parallactic effect, the pointing of the telescope being slightly different on the three occasions, and the relative parallaxes of all the stars on the plate are deduced from measures of the distances between the images of each star so obtained.

The plates employed for this purpose at the Radcliffe Observatory are of the same size as those used in the astrographic telescopes,

viz. 160 mm. × 160 mm., but on account of the greater focal length of the Oxford telescope (22 ft. 6 in.) they cover only one-quarter of the area of the sky taken in by the former. Exposures are given of such a length as will in each case bring up at least 100 stars on the plate.

The regions observed this year have for guiding stars which are approximately central the following objects :—

B.D.	No.	Description.	Yale No.	R.A. 1905·0.	N.P.D. 1905·0.	Mag.	No. of Plates exposed for 1 season.	2 seasons.	3 seasons.
°				h m s	° ′ ″				
+25,	495	Lal. 5761	40	3 2 49·97	64 0 43·7	7·9	5	5	5
...		Nova Persei	42	3 24 44·30	46 25 14·6	...	5	5	...
+41,	750	Lal. 6888-9 pr.	44	3 40 32·63	48 50 10·0	8·2	2	1	1
+15,	651	In Hyades	...	4 29 10·01	74 41 33·1	8·5	4	4	...
+51,	1094	Gr. 990	51	5 30 47·39	38 36 56·5	7·9	6	6	4
+44,	1408	...	...	6 9 55·05	45 15 16·6	8·4	8	8	5
+40,	1758	...	...	6 49 51·08	49 47 32·3	8·6	7	7	4
+48,	1469	Lal. 13,427	57	6 54 22·86	41 28 38·7	8·2	3	3	3
+31,	1684	Lal. 15,290	61	7 47 29·02	59 6 5·4	8·2	8	8	3

Six of these objects are included in the list of stars whose parallaxes have been determined with the Yale heliometer (*Transactions of the Astronomical Observatory of Yale University*, pt. ii., vol. i. p. 196). Their numbers in that list are given in the third column of the above table. To this list of objects should be added 61 Cygni, which has been included in the Oxford programme as much for the sake of affording a check on the accuracy of the methods adopted as for the purpose of obtaining a fresh value to add to the already numerous determinations of the parallax of this star.

On these objects 62 plates have been exposed during the year, representing 313 exposures. All photographs were taken near the meridian, and as near the epochs of maximum parallax factor as possible. These two conditions necessitated the taking of the observations at one season (January to April) in the early evening, and at the other (September to December) in the early morning, hours.

The last three columns of the table, showing the number of plates exposed at one, two, or three seasons, exhibit the state of this work at present.

Arrangements are now being made to bring this parallax investigation to a conclusion, and to employ the 24-inch telescope in the future for determining the proper motion of faint stars down to the 14th magnitude, in accordance with Professor Kapteyn's scheme, entitled "Plan of Selected Areas."

A number of plates other than those taken for parallax ha

been obtained in the course of the year. These include photographs of Brooks' Comet, the great Nebula in Orion, the Pleiades, Præsepe, the great Clusters in Perseus, the Globular Cluster (13 M) in Hercules, the region around the north pole, etc.

The new Radcliffe Catalogue, containing places of 1772 stars, for the epoch 1900, was published and distributed early in the year. For some years past the revision of Main's observations made with the transit circle during the years 1862–1876, and published in vols. xxii. to xxxvi. of the Radcliffe Observations, has been engaging the attention of Herr Hans Osten of Bremen, in connection with Dr Ristenpart's *Geschichte des Fixsternhimmels*. Herr Osten has voluntarily offered his services for this work, and has hitherto carried it on entirely at his own expense. At Dr Ristenpart's request, the original records have in numerous cases been examined at the Radcliffe Observatory, and many discrepancies have thus been removed. The work is now approaching completion, and it is hoped that the publication in the form of a collated catalogue of Main's results may not be much longer delayed.

The preparation of another meteorological volume has engaged the attention of the staff for a considerable part of the year. This volume, which will contain the results of observations made in the years 1900 to 1905 inclusive, is already well advanced.

Meteorological and earth temperature observations have been regularly carried on as heretofore.

University Observatory, Oxford. (*Director, Professor H. H. Turner.*)

The energies of the staff have been almost wholly absorbed during the past year by the heavy work of proof reading, and a certain amount of revision, involved in passing two volumes of the *Astrographic Catalogue* through the press. Vol. i., containing measures of 65,750 stellar images on the 160 plates, with centres in Zone +31°, has been issued to a number of observatories, and vol. ii. (66,718 images; 160 plates; Zone +30°) is in the binder's hands. Vol. iii. is begun, and iv.–vii. will, it is hoped, follow at the rate of two per year; vol. viii. being reserved for discussions, etc.

Temple Observatory, Rugby. (*Director, G. M. Seabroke.*)

The principal object in the erection of this observatory was the ng of a taste for astronomy in the school, and its use for that se increases, and not much time is left for other work. A asures of double-stars have, however, been made. On cloudy s boys are taught to read verniers, to set the transit and al, and to take transits and measure double-stars, using tars for these purposes.

Solar Physics Observatory, South Kensington.
(Director, Sir Norman Lockyer, K.C.B.)

Observations of Sun-spot Spectra.—The Sun was seen on 253 days during 1906, and observations of Sun-spot spectra were made on 122 days, permitting the examination of 225 spots in the region F–D. The records continue to indicate that the lines affected are due to Vanadium, Titanium, Scandium, and some unknown element or elements. Daily photographs (glass negatives) of the Sun's disc are received from Dehra Dûn (India) and Mauritius, the gaps in the Indian record being filled up as far as possible by negatives from Mauritius and Greenwich. The negatives are forwarded to Greenwich for measurement and reduction as they are required. Positives on paper are also received; these are mounted on cartridge paper and bound up into half-yearly volumes for ready reference.

Spectro-heliograph.—The weather conditions were somewhat better than during 1905, being fine enough on 162 days to warrant attempts being made to obtain monochromatic photographs of the Sun in "K" light. The instrument cannot be used satisfactorily during the winter months on account of its position among high buildings. During the period 1906 January 27 to December 1 390 negatives were secured, showing the distribution of "K" radiation on the Sun's disc; with the addition of an occulting disc over the primary slit plate, 95 negatives were obtained, showing the prominences round the solar limb.

By arrangement with the Indian authorities, photographs taken with the spectro-heliographs at South Kensington and Kodaikánal are exchanged, so that the records may be as complete as possible for the year.

Stellar Spectra.—The weather conditions for night observations have been extremely unfavourable during the greater part of the year. The new mounting of the 6-inch Henry prismatic camera has been a great improvement, and the disturbances showing as declination steps appear to have been eliminated. Forty-eight photographs of nineteen stellar spectra have been obtained with the two prisms; twelve stellar spectra with the 9-inch prismatic reflector; the 2-inch quartz-calcite prismatic camera has been employed in photographing ten pairs of stellar spectra, under conditions as nearly constant as possible, for recording extensions of the continuous spectrum in relation to the positions of the stars on the temperature curve of chemical classification. With the 36-inch reflector photographs were taken of the spectrum of the Orion nebula, o Ceti (Mira), and α Orionis.

Publications.—Various papers dealing with stars of peculiar spectra, barometric variations of long duration over large areas, wave-lengths of enhanced lines, and eclipse reductions have been completed during the year, and others on allied subjects are in course of preparation.

Royal College of Science, South Kensington.
(*Assistant, Professor A. Fowler.*)

The observatory exists primarily for the instruction of students, but some of the instruments are also employed in the spectroscopic examination of Sun-spots and prominences. Solar observations were made on 100 days during the year.

Investigations of terrestrial spectra are also carried on, more particularly as regards their application to the interpretation of solar phenomena. Some of the results have been published in the *Monthly Notices* (vol. lxvi. No. 6), and in the *Transactions of the International Solar Union* (vol. i.).

Stonyhurst College Observatory, Astrophysical Department.
(*Director, Rev. W. Sidgreaves.*)

The solar surface has been under observation on all available days during the year, and 210 drawings of spots and faculæ have been made.

The large grating spectrometer has also been employed upon the larger spots when the atmosphere has been calm enough for the steady working of the present heliostat.

A new heliostat is being built for the observatory by Sir Howard Grubb, through favour of the Royal Society's Government Grant Committee. This will carry a 12-inch silver-on-glass reflector, on loan to the observatory by the British Astronomical Association; and the system will be completed by a second reflector of 16 inches diameter, lent by the Royal Astronomical Society.

With this addition the full aperture of the 8-inch objective of the old equatorial telescope will take the place of the present half-filled 4-inch lens, and is expected to add very greatly to the efficient working of the large Rowland grating on the solar surface.

The 4-inch prismatic camera has been employed on almost every available night, but only on a selected number of the brighter stars which had been suspected of showing variable spectra.

Some very good photo-spectrographs of *Mira Ceti* were obtained between November 25 and January 3, both by the Hilger compound prism adapted to the great equatorial, and by the Thorp objective prism on the Cook 4-inch Finder, now mounted as a separate equatorial telescope. But the hopeless clouds and fogs of January have shut out all possibility of following the star's spectrum through the conditions of its declining light.

Wolsingham Observatory. (*Rev. T. E. Espin.*)

The work of measuring with the micrometer neglected double stars, more especially those of Sir John Herschel, has been continued in the past year, and the results presented to the Society. During the year 134 new pairs have been detected and measured. Observations were made on 70 nights during the year.

Sir William Huggins's Observatory, Upper Tulse Hill.

The photography of the spectra of stars and of other celestial bodies, which has been in progress for many years, is being continued.

Experimental work in the laboratory has included further experiments on the radiation of radium, of which some of the results have appeared in the *Proceedings of the Royal Society.*

Sir Wilfrid Peek's Observatory, Rousdon, Lyme Regis.
(C. Grover, Observer in Charge.)

The observatory and astronomical instruments are in good working order. Weather was favourable, and observations were made on 148 nights during the year 1906. Observations of the long-period variable stars have been continued on Argelander's method, and 407 magnitude determinations were made; 18 maxima and 4 minima have been recorded. During the twenty-one years this work has been in progress 9707 magnitude determinations have been made, and 379 maxima and 288 minima have been recorded.

Several of the comets of the year were observed; they were mostly very faint and of little interest.

The planet Saturn was favourably placed during the autumn; and September being remarkably fine, with twenty-three nights' observation, the smaller satellites were better seen than usual, and several interesting configurations were observed.

The minor planet Vesta was observed on several nights in November. It was easily seen with a binocular glass, and its motion followed from night to night in the vicinity of the well-known double star 94 Aquarii.

Transits of stars for regulating the sidereal clock were mostly taken during daylight.

Mr Saunder's Observatory, Crowthorne, Berks.

The measures of 1800 points on a Yerkes negative of the Moon taken 1901 November have been completed. About a quarter of the work of reduction has been accomplished. Rather more than 1500 points have been partly measured on another Yerkes negative taken 1901 August. Of these, 400 have been completely measured, the plate constants have been determined, and about 100 points have been completely reduced. The constants of both these plates have been determined by Professor Turner's method (*M. N.*, vol. lx. pp. 202–205), and the probable errors indicate that a satisfactory degree of accuracy has been attained.

Dr W. E. Wilson's Observatory, Daramona, Co. Westmeath.

The long period of cloudy, wet weather seems to hold. Rain fell on 251 days, giving a total of 35·80 inches. Practically no work was done in the observatory. The experiments were continued with the new instrument, which has been named a Radio-Integrator. This simple instrument gives the daily amount of the vertical component of solar radiation in calories. It has been adopted at Kew, Falmouth, Aberdeen, and Valentia.

Kodaikánal and Madras Observatories.
(Director, C. Michie Smith.)

The first five months of the year were, on the whole, favourable for solar observations, but the remainder of the year was decidedly unfavourable. There were 26 days in the year on which no observations were possible.

Photoheliograms were taken on 317 days, and approximate positions of spots were obtained by projection or by eye observations on 339 days. Up to the end of November it was found possible to send to Greenwich all the solar negatives required to fill in the gaps in the Greenwich and Dehra Dûn set of daily photographs. In all, 297 new groups of spots were recorded during the year. The mean daily number of groups visible varied from 1·8 in October to 7·2 in July, and the average for the year was 4·4.

Spectroscopic work.—Observations of spot spectra were made on 181 days, and prominences were recorded on 269 days. It has not yet been possible to begin photographic work on sun-spot spectra, but as Mr Evershed is expected to join the staff early in 1907, this defect should soon be remedied. The necessary apparatus is available, and the only difficulty is to find the time required for the work.

Spectroheliograph.—Photographs with the spectroheliograph were taken on 277 days. The total number of plates taken was 1163, but of these a considerable number had to be rejected for various reasons. Most of the failures were due to attempts to obtain photographs through clouds or in short breaks, a few were due to bad setting, and a few to defective plates. The instrument has worked admirably during the year, and a large number of excellent photographs both of flocculi and prominences have been obtained. Trouble from unsteadiness of the air has, of course, continued, but it has been considerably reduced, and it is hoped that when the new building for the siderostat, now under construction, is completed, still less trouble will be experienced from this source.

Publications.—Bulletins Nos. iv. to vii. were published during the year, and No. viii. was in type at the close of the year.

At Madras, astronomical observations were confined to those necessary for the time service, which was efficiently maintained. A new Riefler clock has come into use, and has been found most satisfactory.

Perth Observatory, Western Australia.
(Director, Mr W. Ernest Cooke.)

Astrographic.—During the year, 167 plates were taken and passed as satisfactory. In addition, 21 plates have been exposed but not yet developed, owing to the destruction of the réseau, either by growth or chemical action, first noticed on 1906 December 7. A new réseau has been ordered, and upon its arrival the plates which have been already exposed in the telescope will be completed.

There are now only 101 regions to be photographed, and these will probably be finished by July.

No provision has yet been made for measuring the plates, but rearrangements of some kind will probably be made shortly, owing to the creation of a Federal Weather Bureau, and it is hoped that the pressing need for this work will then be recognised by the Government.

Transit Circle.—The whole year has been devoted to the observation and reduction of the 420 secondary standards which are to be used as fundamental stars for the zones 31°–41° S. declination. The observations are completed, and the catalogue will be ready for the printer in a few days. A minimum of 10 complete determinations of nearly all the stars has been secured, and the catalogue, it is hoped, will be printed and distributed very shortly. The following is the result of the year's work in tabular form:—

Nights on which time was determined		210
Observations of secondary standards in R.A.		2718
" " "	N.P.D.	2728
Observations of clock stars in R.A.		991
Separate determinations of	Collimation error	25
	Level	254
	Azimuth	167
	Nadir point	118
Readings of meridian mark		136

Meteorological.—The usual meteorological work was continued, and the annual report for 1905 printed and distributed. The new method of issuing forecasts has proved very successful.

The Federal Parliament authorised the creation of a Federal Weather Bureau, and Mr H. A. Hunt, of the Sydney Observatory, has been appointed Federal Meteorologist. The -vill probably come into operation during the present

Sydney Observatory. (Acting Directo

Early in January 1906 Dr Al
Lieutenant of His Italian Majesty's s
and at this observatory, in the sam
observers, undertook a series of g
ship remained in port. At the F
took magnetic observations on

Dr Hecker of Potsdam in the year 1904. The computations are not reduced yet, but Dr Alessio has hopes that they will compare favourably with those of previous observers.

Another officer, connected with the Carnegie Scientific Investigations, Mr G. Heimbrod, visited Australia in November 1906 for magnetic determinations, and has, in N. S. Wales, used the same position and pier at the Red Hill Branch of the Sydney Observatory, so that he may be able to verify the previous computations.

As stated in the last report, it was found necessary to remove the magnetic instruments out of the influence of the electric energy now so much used for municipal lighting and the tramway service. The magnetic work is now done at the Red Hill Branch by Mr Short, the astronomical photographer, in addition to his duties in astrophotography.

The mechanician, Mr W. I. Masters, has, as opportunity occurred, practically remade some of the instruments in use for many years. The tape chronograph, procured prior to 1865, and which had 88 lbs. to drive it, is now a new instrument, and is driven by 30 lbs. The whole of the working instruments, which are to a certain extent obsolete, are on the list to be renewed when the time can be devoted to them.

The equatorial has not been in regular commission during the year, as some adjustments require to be made.

The seismograph, which had been dismantled owing to the effects of the heavy blasting operations by the Sydney Harbour Trust, was not again put into commission till May 17. The record of the San Francisco earthquake of April 18 was therefore lost.

The buildings of the observatory have been renovated throughout and two new rooms added. This will overcome the congestion in some of the offices caused by the accumulation of many years' meteorological records, which, under the old conditions, for want of space, had to be stacked on the floors of the rooms.

With regard to the future of the observatory, the Commonwealth Government has decided to deal with meteorology only, leaving the astronomical work with the State Government. The future arrangements for its control are still uncertain, and it is not yet decided whether the observatory will remain in the same building or be sent to the clearer atmosphere of Pennant Hills, where the astrographic work is at present carried on.

The astronomical assistants, Messrs Raymond and Merfield, have observed during the year with the transit circle, and the results are as under:—

Number of R.A. observations of stars . . .	2495
" N.P.D. " . . .	2124
" R.A. " of Sun . . .	455
—making a total of R.A. observations . .	2950

The 9 A.M. collimation, level, and nadir observations	308 of e[illegible]
The 9 P.M. level and nadir observations . .	147

Azimuths determined from 137 pairs of circumpolar stars.

The year 1906 has been particularly favourable for observations, a greater number of stars having been observed than for a good many years past. The greater bulk of the observations were made during the winter months, when the definition was first-class. The 1905 Catalogue has been prepared, checked by the junior computers, and only the abstract remains to be made to finally complete it. In addition, the R.A.'s and N.P.D.'s of nearly all the observations made during 1906 have been checked, and the 1906 Catalogue will be put in hand at once.

1605 visitors have registered their names during the year. Of this number, 710 attended the evening lectures with the equatorial and lantern.

The astrographic work at the Red Hill Branch has been carried on by astronomical photographer Mr J. W. Short, who, in addition, has charge of the magnetic work. The magnetic instrument, an old one, has been compared with the modern instrument at the Melbourne Observatory, and, owing to the report received, it has been decided to get a set of new magnets. The Director of the Kew (England) Observatory has been asked to obtain these, and to certify to the errors, etc. before he accepts them. It is hoped that they will be received by April next.

Owing to the instrument being away part of the year in Melbourne the magnetic observations were incomplete, but the declination and dip remain practically constant.

Plates exposed.—Catalogue plates, 31; chart plates, 137. Total number of plates used, 175.

Photographs were also taken of the occultation of *Saturn* in October.

Many of the plates taken have been sent to Melbourne to be measured by the Joint Measuring Bureau (Sydney and Melbourne).

The number of fine nights was 115, partially fine 120, and the remainder cloudy and wet.

Library.—Mr Graham reports that the library received, by way of exchange from other observatories and institutions, 1050 books, pamphlets, and papers, being an increase over that of 1905 by 5 per cent.

The publications distributed were very few—only 200. This is due to the discontinuance of the publishing of the annual rain and meteorological reports. The distributions co[illegible] very unfavourably with those of the year 1905, [illegible] were sent out.

The library is being gradually [illegible] and a catalogue formed.

Lovedale Obser[illegible]
(Dr Ale[illegible]

Observations were made o[illegible] and February. During the great[illegible] was in England, and advantage w[illegible] have

the observations made since 1891 reduced. This work, which is now nearly completed, was facilitated by a grant from the South African Association for the Advancement of Science, to whom, as well as to Sir David Gill, the Director wishes to express his acknowledgments.

Mr Tebbutt's Observatory, Windsor, New South Wales.

For the reasons stated in the Annual Report of the Observatory for 1903, regular observations were discontinued at the close of that year. There is, therefore, little to record for the year 1906. The only astronomical observations made were some measures of the well-known binary *p Eridani* and observations of the lunar occultation of *Saturn* and *Titan* on 1906 October 27. These observations have all been sent to the Society for publication. The usual meteorological observations were also taken.

Notes on some Points connected with the Progress of Astronomy during the Past Year.

Discovery of Minor Planets in 1906.

One hundred and seventeen new planets were discovered or first announced in 1906, as follows :—

Letter and Number.	Date of Discovery.	Discoverer.	Letter and Number.	Date of Discovery.	Discoverer.	Letter and Number.	Date of Discovery.	Discoverer.
SJ ...	Jan. 15	K	HV[a] ...	Mar. 5 (02)	W	US ...	Aug. 22	W
SL ...	22	W	SD[a] ...	Nov. 3 (05)	Ly	UT ...	22	K
SM ...	22	W	TP 591	Mar. 14	K	UU ...	27	W
SN ...	22	W	TQ ...	17	W	UV ...	28	K
SO 582	23	K	TR ...	17	W	UX ...	Sept. 12	K
SR ...	24	W	TS 592	18	W	UY ...	12	K
SS ...	24	W	TT 593	20	K	VA ...	17	K
ST ...	24	W	TU ...	20	W	VB ...	18	K
SU ...	24	W	TV ...	21	W	VC ...	18	K
SV ...	24	W	TW 594	27	W	VD ...	18	K
SW ...	24	W	TX ...	27	W	VE ...	15	M
SY 584	15	K	TY ...	27	W	VF ...	24	W
SZ ...	Feb. 16	W	TY[a] ...	Mar. 27	W	VG ...	24	W
TA 585	16	K	TZ 595	27	W	VH ...	24	W
TB ...	16	W	UA 596	Feb. 21	W	VJ ...	26	K
TC 586	21	W	OO[a]	Aug. 12 (04)	G	VK ...	26	W
TD ...	21	W	UB 597	Apr. 16	W	VL ...	24	M
TE ...	16	M	UC 598	13	W	VM ...	24	M
TF 587	22	W	UJ 599	25	M	VN ...	Oct. 8	K
TG 588	22	W	UK ...	May 13	W	VO ...	8	K
TH ...	22	W	UL ...	29	W	VP ...	11	
TJ ...	16	M	UM 600	June 14	M	VQ ...	11	
TK ...	16	M	UN 601	21	W	VR ...	11	
TL ...	22	M	UO ...	July 30	W	VS ...	11	
TM 589	Mar. 3	K	UP ...	Aug. 13	K	VT ...	8	
TN ...	3	W	UQ ...	22	W	VU ...	17	
TO 590	4	W	UR ...	22	W	VV ...	17	

Letter and Number.	Date of Discovery.	Discoverer.	Letter and Number.	Date of Discovery.	Discoverer.	Letter and Number.	Date of Discovery.	Discoverer.
VW ...	Oct. 17	K	WJ ...	Nov. 11	K	WW...	Dec. 20	K
VX ...	17	K	WK ...	11	K	WX ...	21	K
VY ...	17	K	WL ...	11	K	WY ...	22	L
VZ ...	17	L	WM...	10	K	WZ ...	22	L
WA ...	21	W	WN ...	14	L	XA ...	21	K
WB ...	22	W	WO ...	12	M	XB ...	21	K
WC ...	22	K	WP ...	13	M	XC ...	21	K
WD ...	13	L	WR ...	14	M	XD ...	23	K
WE ...	26	M	WS ...	14	M	XE ...	18	K
WF ...	Nov. 9	K	WT ...	19	M	XF ...	18	M
WG ...	9	K	WU ...	Dec. 7	M	XG ...	18	M
WH ...	11	K	WV ...	7	M	XH ...	18	M

The year of discovery is 1906 unless otherwise noted; the abbreviations (02) (04) (05) stand for 1902, 1904, 1905. The initials represent G, Götz; K, Kopff; Ly, Loewy; L, Lohnert; M, Metcalf; W, Wolf. All the discoveries were made at Heidelberg, except those by M. Loewy (at Paris) and Dr Metcalf; the latter gentleman's observatory is at Taunton, Mass., U.S.A., and he has adopted the method of guiding the telescope so as to move at the rate of an average minor planet moving parallel to the ecliptic. In this way fainter planets can be photographed, since their trails are much shorter. Planets are distinguished from photographic defects by two exposures, with a shift in declination between them.

The following planets not numbered at the date of the last report have since received permanent numbers: QX 570, QZ 571, RB 572, RC 573, RD 574, RE 575, RF 576, RH 577, RZ 578, SD 579, SE 580, SH 581, SP 583.

The following errata in the last report (vol. lxvi. p. 217) may be pointed out:—RT, *not* RN, is identical with 477 Italia; HB, *not* HA, is identical with 526 Jena; *for* Oello, Genna, *read* Ocllo, Genua.

The following planets do not receive permanent numbers, not having been sufficiently observed:—RG, RJ, RL, RM, RN, RO, RP, RQ, RR, RS, RU, RV, RW, RX, RY, SA, SB, SC, SD², SG, SJ, SL, SM, SN, SR, SS, ST, SU, SV, SW, SZ, TB, TD, TE, TH, TI, TK, TL, TN, TQ, TR, TU, TV, TX, TY, TY², UK, UL, UO, UP, UQ, UR, US.

The following identities have been established:—CR (possibly) 488, CX 554, KR = KV, KY 529, SF 488, SK 275, SQ 411, SX 88, UD 181, UE 410, UF 394, UG 149, UH 431, UW 66, UZ 408, WQ 167, 469 = 488; the number 469 has been transferred to GE (discovered 1901).

The following planets have been named:—400 *Ducrosa*, 459

Signe, 461 *Saskia*, 463 *Lola*, 464 *Megaira*, 465 *Alekto*, 466 *Tisiphone*, 467 *Laura*, 468 *Lina*, 469 *Argentina*, 471 *Papagena*, 473 *Nolli*, 474 *Prudentia*, 479 *Caprera*, 480 *Hansa*, 481 *Emita*, 490 *Veritas*, 492 *Gismonda*, 495 *Eulalia*, 500 *Selinur*, 501 *Urhixidur*, 502 *Sigune*, 513 *Centesima*, 514 *Armida*, 515 *Athalia*, 524 *Fidelis*, 525 *Adelaide*, 526 *Jena*, 527 *Euryanthe*, 528 *Rezia*, 529 *Preciosa*, 530 *Turandot*, 531 *Zerlina*, 536 *Merapi*, 538 *Friederike*, 539 *Pamina*, 540 *Rosamunde*, 541 *Deborah*, 543 *Charlotte*, 545 *Messalina*, 546 *Herodias*, 547 *Praxedis*, 548 *Kressida*, 549 *Jessonda*, 550 *Senta*, 551 *Ortrud*, 552 *Sigelinde*, 553 *Kundry*, 555 *Norma*, 556 *Phyllis*, 557 *Violetta*, 558 *Carmen*, 559 *Nanon*, 560 *Delila*, 561 *Ingwelde*, 562 *Salome*, 563 *Suleika*, 564 *Dudu*, 567 *Eleutheria*, 581 *Tauntonia*.

The interesting planets *Thule* and *Andromache* were photographed at Heidelberg, also 294 *Felicia*, which had not been seen since 1891.

It is hardly necessary to say that 1906 breaks all records for the number of discoveries; it has, indeed, practically doubled the previous record of 59 in 1904. It is, of course, likely that a certain number of the planets described as new will be subsequently found to be old ones re-observed, but in any case the number of actual discoveries must be very large, showing that the hopes expressed a few years ago that the zone was approaching exhaustion were decidedly premature. As if to show that the search for planets cannot be abandoned without the risk of losing some very interesting objects, one of the bodies discovered last year has a very remarkable orbit. This is TG 588, which Dr Berberich has found to have a mean distance almost (perhaps exactly) the same as that of *Jupiter*, and a large eccentricity, so that its aphelion distance is 6·2, an entire astronomical unit beyond *Jupiter's* mean distance. The planet's heliocentric longitude at discovery was $55\frac{1}{2}^\circ$ greater than that of *Jupiter*, from which Dr Charlier concludes that it is very probable that the Sun, *Jupiter*, and 588 illustrate Lagrange's proposition that three masses at the angles of an equilateral triangle will, if properly projected, continue to move with unchanged relative configuration about their common centre of gravity. Since the orbits of both *Jupiter* and 588 are eccentric, and their planes inclined to each other at an angle of about 10°, the above conditions are not rigorously fulfilled; but provided certain limits are not exceeded, they could oscillate about the equilateral position as a mean. The re-observation of 588 may be expected shortly when it will presumably be possible to ascertain whether its motion is the same as that of *Jupiter*, in which case Dr Cl hypothesis is probably correct. Dr Berberich's elements are given. Epoch 1906, Feb. 22·5, Berlin M.T., M 4
ω 120° 25′ 50″, Ω 315° 34′ 7″, i 10° 20′ 53″, ϕ
μ 295″·13, log a 0·71999.

The *Berliner Jahrbuch* for 1908 gives t
about the observation of the minor planets
1906 :—388 planets had been observed at

31 at 3 oppositions, 53 at 2 oppositions, 97 at 1 opposition only. It is of interest to give the actual list of these lost planets from 1 to 400 inclusive:—99, 132, 155, 193, 220, 285, 290, 293, 309, 310, 315, 316, 320, 323, 330, 353, 357, 368, 392, 396, 398, 400.

A. C. D. C.

The New Satellites of Jupiter and Saturn

During the past year Dr F. E. Ross has published the following revised elements of *Jupiter* VII., referred to the Earth's equator.

Epoch. 1906 Jan. 0·0 G.M.T.

$g = 18^\circ{\cdot}9$	$e = 0{\cdot}208$
$\varpi = 118^\circ$	$n = 1^\circ{\cdot}386$
$\Omega = 291^\circ$	$\log.\ a = 8{\cdot}8946$
$i = 25^\circ\ 28'$	Period $259^{d}{\cdot}7$

Recent Greenwich observations seem to show that the above period is somewhat too long; probably the value 257 days is nearer the truth.

A successful search has been made on the Harvard plates for images of *Jupiter* VI. in the years preceding its discovery (*Harvard Annals*, vol. lx. No. 11). The following positions are given:—

Date.	Satellite—*Jupiter* R.A.	Satellite—*Jupiter* Dec.
d	m s	′ ″
1894 Jan. 25·563 G.M.T.	−3 30·75	+2 51·2
1899 June 26·561 ,,	+2 54·42	+6 16·3
,, ,, 27·588 ,,	+2 56·01	+6 0·7
,, ,, 30·566 ,,	+2 59·30	+5 20·3
,, July 1·594 ,,	+3 0·20	+5 7·4
,, ,, 12·605 ,,	+2 56·38	+2 7·7

A position given for 1894 February 2 is omitted here, since it is inconsistent with that for January 25, and the identity of the object observed with the satellite seems doubtful. The chief value of the above positions will be to improve the value of the sidereal period, and possibly also the secular changes in the elements. For this purpose it will be necessary to compute the perturbations for the above dates. However, their effect divided by so many revolutions cannot be serious, and the values of the period obtained by neglecting them are probably near the truth: these are $250^{d}{\cdot}3$ from the 1894 position, $250^{d}{\cdot}5$ from the assemblage of the 1899 ones: as the recent observations appear to indicate $250^{d}{\cdot}7$, it is probable that the value $250^{d}{\cdot}5$ is within $0^{d}{\cdot}2$ of the truth. The periods of VI., VII. are so nearly equal that some thirty years are required for VI. to gain a revolution on VII.

Professor Pickering notes in the same number of the *Annals*

that "the tenth satellite of *Saturn*, which is an exceedingly difficult object, can be seen when within 2′ of the planet."

Astr. Nachr. No. 3143 contains an ephemeris of *Jupiter* VI. calculated by J. E. Martin, of Washington, from unpublished elements by Dr F. E. Ross, extending from 1906 August 20 to 1907 April 17. The discordances between prediction and observation are small, as is also the case with Dr Ross' ephemeris of *Phœbe*, of which numerous observations have been obtained at Arequipa during 1906.

The following table gives a summary of the *Phœbe* observations:—

Date.	Obsd.—Tab. R.A.	Obsd.—Tab. Dec.	Number of Nights.
	s	′	
1906 May 19·4	−0·3	−0·3	2
June 27·4	−0·9	0·0	4
Aug. 16·0	−0·8	−0·2	3
,, 23·8	−0·9	0·0	3
Sept. 14·6	−1·6	−0·1	3

A. C. D. C.

The Comets of 1906.

The number of comets discovered and observed during the past year has been unusually large. Some of the particulars connected with their history can be most conveniently exhibited in a table.

Comet's Designation at Discovery.	Date of Discovery.	Name of Discoverer.	Date of last known Observations.	Date of Perihelion Passage.	Comet's Catalogue Designation.
	1905.		1906.	1905.	1905.
Comet 1905 *b*	Nov. 17	Schaer.	Jan.	Oct. 25	V.
				1906.	1906.
,, 1905 *c*	Dec. 6	Giacobini.	Mar.	Jan. 22	I.
	1906.			1905.	1905.
,, 1906 *a*	Jan. 26	Brooks.	Apr.	Dec. 22	[illegible]
,, 1906 *b*	Mar. 3	Kopff.	May.	Oct. [illegible]	[illegible]
				I[illegible]	[illegible]
,, 1906 *c*	,, 17	Ross.	Apr.	F[illegible]	[illegible]
,, 1906 *d* (Finlay)	July 16	Kopff.	Still visible.	[illegible]	[illegible]
,, 1906 *e*	Aug. 22	Kopff.	Oct.	[illegible]	[illegible]
,, 1906 *f* (Holmes)	,, 28	Max Wolf.	Still visible.	[illegible]	[illegible]
,, 1906 *g*	Nov. 11	Thiele.	,,	[illegible]	[illegible]
,, 1906 *h*	,, 14	Metcalf.	,,	[illegible]	[illegible]

Necessarily, the orbits are not yet definitive, so that the catalogued order cannot be regarded as finally settled. Subsequent discoveries may further disturb the designation. But the divergences from the order of discovery are sufficiently remarkable, and must not be overlooked.

The first two comets on the list were mentioned in the last Annual Report. Comet 1905 V. calls for no further remark. As anticipated, comet 1906 I. was observed at the Cape of Good Hope, and subsequently at other observatories in Europe and America.

Comet 1905 VI. is of the ordinary parabolic class, and possesses no points of interest.

Comet 1905 IV. was actually photographed at the Königstuhl Observatory on 1905 January 14, or 413 days earlier than Dr Kopff's announcement. The orbit is remarkable for the large perihelion distance (3·3 R). This distance has been exceeded only in the case of the comet of 1729. The motion may be regarded as parabolic, though an ellipse of 1153 years' period has resulted from computations in which no assumption was made regarding the excentricity.

Comet 1906 II. was discovered in the southern hemisphere, and was very unfortunately situated for European observatories. The observations, few in number, extend over little more than three weeks. The elements, though uncertain, appear to be of the ordinary parabolic class. This is the more probable owing to the large inclination, the motion being nearly perpendicular to the ecliptic.

Comet 1906 V. is the third return of the comet of Finlay, which passed undetected in 1899–1900. The accumulated error in the mean anomaly in this long interval, arising from neglect of perturbations and other causes, made the ephemeris calculated by M. Fayet, from the elements of M. Schulhof, somewhat uncertain. The position with regard to our horizon is not very favourable for observation, but the opportunity these observations afford for correcting the mean motion is most fortunate, since, in the next revolution, the comet will approach (in 1910) so close to *Jupiter* as to suffer very considerable perturbations. Observations at the next return will afford very exact means for determining the mass of that planet.

Comet 1906 IV. proved to be one of the family of short-period comets. The observations did not begin till after the perihelion passage, and could not be pursued very long, but the elliptic character is well marked.

Comet 1906 III. is the second return of Holmes' Comet. A very few observations, and only with the large equatorials of the Lick and Yerkes Observatories, were obtained in 1899–1900, but Dr Zwiers found these sufficient to correct the elements, and, after allowing for the effect of perturbation by *Jupiter*, he was able to supply an ephemeris which represented the path of the comet with great accuracy. From this ephemeris Dr Max

Wolf succeeded in photographing the comet on August 28, and though the object was, and has remained, very faint, it is probably still under observation in the largest telescopes.

Comet 1906 VII. is a tolerably bright comet, still visible, whose motion can be represented by a parabola.

Comet 1906 VI. is a faint object, discovered on a photographic plate by Mr Metcalf, of Taunton, U.S.A. The observations have not been very numerous, and, like some others mentioned, have all been post-perihelion, but there is no doubt the object belongs to the family of short-period comets. The most probable period is about $7\frac{1}{2}$ years; and the elements bear a very strong resemblance to those of Faye, Wolf, and some others.

The orbits of the following comets have been definitively determined during the year:—

Comet.	Character of Orbit.	Calculator.	Authority.
1742 I.	Parabolic	B. Cohn	*Ast. Nach.* No. 4111.
1819 II.	Elliptic	Peck	*Ast. Journal*, Nos. 584–5
1826 V.	Parabolic (?)	Hnatek	*Denksch. Wien Ak.* 77
1844 III.	Elliptic	Fayet	*Thèses, Paris*, 1906
1864 III.	Elliptic	Schroeter	*Ges. d. Wiss. Christ.* 1905
1874 II.	Parabolic	Burggraf	*Sitzb. Wien Ak.* 1904
1883 I.	Hyperbolic	Hellebrand	*Ast. Nach.* No. 4090

The result of the calculation in the case of 1826 V. is not very satisfactory, since some of the outstanding residuals are large. 1844 III., according to Bond's calculations (*Ast. Journal*, 103), gave marked evidence of hyperbolic motion. Fayet's new reduction of the observations does not confirm this suggestion. In the case of 1883 I. a parabola would amply satisfy the observations; the hyperbola is a computational result.

W. E. P.

Progress of Meteoric Astronomy in 1906.

Meteoric observation does not appear to have made subst[illegible] progress in 1906. Observers were few, and no specially [illegible] showers attracted attention. The *Perseids* returned at a [illegible] of very unsettled weather, and the *Leonids* and *Andr*[illegible] failed to present active appearances.

The *Böotids* (or *Quadrantids*) were vis[illegible] the early [illegible] of January 3, furnishing some brilliant [illegible] [illegible]ed mete[illegible] the display could not be closely watched a[illegible] as clouds [illegible] gathered over the sky.

The *Lyrids* were well observed by Mr [illegible]ook, at [illegible]

near Huddersfield, on April 21. In 3^h 40^m, between 9^h 20^m and 14^h, he counted 37 meteors, including about 20 *Lyrids*, from a radiant apparently double at $267\frac{1}{2}^\circ + 35^\circ$ to $275^\circ + 31^\circ$. The mean position at $271^\circ + 33^\circ$ agrees exactly with the best previous determinations of the shower-centre. The *Lyrids* seen by Mr Brook were quick, and a proportion of them left streaks. On April 20, during nearly an hour, only one meteor was recorded, so that the maximum appears undoubtedly to have occurred on April 21, and to have furnished some similar characteristics to the display of 1901 April 21.

The August *Perseids* returned in about average strength, but partial moonlight and stormy weather prevented much observation until August 12, 13, and 14, when a fair number of meteors were seen by Miss Irene Warner, of Horfield, Bristol, as follows:—

Aug. 10.	$9\frac{3}{4}^h$ to 11^h, 15 meteors.		Many clouds.
" 12.	11 to 12, 16 meteors.		Partly clear.
" 13.	30 meteors per hour.	2/3rds *Perseids*.	Cleared after storms.
" 14.	20 meteors per hour.	1/3rd *Perseids*.	Cleared after storms.

On October 22, 10^h 15^m to 13^h 25^m, Mr C. L. Brook saw 26 meteors, including several *Orionids*. At 12^h 16^m a third mag. meteor was doubly observed by Mr Brook and by Mr J. P. Kenyon at Stockport from a radiant at $98^\circ + 14^\circ$. This was an example of the ξ *Geminids*, and the latter system at the October epoch appears to have furnished a richer display than the *Orionids* in recent years.

About the middle of November a few of the usual streaking *Leonids* were remarked, and several brilliant fireballs, including *Leonids* and *Taurids*, were reported. On November 16, at 13^h 20^m, what appears to have been a magnificent *Leonid* was seen at Hampstead, N.W., and Walton-on-Thames. It gave a flash like lightning and left a streak for several minutes, but the observations are not sufficiently accurate or numerous to afford useful deductions as to the real path. On the evening of November 17, between 11^h 9^m and 11^h 40^m, Mr Brook registered three *slow-moving Leonids*. One of these was also noted by Mr Kenyon, and it had a velocity certainly not exceeding 30 miles per second. It is frequently found that the swifter class of meteors during horizontal flights exhibit unduly slow motion, as though encountering great atmospheric resistance. As another case of the same kind, the very long-pathed *Aquarid* observed by Professor Herschel and Mr Bridger on 1900 May 3 may be mentioned; this object had a visible flight of 155 miles and a velocity of 28 miles per second, which is much less than the theoretical speed of about 40 miles per second.

The *Leonids* were well observed by Mr C. P. Olivier, at the

Leander McCormick Observatory, University of Virginia, U.S.A., on November 16, and the following numbers were counted:

Local Time.	*Leonids.*	Others.	Total.	Remarks.
h m h m 10 12 0	1	7	8	Clear. Leo just rising.
12 0–13 0	4	10	14	,,
13 0–14 0	9	8	17	,,
14 0–14 11	3	1	4	,,
14 36–15 0	3	3	6	,,
15 0–16 0	14	12	26	,,
16 0–17 0	14	10	24	Clouds near horizon.
17 0–17 21	4	2	6	Part cloudy.
	52	53	105	

Eight of the *Leonids* were equal to first magnitude stars. The prevailing colour was yellow, but there were many green and a few red ones. The finest meteor Mr Olivier ever observed came at $17^h\ 51^m$. It was equal to the moon at first quarter, and ended at *ψ Virginis* after a path of only 3°. From 12 meteors plotted near the radiant, the centre came out at $151° + 22°$. The streaks left by some of the meteors were very persistent, and the display was considered very good, considering its lateness. Several minor radiants were in strong activity. These details are given in *Popular Astronomy* for January 1907.

Mr Brook witnessed a fairly active return of the *Geminids* on December 12 (a few were also noticed on December 13), and bright meteors were seen on December 12, $13^h\ 35^m$, and December 13, $6^h\ 40^m$. On Dec. 12, during watches amounting to $2^h\ 42^m$, between $10^h\ 35^m$ and $13^h\ 42^m$, 51 meteors were seen (of which 26 were registered), and many others must have escaped observation.

A considerable number of fireballs and large meteors of various kinds have been reported during the year, but, except in a few instances, the descriptions were not so precise and complete that the real paths might be derived. It is to be hoped that more attention will be given to this particular branch of observation, and every effort made to secure the necessary data for ascertaining the heights, velocities, and radiants of these interesting bodies. The usually make their apparitions at times when least expected, a are often seen accidentally, but their positions and direct amongst the stars should always be registered with the u accuracy possible under the circumstances.

In 1906 the following real paths were computed for m seen in this country :—

Date.	G.M.T.	Brightness.	Height at First. Miles.	Height at End. Miles.	Length of Path. Miles.	Velocity. Miles per Sec.	Radiant Point. α δ	Observers.
1906.	h m							
Jan 27	8 33	= ☽	59	45	42	24	214 + 53	13
Feb. 13	10 42	♀	61	28	36	...	105 + 51	3
Mar. 23	10 52	½ = ☽	68	45	47	...	218 + 10	3
April 15	8 6	3 × ♀	65	28	39	18	153 + 33	3
16	8 53	> ♃	69	22	53	15	187 + 33	3
21	13 4	*Sirius*	89	56	32	25	275 + 35	2
Aug. 5	10 33	> ♀	65	52	24	30	38 + 55	3
Sept. 1	10 0	> 1	79	40	52	...	320 + 12	2
15	10 24	3 × ♃	77	43	72	21	245 + 30	2
27	6 59	> ♃ > ♀	63	45	46	...	345 + 2	2
Oct. 22	12 16	3 − 1	75	56	44	36	98 + 14	2
Nov. 17	11 9	1	77	66	91	30	150 + 21	2
23	8 5	> ♀	59	36	52	21	46 + 5	3

Jan. 27. A very brilliant object, satisfactorily observed.
Mar. 23. The radiant point is uncertain.
April 21. The radiant may have been at $294° \pm 0°$ and height of the meteor 87 to 79 miles.
Aug. 5. A fine, flashing *Perseid*, leaving a streak visible for 20 secs. though the full moon was shining.
Sept. 1. The radiant possibly at $314° + 10°$.
Nov. 17. A bright *Leonid*, with abnormally slow motion.

Fall of a Meteorite at Sea.—Capt. Anderson of the vessel *African Prince*, writing to his principals, says, "We sometimes hear of vessels disappearing during a passage of fine weather and in the open sea, free from navigation dangers. My experience on the voyage from New York has suggested to my mind that ships may have occasionally been lost by meteors falling on them.

"On the evening of Oct. 17, 1906, I was on the bridge with the second officer, when suddenly the dark night became as light as day and an immense meteor shot perpendicularly towards the earth. Its train of light was a broad electric coloured band, gradually turning to orange, and then to the colour of molten metal. The meteor entered the water with a hissing noise close to the ship, and the consequence, had it struck our ship, would have been annihilation."

W. F. D.

Solar Activity in 1906.

Sun-spots.—The general character of the sun-spot acti
1906 has been that of the last phase of maximum. The
been a distinct falling off in the mean daily spotted area
pared with 1905, though spot-groups have still been nu
especially in the northern hemisphere. But the giant gro

characteristic of 1905 have been much less frequent, and the tendency has been rather to develop long processions of groups of moderate size following each other at short intervals along a parallel of latitude, than to concentrate into great disturbances. Still, groups of the first order of magnitude were seen on four occasions during the year, viz.—January 21–30, March 16–27, July 27–August 5, and December 12–25—two great disturbances being on the disc at the same time during the last of these periods. There was a very striking secondary minimum in October, the sun being free from spots on no fewer than nine days in that month; but a distinct revival set in in November, and the year closed with a period of great activity. The mean daily total spotted area will probably work out as about three-fourths that for 1905, so that the crest of the wave seems to have been passed; it may provisionally be taken as being placed in October 1905. Faculæ, on the other hand, have been about as numerous as in 1905, and have, in general, showed no great fluctuation from month to month.

The mean latitude of the spotted area continued to be about 13° from the equator; that is to say, the zone usually occupied at maximum has already been passed. E. W. M.

The Prominences.—The daily frequency of prominences deduced from spectroheliograms taken on 45 days during the year 1906 was practically the same as that deduced for the preceding year from photographs secured at the Solar Physics Observatory, South Kensington. The pictures were taken through the "K" line (calcium) of the solar spectrum, and the lower limit for height of prominences accepted was 20″. The preponderance of activity in both years was manifested in the northern hemisphere of the Sun, the figures obtained being as follows:—

	1905.	1906.
North hemisphere	3·4	3·9
South "	3·0	2·6
Total daily frequency ...	6·4	6·5

The decrease in the daily frequency for the southern hemisphere is noticeable, though the increase in the northern more than counterbalances it. The most active prominence zones were in the spot latitudes, and in belts about 30° higher. Many e[illegible] -mi-nences were registered, while 6·5 per cent. of the [illegible] the whole limb were within 10 degrees of the Su[illegible]

From June 6th to the 11th a strong, persis[illegible] group of prominences was shown clustered rou[illegible] the Sun. On a negative taken on the 27th [illegible] broken flame of bright calcium, exte[illegible] radially from the photosphere, wa[illegible] western limb in the spot latitude of [illegible] rapidly disappeared, for on an [illegible] forty minutes afterwards no trac[illegible]

The prominence activity for the first six months of 1906 was given by the late Professor Mascari from visual observations at Catania, the lower limit for their height being taken as 30″. A moderate increase of activity is indicated for this half-year over the mean of the preceding year. The mean daily frequencies are as follows:—

1905 . . .	3·02
1906 (first six months)	3·92.

This increase is somewhat greater than that indicated by the South Kensington photographs, though the actual daily frequency, as given by the negatives, is much higher.

Mr Evershed, in his last report to the Society, gave +75° and −75° for the prominence limits in 1905. A remarkable feature of the distribution in 1906 was the prominence activity in the neighbourhood of the solar poles. In the observations of Mascari prominences within 10° of the solar poles are recorded in the years just preceding the spot maximum of 1884, but since that date they are sharply restricted to the year 1895. Their almost complete absence in the remaining years is striking. W. J. S. L.

Double Stars.

The same classification is adopted as in previous reports. The abbreviations are:—

M. N.: Monthly Notices R.A.S.
A. J.: Astronomical Journal.
L. O. B.: Lick Observatory Bulletin.
A. N.: Astronomische Nachrichten.
B. A. A.: Journal of British Astronomical Association
A. S. P.: Publications of Astronomical Soc. of Pacific.

Observations—

Rev. T. E. Espin. M. N. lxvi. 7. A catalogue of 42 double stars in the zone +30° to +40°.

Royal Observatory, Greenwich. M. N. lxvi. 8. Measures double stars made in the year 1905 with the 28-inch refractor.

John Tebbutt. M. N. lxvi. 9. Measures of 6 south binaries.

Rev. T. E. Espin. M. N. lxvii. 3. A catalogue of 80 ne double stars in the zone +29° to +34°.

Rev. T. E. Espin. M. N. lxvii. 3. M[illegible] of 80 Hersch and 27 miscellaneous stars made in 1906.

J. Nangle. B. A. A. xvii. 1. Measur[illegible]

H. E. Lau. A. N. 4078. A set of m[illegible] laneous pairs and 68 Struve pairs made for *Me*[illegible]

G. van Biesbroeck. A. N. 4107–8. A fi[illegible] made with the 12-inch refractor at Heidelbe[illegible] pairs and 185 Struve pairs made at the [illegible]

H. E. Lau. *A. N.* 4111. A continuation of the previous measures, containing 52 Struve and 28 miscellaneous pairs.

E. E. Barnard. *A. N.* 4128. A fine set of measures of 61 *Cygni* on 144 nights (last 1906 July 7, 127°·8, 22″·72). These do not confirm Dr Wilsing's hypothesis of a periodic oscillation in distance.

E. E. Barnard. *A. N.* 4128. Measures of Σ 2398, Σ 1110, and Σ 2220.

W. Doberck. *A. N.* 4130. A fine set of measures of 200 pairs made at Hong-Kong in 1904–5.

H. E. Lau. *A. N.* 4134. Measures of ξ *Ursæ Majoris* on 61 nights in 1905–6.

R. G. Aitken. *L. O. B.* 93. Catalogue of 350 new doubles, of which 267 are separated less than 2″. This makes 1250 of this class of double star discovered by Professor Aitken at Lick.

H. Morgan. *A. J.* 584–5. Measures of 19 double stars with the 12-inch Morrison refractor.

R. G. Aitken. *A. S. P.* 108. Measures of Hu 1176, A 570, A 691, and discovery of Σ 2348 as a close double.

Calculation—

W. Bowyer and *H. Furner.* *M. N.* lxvi. 7. On the orbit and relative masses of 85 *Pegasi* (β 733). Faint star four times the mass of the bright star.

A. C. D. Crommelin. *M. N.* lxvii. 2. Proper motion of *Castor.*

T. Lewis. *Memoirs R.A.S.* vol. lvi. Measures of the stars of Struve's *Mensuræ Micrometricæ* collected and discussed.

G. D. Hirst. *B. A. A.* xvii. 1. Note on α *Centauri.*

F. W. Dyson. *B. A. A.* xvii. 1. Review of *Memoirs R.A.S.*, lvi. Struve double stars.

A. C. D. Crommelin. President's Address. *B. A. A.* xvii. 1. Exposition of *Memoirs R.A.S.*, lvi.

W. Doberck. *A. N.* 4110. Elements of orbit of 85 *Pegasi.*

W. Doberck. *A. N.* 4115. Elements of orbit of 70 *Ophiuchi.*

H. D. Curtis. *L. O. B.* 98. The system of *Castor.* Radial velocity curves showing both α^1 and α^2 to be spectroscopic binaries—α^1 the fainter, 3·7 mag., has period 2·928 days; α^2, mag. 2·7, has period 9·219 days.

R. G. Aitken. *L. O. B.* 101. Orbit of β 612 (period 34·4 years).

A. Hall. *A. J.* 588. Note on μ^1 *Herculis.*

R. G. Aitken. *A. S. P.* 104. Note on 13 *Ceti* (period 7·1 years).

R. G. Aitken. *A. S. P.* 106. Note on 95 *Ceti*, Σ 554, β 971, and β 163.

R. G. Aitken. *A. S. P.* 110. Review of *Memoirs R.A.S.* lvi. Struve double stars.

E. W. Maunder. *Knowledge*, 1906 September. A great catalogue of double stars (review).

Miss A. M. Clerke. *Observatory* No. 369. Note on spectral changes of ζ Boötis.

T. Lewis. *Observatory* No. 373. Colours and magnitudes of double stars.

W. J. Hussey. *Observatory* No. 376. A memoir on double stars (review).

R. T. A. Innes. *Observatory* No. 379. Note on κ *Toucani.*

S. W. Burnham. *Popular Astronomy*, xiv. 9, 1906, November. Review of *R.A.S. Memoir*, lvi. T. L.

Variable Stars.

Progress in the discovery of new variable stars has continued during 1906, the highest provisional number allotted so far to a new discovery being 1906, 122 (*A. N.* 4131), and this number may be taken to represent, at the least, the number of new stars found in the year.

Harvard College Observatory is in the front rank in this line of research. The following announcements, among others, have been made :—

H.C.O. Circular.	Number of New Variables.	Remarks.
107	25	All very faint. Chiefly in *Orion* and *Cygnus.*
111	13	Discovered from their peculiar spectra. Principally in N. hemisphere.
115	22	In *Carina.* One is *Algol* type.
117	1	C.D.M. – 30° 16169. Varies 1 mag. in two days. *Algol* type.
120	31	All in S. hemisphere, and principally in *Crux*, *Centaurus*, etc. One of them is *Algol* type. Nearly all faint.
121	1	*Nova Velorum.* Variation $<11{\cdot}5$ to $9{\cdot}8$.
122	36	Principally in *Carina* and *Centaurus.* Six probably *Algol* type.

Mr A. Stanley Williams continues his studies of certain known variables (*A. J.*).

In *H. C. O. Circular* 116, Professor E. C. Pickering, with characteristic energy, proposes that as the number of known variables now amounts to over 3000, the time is ripe for the compilation of a D. M. of variable stars, with the view of ascertaining the proportion of the different classes, and their general distribution in the heavens. He estimates the number of the stars of the 16th magitude and brighter at fifty millions. His project involves the examination of all these stars by the method of superposing one photographic plate on another taken at a different time from the first. It is, of course, a huge task, but with the rapid and sweeping method indicated it is considered as not beyond the powers of the present generation of observers. He considers that

although co-operation of astronomers is necessary, there is at present no need for a formal organisation and allotment of work. The field is so large as to admit of different observers working independently; it is only required that their work should not overlap or clash.

Dr A. W. Roberts has published an important paper on "A Method of determining the absolute Dimensions of an *Algol* Variable Star" in *Monthly Notices*, vol. lxvi. No. 3.

The Carnegie Institution of Washington has recently issued *Researches in Stellar Photometry*, made chiefly at the Yerkes Observatory, by J. A. Parkhurst, during the years 1894–1906. This work refers to 12 variable stars, 10 of which may be called recently discovered stars. The object proposed was (1) the accurate determination of the complete light curves, and (2) the behaviour of the variables during their faint stages. The number of visual observations is 1405. Special attention has been given to the magnitudes of the comparison stars which have been ascertained from photometric measures. With each star are given plates showing the magnitude curve, the light curve, and the mean light curve; also a very clear chart of the vicinity of the variable, showing all the stars in the field, the comparison stars being lettered. The work was done with three different telescopes, of 6, 12, and 40 inches aperture. The break of continuity in the visual observations resulting from the use of these three instruments "was kept within limits by the use of the photometric magnitudes of the comparison stars." The work must occupy an important place in the research in the long-period variables. A point which strikes one is that the figures representing the light curves might have been more opened out, *i.e.* the ratio length representing magnitude to that of time being made greater, by which the minor fluctuations superposed on the main curve would have been rendered more apparent to the eye.

Vol. xv. of the *Memoirs* of the British Astronomical Association has just been published, being the sixth report of the Variable Star Section, by Col. E. E. Markwick, C.B. It contains in detail 5717 observations of 26 well-known long-period and two irregular variable stars, made by twenty-one members in the years 1900–1904 inclusive. There are appended plates showing at a glance the light curves of these stars, as deduced from the observations drawn on the scale—length representing one magnitude (ordinate) = that representing 20 days (abscissa). The theoretical curves are also given, so that the difference "O–C" is readily visible to the eye for any specified date. E. E. M.

Stellar Spectroscopy in 1906.

Nebulæ.—The apparent persistency of unaltered form in some of the large gaseous nebulæ has frequently attracted attention. A great part of the luminosity of such nebulæ is recognised as being due to the glowing of hydrogen, helium, and another unidentified

vapour. In a characteristic note contributed to the *Observatory* (October 1906, p. 380), one of our honorary members, Miss Agnes M. Clerke, whose recent death is deplored in a circle far wider than that of our own Society, has suggested an attempt to bring the work of the physicists, J. J. Thomson, Ramsay, and Rutherford, into line with the observations of astronomers.

In the nebula near π and δ Scorpii recently photographed by Professor Barnard on Mount Wilson (*Astroph. Jour.*, xxiii. 144), most of the larger stars involved have spectra of the *Orion* type, with the characteristic absorption lines of helium.

New Stars.—A star, exhibiting the photometric peculiarities of a Nova, has been discovered by Miss Leavitt at Harvard College Observatory. Professor E. C. Pickering, in announcing it (*Ast. Nach.*, 173, 295, *H. C. O. Circular*, No. 121) as Nova *Velorum*, says that it is not impossible that it may again become sufficiently bright for its spectrum to be obtained, and adds that "even without such proof there can be little doubt that the object observed is actually a Nova."

The spectrum of Nova *Aquilæ* No. 2 has been observed by Mr J. H. Moore at the Lick Observatory (*Astroph. Jour.*, xxiii. 261).

Classification of Stellar Spectra.—The published notes on this branch of the subject during the year relate almost entirely to the relation between the spectra of sun-spots and stars; a study of great importance, and likely to throw much light on methods of classification of stellar spectra. The following papers may be noted:—

"Spectra of sun-spots and *Arcturus*," W. S. Adams (*Astroph. Jour.*, xxiv. 69). Attention should also be called to Sir N. Lockyer's paper (*Proc. R.S.*, lxxiv. 53). "Spectra of sun-spots and Third type Stars," G. E. Hale and W. S. Adams (*Astroph. Jour.*, xxiii. 400). "Spectra of sun-spots and Fourth type Stars," W. M. Mitchell (*ibid.* xxiii. 211). "On the relation between Stellar Spectral types and the intensities of certain lines in the spectra," S. Albrecht (*Astroph. Jour.*, xxiv. 333). "Enhanced lines of Iron in the region F–C," A. Fowler (*M. N.*, R.A.S., lxvii. 154). "Preliminary paper on the cause of the characteristic phenomena of sun-spot spectra," G. E. Hale, W. S. Adams, H. G. Gale (*Astroph. Jour.*).

Here also we may note a paper by Miss Agnes Clerke—probably the last written by her before her death—on sun-spot spectra (*Observatory*, January 1907, p. 55).

Studies of Special Stars.—At a time when the maximum in the brightness of *o Ceti* is shown by Father Sidgreaves (*Observatory*, January 1907, p. 49) to be again accompanied by extreme brilliance of the hydrogen lines, and by peculiar behaviour of the H_β line in particular, it is of interest to record Professor Frost's observations of *Pleione* (*Astroph. Jour.*, xxiii. 268). The hydrogen lines in *Pleione* have been frequently seen bright and narrow, superposed on broader dark lines. Frost now observes that the

bright lines are no longer visible. Professor Pickering corroborates this observation in 1906, at any rate with respect to H_β, which was often recorded as bright in 1896.

Sir N. Lockyer has a note in the *Proc. R.S.*, **77**, 550, on peculiarities in the spectra of *α Andromedæ, θ Aurigæ, α Canum Ven.*, and *ε Ursæ Majoris.*

Herr Ludendorff (*Ast. Nach.*, **173**, 1) publishes remarks upon the spectra of *R. Coronæ Borealis*, 12 *Canum Ven.*, and 72 *Ophiuchi.* The first-named star has a spectrum similar to that of *α Persei*, except that the H_γ line seems to be absent. Professor Frost corroborates the absence of the hydrogen lines in 1903.

A list of 24 stars with peculiar spectra is given by Professor Pickering (*H. C. O. Circular*, No. 110, and *Ast. Nach.*, **171**, 139, *Astroph. Jour.*, xxiii. 257). Reference is here made to *R Cygni*, which is also the subject of a note by Miss Clerke in the *Observatory*, April 1906.

Radial Velocity of Stars.—In a note on sun-spot lines in spectra of Red Stars, G. E. Hale and W. S. Adams give parenthetically the velocity of *α Orionis* + 26·7 km/sec. (*Astroph. Jour.*, xxiii. 402).

Variable Radial Velocity.—Notes on variability of velocity are given for the following stars :—

(The numbers prefixed in the first column refer to Campbell and Curtis's First Catalogue, *L.O.B.*, No. 79. Here also it may be remarked that Professor Campbell gives (*L.O.B.* No. 107) a list of changes to be made in the first catalogue.)

29 ε Aurigæ	Ludendorff	*Ast. Nach.* 171, 49
80 α Draconis	,,	,, 171, 127
12 β Arietis	,,	,, 171, 149
γ Cassiopeiæ	Hartmann	,, 173, 101
RZ Cassiopeiæ	,,	,, 173, 101
65 *o* Leonis	Zurhellen (Bonn)	,, 173, 353
* Orionis B.D. − 1° 1004	Frost	*Astroph. Jour.* xxiii. 264
29 Canis Majoris	,,	,, ,, 265
30 (τ) Canis Majoris	,,	,, ,, 265 (footnote)
μ Orionis	,,	,, ,, 266
T Monocerotis	,,	,, ,, 266
80 α Draconis	,,	,, ,, 267
129 β Cephei	,,	,, xxiv. 259
1 Geminorum	Campbell and Moore	*Pub. A.S.P.* 18, 308, and *L.O.B.* No. 107.
* Ophiuchi (Draper C. 7579)	,, ,,	,, ,,
δ Sagittæ	,, ,,	,, ,,
o_2 Cygni	,, ,,	,, ,,
ε Cygni	,, ,,	,, ,,
ζ Cygni	,, ,,	,, ,,
ι Capricorni	,, ,,	,, ,,
SU Cygni	Maddrill	*Pub. A.S.P.* 18, 252, and *L.O.B.* No. 107.
α Scorpii (Antares)	Wright	*L.O.B.* 107

7 α Ursæ Minoris (Polaris)	Campbell	*L.O.B.* 107
β Reticuli	Wright	,,
m Velorum	,,	,,
ν Centauri	,,	,,
υ_2 Centauri	,,	,,
δ Capricorni	Slipher	*Astroph. Jour.* xxiv. 361
τ Ursæ Majoris	Moore	,, xxiii. 263
λ Hydræ	,,	,, ,,
μ Ursæ Majoris	,,	
Y Ophiuchi	Albrecht	*Pub. A.S.P.* 18, 66
U Aquilæ	,,	,, 18, 142

Orbits of Spectroscopic Binaries.—The binary character of the fainter companion of *Castor* was discovered by Belopolsky in 1896; his later work led him to the view that there was a rotation of the line of apsides. The observations made at the Lick Observatory have been fully discussed by Mr H. D. Curtis (*Astroph. Jour.*, xxiii. 351), and he is led to abandon the assumption of any rotation in the line of apsides and to adopt an orbit nearly circular, $e = 0{\cdot}01$. In October 1904 Mr Curtis announced the variation in the radial velocity of the brighter component; and he deduces for it an orbit which exhibits signs of considerable eccentricity, $e = 0{\cdot}50$. Doberck, in his last discussion of the visual system, arrives at the conclusion that the period 347 years is the most probable. The relative motion of the two component systems derived by Mr Curtis is 7·14 km/sec.; and the periods are 9·2188 days and 2·9283 days for the bright and the faint components respectively. Mr Curtis has searched for variability in the light of the brighter component, but has not detected any.

From a discussion of radial velocities deduced at the Lick Observatory from 56 photographs of the spectrum of λ *Andromedæ*, Mr K. Burns has calculated an orbit with a period of 2·05 days (*Astroph. Jour.*, xxiv. 345). Further observations are required to decide whether there is a change in the elements.

Radial Velocities from Objective Prism.—The importance of devising a wholesale method of gathering information about radial velocities is widely recognised. Three notes on practical suggestions have been made in the last year.

Professor E. C. Pickering (*Astroph. Jour.*, xxiii. 255) recalls the method suggested by him in *Harvard Circular* No. 13—two exposures with the prism reversed. Mr G. C. Comstock (*Astroph. Jour.*, xxiii. 148) suggests the use of a single object-glass with a direct-vision objective prism in front of each half of the object-glass, the prisms being fixed with their refracting edges in opposite directions.

Mr De Lisle Stewart (*Astroph. Jour.*, xxiii. 396) describes a form of instrument with two object-glasses, each having an objective prism in front of it; by appropriate inclination of the axes of the lenses, and by separation of their centres, the two spectra may be set side by side on one photographic plate in opposed directions.

Reduction of Observations.—Dr H. K. Palmer (*Astroph. Jour.*, xxiv. 51) contributes a note on a short method of computing an approximate value of the reduction to Sun in radial velocity determinations. Dr Hartmann (*Ast. Nach.*, 173, 97) remarks upon the method, suggests the utilisation of the distance of the star from the apex of the Earth's motion, measured on a great circle, and describes a practical method of obtaining the required result by readings from a globe. H. F. N.

The Astrographic Chart and Catalogue.

There is little to record in connection with the Astrographic Chart beyond what may be gathered from the Proceedings of Observatories in another part of this report. The publication of the measures of stars of the northern zones is progressing. The printing of the rectangular co-ordinates of the Greenwich zone, Dec. $+64^\circ$ to the Pole, is practically complete. Oxford University Observatory has published its first volume of measures Dec. $+29^\circ$ to $+31^\circ$, a second volume is on the eve of publication, a third is being printed, and five more volumes will complete the Oxford section. Nothing has issued from the French observatories during the year 1906, although the work of publication already begun at Paris, Bordeaux, Toulouse, and Algiers is no doubt proceeding. M. Loewy mentioned in his annual report that the work at Paris had been delayed by the death of M. Paul Henry, and possibly the death of M. Rayet at Bordeaux has caused a similar hindrance. M. Henry has been succeeded in the superintendence of this branch at Paris by M. Puiseux.

Three volumes were published from Potsdam in the years 1899–1903, and it appears that four more volumes of measures were prepared for printing some time ago, but the publication was delayed by the computation of the plate-constants. As to the zone 47° to 64°, which is divided between the Vatican Observatory at Rome and the Catania Observatory, nothing has appeared since the first volume was issued from Rome in 1903. Father Hagen, the newly appointed Director of the Vatican Observatory, paid a visit to Greenwich last summer for the special purpose of making himself acquainted with the details of the astrographic work.

Of the zones which cover the sky south of the equator,* that between Dec. 32° and 40° undertaken at a later date than most of the others by the Observatory at Perth, West Australia, seems to be making small progress. The staff of this observatory is evidently inadequate to effect the measurement of the plates unless some other work of the observatory is discontinued. The

* Since the above has been in type a letter has been received from M. Valle, Director of the Tacubaya Observatory, Mexico, saying that 1200 of the 1260 catalogue plates required to cover the zone -10° to -16° have been taken, and that the computation of the plates, declination -15° and -16°, between R.A. 0^h and 4^h, will be completed, and the results printed in a few weeks. Some progress has also been made in taking the chart plates.

exact state of the work between this zone and the south pole will be learned from the reports of the British Colonial Observatories. A suggestion has been made that another conference should be held at a near date; should this take effect, information gathered as to the position of affairs at all the observatories might indicate any rearrangements necessary to bring the whole work to a conclusion, and discussion might lead to resolutions as to the procedure which should follow the publication of the measures.

Enlargements of the Chart Plates have been distributed during the year by the Directors of the Paris, Algiers, Toulouse, Bordeaux, San Fernando, and Greenwich Observatories. The total number issued from these observatories is now

Paris 271	Bordeaux 64
Algiers 276	San Fernando 95
Toulouse 122	Greenwich 538

H. P. H.

Star Catalogues.

A number of important star catalogues have appeared during the year. In this note it is only possible to enumerate them and indicate their scope.

Catalogue of Zodiacal Stars for 1900 *and* 1920.—This is published as vol. viii. part iii. of the Astronomical papers of the American *Ephemeris*. The catalogue contains 1607 stars, and is prepared principally for use in occultations of stars by the Moon. In determining positions of the stars and their proper motions, the more important catalogues from 1755 to 1900 have been used, after being reduced to the system of Newcomb's Catalogue of Fundamental Stars. This catalogue contains 1000 stars which may be regarded as fundamental. The positions of the remainder will be improved very considerably by recent observations of zodiacal stars.

Cape General Catalogue for 1900.—This catalogue contains 3365 stars which are north of the zenith at the Cape and 995 south of the zenith. The catalogue contains the results of observation at the Cape of 2798 zodiacal stars contained in a list circulated by Sir David Gill, and adopted at the Paris Conference of 1896 as forming a suitable basis for a catalogue of zodiacal stars in connection with heliometer observations of planets, occultations, etc. The remainder of the stars are, generally speaking, stars brighter than $8^{m}\cdot5$, which are not found in any catalogue of precision. The stars are generally observed five times, the epoch being about 1902 or 1903. The right ascensions are corrected for personal equation depending on magnitude. The declinations are reduced to the system of Newcomb's Fundamental Catalogue.

Radcliffe Catalogue for 1900.—The Radcliffe Catalogue of 1772 stars continues the programme of the Radcliffe Catalogue for 1890. The 1890 catalogue contained all stars down to $7^{m}\cdot0$

from the equator to -25° dec.; the present catalogue contains all stars to $7^{m}\cdot0$ between the equator and $+5^{\circ}$ dec. A large number of the zodiacal stars contained in the Zodiacal Catalogue of the American *Ephemeris* and in the Cape Catalogue are also observed here. Attention may be called to the determination of pivot errors, and the new determination of the division errors given in the introduction.

Henderson's Catalogue for 1840.—*The Annals of the Edinburgh Observatory*, vol. ii., consists of a reduction by Dr Halm of the observations made at the observatory on Calton Hill by Henderson and his assistant Wallace. The catalogue contains 3595 stars from the pole to dec. -20°, nearly all brighter than $7^{m}\cdot0$, observed between 1835 and 1845 with a transit instrument by Repsold and a mural circle by Troughton & Simms. The principal differences between the new and old reductions are, in the right ascensions, the elimination of the effect of the heat of the illuminating lamp on the level and azimuth, and in the declinations, the correction for division errors. The positions of the clock stars, etc. are taken from Auwers' Fundamental Catalogue.

Ambronn's Catalogue of all Stars to $6^{m}.5$ *for* 1900·0.—This compilation from the Catalogues of the *Astronomische Gesellschaft*, the Argentine General Catalogue, and other sources, contains 7796 stars, whose positions are given to $0^{s}\cdot1$ in R.A. and $1''$ in dec., and may be very useful where approximate places of bright stars are required.

The catalogues referred to above are in the main concerned with fairly bright stars. The catalogues which follow contain fainter stars.

Cape Catalogue of Astrographic Standard Stars for 1900·0.—The purpose of this catalogue is to provide accurate positions of stars to serve as reference stars for the Cape Section of the Astrographic Catalogue. It contains 8560 stars between the declinations -40° and -52°. All are observed three times, many of them five times, the observations being all made in the four years 1896–1899. The accidental probable error of a single observation is given as $\pm^{s}\cdot024$ sec δ in R.A. and $\pm0''\cdot28$ in declination, so that the accidental error of a result depending on three or five observations is very small. The point of special interest in the reductions is the application of zone corrections to each series of observations derived by comparison of 10 or 12 selected stars each night with the means of these stars from all observations. Magnitude equation has been applied separately for each observer.

Extensive comparisons are made with earlier catalogues, from which proper motions of a large number of stars have been deduced. These proper motions depend to a large extent on the Cordoba Catalogues, and it is stated that they cannot be used for purposes of cosmical research without further investigation of the systematic errors of that catalogue. The proper motions are, however, sufficiently accurate for the immediate purpose of the

catalogue, which is that of forming a secure foundation for the Cape Section of the Astrographic Catalogue.

Astronomische Gesellschaft Catalogue (*2nd Section*), *Strassburg Zone*.—This catalogue, which forms the first division of the A. G. Southern Catalogue ($-2°$ to $-23°$), extends from dec. $-2°$ to dec. $-6°$. It contains 8204 stars, being all the stars to $9^{m}\cdot0$ and fainter ones according to the A. G. programme. The positions are given for 1900·0 (not 1875·0 as in the N. Section of the A. G. C.), and are reduced to the system of Auwers' Southern Fundamental Catalogue. The average number of observations on which a catalogue place depends is nearly three, and the probable errors are about $\pm^{s}\cdot021$ and $\pm0''\cdot27$, being uniform in declination for all magnitudes, but in right ascensions ranging from $\pm^{s}\cdot017$ for bright stars to $\pm^{s}\cdot024$ for those fainter than $9^{m}\cdot0$.

Astronomische Gesellschaft Catalogue (*2nd Section*), *Wien, Ottakring Zone*.—This catalogue, forming the second division of the A. G. Southern Catalogue, was published in 1904, but has not yet been referred to in the Council reports. It extends from $-6°$ to $-10°$ dec. and contains 8468 stars. The probable accidental error seems to be about the same as in the Strassburg Zone.

In this connection it may be noticed that the reductions of the observations for the remaining zones, Cambridge U.S. ($-10°$ to $-14°$), Washington ($-14°$ to $-18°$), and Algiers ($-18°$ to $-23°$), are being rapidly pushed forward. F. W. D.

Universal Time.

Since the last mention of the subject of Universal Time in this Report (1905 February), the time system of India has been altered as there indicated. From 1905 July 1 the standard time of India has been $5\frac{1}{2}$ hours fast on Greenwich, that of Burmah $6\frac{1}{2}$ hours fast, but the time-ball of the Colaba Observatory, Bombay, is dropped at exactly 3^{h} a.m. Greenwich time.

It has been enacted by the Council at Port Louis that from 1907 January 1 the standard time throughout Mauritius and its dependencies shall be the time of the meridian sixty degrees east of Greenwich, except for the Chagos archipelago, where the standard time 5 hours east has been adopted from the same date. The time 4 hours fast on Greenwich is now also standard in the colony of the Seychelles. H. P. H.

Geodesy.

The new gravimetric Survey of India, begun in 1904, is still proceeding, and some results of a preliminary nature have appeared during the last two years in the publications of the Royal Society. The history of the subject of gravity in India begins in the year 1865, between which year and 1873 observations were made by

officers of the Survey at 31 stations in India with the Royal Society's invariable seconds pendulums. The results of these operations showed considerable discordances from the values of gravity as computed by Clairaut's law, and their physical meaning was for some time a matter of discussion. At 15 inland stations under 2000 feet above sea-level the number of swings of a "seconds" pendulum was 2·27 less per day than that computed from theory. At 4 stations between 2000 and 7000 feet high the defect was as much as 5·09, and at one station, in the Himalayan table-land, 15,400 feet above sea-level, the invariable pendulums made 21½ swings per day less than was expected. The deficiency of gravity which was then found to exist in Himalayan regions was attributed by different authorities to different causes, but lately there seems to have been some doubt as to the actuality of the fact itself, for at the International Conference of the Geodetic Association at Copenhagen in 1903, a resolution was passed for submission to the Government of India to the effect that it was desirable to make an accurate determination of the distribution of gravity in the mountainous country and in the plains of India. This work was begun in India in 1904, with a set of half-second pendulums of the Von Sterneck pattern, which were previously swung at Kew in 1903, with the primary object of standardising the instruments. As, however, on the suggestion of the Astronomer Royal, a series of swings were also made at Greenwich, the results gave a difference of the value of gravity at the two observatories. This observed difference * of 0·014 dynes excess at Kew is rather larger than the theoretically computed difference, which is +0·005 if the corrections for the known geological strata around the two places be included, but the accordance between computed and observed is nearer than in previous determinations made with the older form of instrument, and there is close agreement between two sets of operations that have been made with half-second pendulums.

The results found in India by Major Lenox-Conyngham, so far as yet published,† are remarkable. At Dehra Dûn, Madras, Bombay, Mussooree, the force of gravity, as recently determined, is larger than the earlier value by a considerable quantity, so that Colonel Burrard feels justified in saying that "the idea that gravity is exceptionally weak throughout India as compared to Europe can no longer be upheld, and the so-called marked negative variation has been found to rest on erroneous data."

The same memoir in the *Philosophical Transactions* contains an account of observations made to determine the local deflection of gravity over India. An earlier publication ‡ on this subject showed that local deflections of gravity could be classified in groups, and to test this, observations for latitude have been made

* *Proc. Roy. Soc.*, 1906 November 5.

† "On the Intensity and Direction of the Force of Gravity in India," by Lieut.-Col. Burrard, R.E., F.R.S., *Phil. Trans.* 1905.

‡ Survey of India Department, Professional Paper No. 5, 1901.

under Col. Burrard's direction at various stations along a meridian, or rather along each of several meridians, with a result which proved to be very much as expected. In the mountainous regions of the Himalayas the plumb-line is deflected northward by more than 30″; in the plains the deflection is in the same direction, but much less in amount. Further south, and parallel with the Himalayas, is a tract of country 4000 miles long and 200 broad, where the plumb-line is deflected in the opposite direction, *i.e.* towards the south; and having passed through that zone of southerly deflection into the Indian peninsula or into north-west India, the plumb-line is again found to be deflected northward. Discussing these determinations, the conclusions are arrived at that there is a local deflection of gravity at Kalianpur, the origin of the Survey, and that a spheroid which has the ellipticity of Bessel, 1/299·15, but the semi-major-axis found by Col. Clarke, 6,378,190 metres, best represents the facts of Indian geodesy.

Details of the geodetic work done in South Africa will be found in the report of the Cape of Good Hope Observatory. It appears that the measurement of the meridian 30° east of Greenwich is complete, or nearly so, as far north as latitude 13° 58′ S. And here it may be mentioned that the Survey department of Egypt, under the directorship of Captain H. G. Lyons, D.Sc., F.R.S., has begun the preliminary work of the geodetic triangulation of Egypt, which it is hoped will ultimately be connected with the triangulation in the southern part of the Continent (an intermediate section being measured by Germany), and so complete the measurement of this 30th meridian from 34° S to 30° N. As Captain Lyons says in his Report for 1905, it seems only fitting that the country where Eratosthenes (B.C. 275–196) did the first geodetic work, should take a share in the modern re-determination of the earth's form.

During the year a volume has been published by the Astronomer Royal containing the details of several determinations of longitude made by the Greenwich staff in the years 1888–1902. The results given in the first part of this, the difference of longitude between Greenwich and Paris, have already appeared in the *Monthly Notices.* The second part comprises the determination of a trans-Atlantic arc, Greenwich-Montreal, and the measurement of the arcs between Greenwich and Waterville and Killorglin in the south-west of Ireland, which are of some interest. This point of the United Kingdom is of geodetic importance, since it forms the western end of the great European arc in the parallel of latitude 52° N, and in the middle of the last century three separate determinations of longitude agreed in showing that the astronomical value of the arc was greater than the geodetic when Clarke's first values ($a = 20,926,348$ feet, $b = 20,855,233$ feet) were used for the spheroid assumed to represent the Earth's surface. The suggestion was made at the time that the difference was due to local attraction at the western stations, but the amount was larger than was consistent with any reasonable hypothesis. The new determinations

continue to show the astronomical value greater than the geodetic, the same spheroid as before being assumed in the computation of the latter, but the difference is not so large. The excess in the case of Waterville is $0^s.37$, in the case of Killorglin $+0^s.15$.

In a short paper * on the size of the Earth, Professor Helmert combines the values of recent arcs which connect Greenwich with the Continent and the arc Greenwich–Killorglin, with previous discussions of the great European arc, and arrives at a correction of +660 metres to the *a* found by Bessel; or, in other words, the derived equatorial radius of the Earth is 6,378,057 metres, or 20,925,572 feet. Professor Helmert gives this result with some reservation, apparently with reference to the fact that the question of local attraction at the Western stations has been treated only by an approximate method.

The triennial Conference of the International Geodetic Associations was held in 1906 at Buda Pest; the subjects of the measurement of the arc in Peru by French Geodesists, the survey of Africa, and Indian geodesy formed part of the proceedings. Sir George Darwin expressed the wish that it might be possible to hold the next meeting of the Association at Cambridge, regretting that it was not in his power to do more than make the suggestion. A formal invitation to England must proceed from the British Government. H. P. H.

* "Die Grösse der Erde," *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*, 1906, xxvii. p. 523.

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Batavia, Royal Society of Sciences.
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Berlin, Royal Prussian Academy of Sciences.
Bologna, Royal Academy of Sciences.
Bologna Observatory.
Bombay Branch of the Royal Asiatic Society.
Bombay, Government Observatory.
Bordeaux Observatory.
Bordeaux, Society of Physical and Natural Sciences.
Boston, American Academy of Arts and Sciences.
Brazil, Sociedade Scientifica de São Paulo.
Brussels, Belgian Astronomical Society.
Brussels, Royal Academy of Sciences of Belgium.
Buda-Pesth, Hungarian Academy of Sciences.
Buda-Pesth, Royal Hungarian Institute for Meteorology and Terrestrial Magnetism.
Calcutta, Asiatic Society of Bengal.
Canada, Department of Marine.
Canada, Geological Survey.
Canada, Royal Society.
Cape of Good Hope, Royal Observatory.
Cape Town, South African Philosophical Society.
Catania, Italian Spectroscopic Society.
Catania, Royal Observatory.
Colorado College Observatory.
Copenhagen, Royal Danish Academy of Sciences.
Cracow, Academy of Sciences.
Delft, Geodetic Commission of the Netherlands.
Dijon, Academy of Sciences.
Egypt, Survey Department.
Geneva Observatory.

Göttingen, Royal Observatory.
Göttingen, Royal Society of Sciences.
Granada Observatory.
Groningen, Astronomical Laboratory.
Halle, Imperial Lepold-Caroline Academy.
Hamburg Observatory.
Harvard College Astronomical Observatory.
Heidelberg, Astrophysical Observatory.
Heidelberg, Astronomical Institute.
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International Bureau of Weights and Measures.
Kasan, Imperial University.
Kodaikánal Observatory.
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Leipzig, Prince Jablonowski Society.
Leipzig, Royal Society of Sciences of Saxony.
Lick Observatory.
Lisbon Observatory.
Lowell Observatory.
Lund, Astronomical Observatory.
Madrid, Geographical Institute.
Madrid Observatory.
Madrid, Royal Academy of Sciences.
Manila, Philippine Weather Bureau.
Mauritius, Royal Alfred Observatory.
Melbourne Observatory.
Mexico, Scientific Society "Antonio Alzate."
Missouri University.
Moncalieri Observatory.
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Munich, Royal Observatory.
Natal Observatory.
Naples, Royal Academy of Sciences.
Neuchâtel Observatory.
New York, Columbia College Observatory.
O Gyalla, Central Meteorological and Magnetical Observatory.
Oporto, Polytechnic Academy.
Paris, Academy of Sciences.
Paris, Astronomical Society of France.
Paris, Bureau of Longitude.
Paris, École Polytechnique.
Paris, Mathematical Society of France.
Paris Observatory.

Paris, Philomathic Society.
Pennsylvania University.
Perth Observatory, Western Australia.
Philadelphia, American Philosophical Society.
Philadelphia, Franklin Institute.
Philadelphia, University Observatory.
Pola, Imperial Hydrographic Office.
Potsdam, Astrophysical Observatory.
Potsdam, Central International Geodetic Bureau.
Potsdam, Royal Prussian Geodetic Institute.
Poughkeepsie, Vassar College Observatory.
Prague, Imperial Observatory.
Pulkowa Observatory.
Pulkowa, Physical Observatory.
Queensland Government.
Rio de Janeiro Observatory.
Rome, Royal Academy *dei Lincei*.
Rome, Vatican Observatory.
St Petersburg, Imperial Academy of Sciences.
San Fernando, Observatory of Marine.
San Francisco, Astronomical Society of the Pacific.
Sophia University.
Stockholm Observatory.
Stockholm, Royal Swedish Academy of Sciences.
Stockholm, Swedish Geodetic Commission.
Sydney Observatory.
Tacubaya National Astronomical Observatory.
Tōkyō Astronomical Observatory.
Toronto, Canadian Institute.
Toronto University.
Toulouse, Academy of Sciences.
Toulouse, Meteorological Commission.
Toulouse Observatory.
Turin, Royal Academy of Sciences.
Transvaal Meteorological Department.
Uccle, Royal Observatory of Belgium.
United States Coast and Geodetic Survey.
Upsala, Royal Society of Sciences.
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Vienna, Imperial Military Geographic Institute.
Wanganui Astronomical Society.
Warsaw, Observatory of the Imperial University.
Washington, Bureau of Standards.
Washington, Navy Department.
Washington, Philosophical Society.
Washington, Smithsonian Institution.
Washington, United States Naval Observatory.
Yale University Astronomical Observatory.
Yerkes Observatory.

Zi-Ka-Wei Astronomical Observatory.
Zürich, Central Meteorological Institute of Switzerland.
Zürich, Geodetic Commission of Switzerland.
Zürich, Natural History Society.
Editors of the "American Journal of Mathematics."
Editors of the "American Journal of Science."
Editor of the "Astronomical Journal."
Editor of the "Astronomische Nachrichten."
Editor of the "Astronomischer Jahresbericht."
Editors of the "Astrophysical Journal."
Editor of the "Athenæum."
Editors of the "Bulletin des Sciences Mathématiques."
Editor of the "English Mechanic."
Editor of "Himmel und Erde."
Editor of "Indian Engineering."
Editor of "Nature."
Editor of "Naturwissenschaftliche Rundschau."
Editors of "The Observatory."
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ADDRESS

Delivered by the President, Mr William H. Maw, on presenting the Gold Medal of the Society to Professor Ernest William Brown, February 8, 1907.

THE Gold Medal of the Royal Astronomical Society has this year been awarded to Professor Ernest William Brown for his "Researches in the Lunar Theory," and it is my duty to put before you on the present occasion the grounds for this award. In attempting this, I fear it will be impossible for me to do adequate justice to the importance of our medallist's work. This work is of so special a character, and its development has been marked by the introduction of so many original devices and methods of calculation, that an address such as this is quite unsuited for an examination of the details of the research. All I can hope to do is to put before you the broad features and general extent of Professor Brown's work, and to record the success which he has attained.

Professor Brown is the seventh astronomer to whom the Gold Medal of the Royal Astronomical Society has been awarded for work in connection with the Lunar theory. His predecessors were: Baron Damoiseau (1831) for his "Memoir on the Theory of the Moon," and for his Lunar Tables; M. Jean Plana (1840) for his work entitled "Théorie du Mouvement de la Lune"; Professor P. A. Hansen (1860) for his Lunar Tables; Professor J. C. Adams (1866) for his "Contributions to the Development of the Lunar Theory"; M. Delaunay (1870) for his "Théorie de la Lune"; and Dr G. W. Hill (1887) for his "Researches on the Lunar Theory." This is a long list of illustrious workers, and with them our present medallist is well qualified to rank.

In a paper entitled "Theory of the Motion of the Moon, containing a New Calculation of the Expressions for the Co-ordinates for the Moon in Terms of the Time," published in volume liii. of our *Memoirs*, our medallist has stated so clearly the nature of the problem on which he has been engaged that I may be permitted to quote from his introduction. He says:—"The formation of numerical expressions deduced as a consequence of the Newtonian laws of motion and gravitation which shall represent the position of the Moon at any time may be roughly divided into three stages. As a first step, we consider each of the three bodies—the Sun, the Earth, and the Moon—as a sphere of mass equal to its actual mass, and arranged in concentric layers of

equal density. The Earth (or the centre of mass of the Earth and Moon) is supposed to move round the Sun in a certain ideal elliptic orbit, and all disturbances of this orbit and of the Moon from any other source than the ideal Sun and Earth are neglected. This first stage constitutes nearly the whole of the labour of solving the problem of three bodies as far as the particular configuration of the Sun-Earth-Moon system is concerned. When this is done, we proceed to the second step, which involves the determination of the effects due to the difference between the actual and the ideal motions of the Earth and Sun, to the influence exercised by the other bodies of the solar system, and to the differences between the real and ideal arrangements of the masses of the bodies. The calculations so far may, theoretically at least, be made without any knowledge of the configuration of the system at any given time or times, beyond a general idea of the order of magnitude of certain of the constants involved. The third and final stage consists in a determination by observation of the various constants which have entered into the theory and the substitution of their values, so as to obtain numerical expressions for the co-ordinates in terms of the time."

As we shall see later, it is the completion of the first of these stages which has primarily been the object of Professor Brown's past labours; and as a result he has, after arduous work extending over the past fifteen years, completed the solution of the problem of three bodies for the case of the Sun-Earth-Moon with an accuracy very far in excess of that attained by any of his predecessors in this line of research.

Our knowledge of the motion of the Moon has accumulated during long ages; but it is, of course, only since the time of Newton, or, say, during the past two and a half centuries, that the Lunar theory has had any existence. Our earlier information as to irregularities in the Moon's movements was knowledge derived from observations, and it did not include any explanations of the causes to which these irregularities are due. Before the discovery of gravitation, all that could be done with the motion of the Moon was necessarily empirical. But even Newton, the discoverer of this principle, contented himself with the variation, the motions of the perigee and node, and the largest inequality of the latitude. Analytical expressions giving the position of the Moon in space were not seriously attempted until the middle of the eighteenth century, when three men simultaneously concerned themselves with the problem. They were Clairaut, D'Alembert, and Euler. The last-named, in 1753 and 1772, produced three Lunar theories nearly independent of each other, and of the third of these I shall have more to say later.

Of the six inequalities which affect the Moon's position by an amount capable of being discovered by naked-eye observations, viz. the effect of solar attraction in enlarging the Moon's orbit, the revolution of the line of apsides, the regression of the nodes, the evection, the variation, and the annual equation, only one—that to

which the name of "evection" was given by Boulliaud in the seventeenth century—was known to the ancients. It appears to have been first noticed by Hipparchus, about 150 B.C., when he was engaged in endeavouring to determine the Moon's distance, but it was first taken systematically into account by Ptolemy, to whom, indeed, its discovery has been attributed.

The variation, which has a fortnightly period and a zero value at syzygies and quadratures, does not affect the time of an eclipse, and thus escaped the notice of the Greek astronomers. It appears, however, to have been detected by an Arabian astronomer, Aboul Wefa, in the tenth century, but was lost sight of until rediscovered by Tycho Brahé about the end of the sixteenth century. To Tycho Brahé is also due the detection of the annual equation, although he gave it an erroneous value. A more correct value was determined by Horrocks, but its true character appears to have been first appreciated by Flamsteed.

Of smaller perturbations the number is almost endless—seventy such perturbations are, I understand, taken into account in the calculations of the Moon's longitude made for the American Ephemeris, and about half that number in the computations for latitude—but of these the most important, and in many ways the most interesting, is the secular acceleration of the Moon's mean motion; and on this I desire to say a few words. As early as 1693, Halley, after a consideration of the records of a number of ancient eclipses, arrived at the conclusion that these records could only be satisfied by assuming a progressive shortening of the Lunar month. A long period elapsed before this suspicion was confirmed, but in 1749 Dunthorne contributed to the Royal Society a paper discussing all available observations bearing on the subject, and the matter was further investigated by Mayer, Bouvard, and Burg. The explanation of the acceleration, however, long evaded the efforts of the mathematicians, but later the problem was taken up by Laplace, who at length, on November 19, 1787, announced to the Académie des Sciences his discovery that—"The secular equation of the Moon is due to the action of the Sun on the Satellite, combined with the secular variation of the eccentricity of the terrestrial orbit."

Laplace's first value for this acceleration was $11''\cdot135$ per century: a value subsequently reduced to $10''\cdot18$. The cause of this inequality acts on the Moon as gravity on a falling body, and its effect, therefore, is as the square of the time; but in carrying the calculations back to the time of the Chaldean observations, it was found necessary to add a small term depending on the time cubed. Thus if t = the number of centuries from the assumed epoch, the acceleration, according to Laplace, was equal to $10''\cdot1819\, t^2 + 0''\cdot01854\, t^3$.

These deductions of Laplace were verified in 1820 by two of our medallists, Damoiseau and Plana, and also by Carlini, their approximations being carried to a higher order than those of Laplace. The value deduced by Damoiseau was $10''\cdot72$, and by

Plana $10''\cdot58$. These values were all too large; but the error was at once fortunate and unfortunate. It was fortunate in so far that the values arrived at fairly satisfied the discrepancies between the records of ancient eclipses and the results of modern observations, and led to the acceptance of the explanation and the upholding of the gravitational theory; but unfortunate inasmuch as this close agreement had the effect of stopping for the time further research. Moreover, when some twenty years later Hansen took up the matter, he also arrived at a large value which he announced in 1842 as $11''\cdot93$, reduced in 1847 to $11''\cdot47$.

Things remained thus until another of our medallists, Professor Adams, took up the problem; and on June 16, 1853, in a paper read before the Royal Society, pointed out an important error in the work of Plana and Damoiseau, and showed that the correction of this error most materially reduced the value of the acceleration. Three years later, Plana was induced by Adams's investigations to re-examine a portion of his own work; and in April 1856 he admitted the imperfection of his theory, and deduced a result agreeing with that of Adams. A little later, however, he withdrew this admission, and deduced a value differing both from that of Adams and his own original result.

Adams's investigations led to strong discussions between the chief mathematicians of the time, and his deductions were not at once accepted. They were opposed by de Pontécoulant, and in 1857 Hansen published his *Tables de la Lune*, in which the value adopted for the secular acceleration was $12''\cdot18$. In 1859, however, the investigation of this part of the Lunar theory was taken up by Delaunay, another of our medallists. Adams had shown that in a certain series in m^2, m^4, etc. (the term m^3 being absent) the term m^4 had been wrongly calculated, and that as a result the numerical value of the secular acceleration must be approximately halved. Delaunay, carrying his calculations to m^4, obtained exactly the same result as Adams; a result which he announced in January 1859. This induced Adams to publish the value he had already obtained, using terms involving m^5, m^6, and m^7, the result being to give the coefficient of secular acceleration the value $5''\cdot7$, a result subsequently reduced to $5''\cdot64$. Again Delaunay took up the matter, and on April 25, 1859, he communicated to the Académie des Sciences the result of his investigations, confirming Adams's new term: and, by carrying the approximations to the 8th order, deducing the value $6''\cdot11$.

Into the further stages of this important controversy it is unnecessary to enter, but ultimately the results of Adams were accepted. The acceptance of his value, or of a close approximation to it, leaves the remaining discrepancy between theory and the results of observation to be explained by some other cause or causes; a work with which Mr Cowell is at present prominently identified.

The next important step to be noticed in the development of the Lunar theory is the epoch-marking work of Dr G. W. Hill.

The features of that work were most admirably dealt with in the Address delivered by Dr Glaisher on the occasion of the Gold Medal being awarded to Dr Hill in 1887, and it would be quite impossible to summarise them here. It need only be said that—founded to some extent on a suggestion contained in the paper (to which reference has been already made) published by Euler in 1772, namely, that of employing moving rectangular co-ordinates, but embodying entirely novel methods of development, of the highest interest from the point of view of both the pure mathematician and the astronomer—Hill's work opened out a new region for theoretical research, at the same time introducing great simplifications in methods of practical calculation.

Many other distinguished names will no doubt occur to you as associated with the development of the Lunar theory, such as Airy, Donkin, Cayley, and Newcomb, but time will not permit of my entering into the details of their work on the present occasion. I must pass on to consider some of the salient features which have marked the development of the Lunar theory; and in doing this I desire to express my indebtedness to Dr G. W. Hill for much valuable information relating to this branch of my subject, which he has most kindly placed at my disposal.

The incessant call for greater precision in dealing with the motions of the Moon has led to frequent repetitions of treatment of this subject, so that we are now in possession of ten or eleven Lunar theories, each professing to go over the whole ground. The advance in precision is obvious from the fact that while Euler contented himself with about thirty periodic terms in the longitude, Professor Brown has nearly four hundred. As would be expected, each investigator has adopted such a method as, in his judgment, would lead most promptly to the desired end. These judgments, however, from the necessity of the case, can be only probable conclusions. Thus the Lunar theories we have now before us exhibit much variety.

The broadest division of these theories which can be made is into the two classes of *literal* and *numerical*. For the first, all the quantities on which the result depends are represented by algebraic symbols; while in the second, the investigators have attempted to shorten their labour by introducing at the outset the numerical values of as many of the quantities as the nature of the problem admitted. The much greater labour involved in the elaboration of a *literal* Lunar theory has brought it about that of our ten or eleven treatments of this subject, only three are *literal*, viz. those of Plana, de Pontécoulant, and Delaunay. But this method of treatment is much more satisfactory to the mathematical mind, while it also possesses obvious advantages over the numerical method. In the earlier days of the treatment of our problem it seems to have been thought nearly impracticable to adopt the literal method, and to Plana must be accorded the merit of having first elaborated such a theory. Professor Brown's theory is partially literal and partially numerical. Of the five parameters involved in the pre-

sentation of the Lunar co-ordinates, four are left indeterminate, and it is only the ratio of the month to the year that receives a definite numerical value from the beginning.

We next have to note other varieties of treatment. In the earlier period it was a favourite method to adopt the true longitude of the Moon as the independent variable; that is, in the first instance, the mean longitude, latitude, and reciprocal of the radius were determined in terms of the true longitude, and there remained the task of inverting these formulas. This was the method of all the early elaborations, except the last theory of Euler, to which reference has already been made. On many grounds it could, no doubt, be defended; but it seems that it was Poisson who first threw discredit on it. His pupil, de Pontécoulant, adopted the time as the independent variable, and so also did Hansen.

I must now direct attention to the most prominent of the peculiarities which mark our medallist's elaboration of the problem. This may be stated as the complete utilisation of the circumstance that, setting aside the rates of motion of the elements of the arguments of the periodic terms, the analytical expressions of the three co-ordinates are capable of being separated into portions, each factored by certain powers and products of the four parameters which Professor Brown has left indeterminate in his formulas. These factors have been termed "characteristics." The advantage of considering these portions is, that each can be determined independently of the adjacent, so to speak, portions; while the treatment depends on the portions of lower degrees, which have been precedently treated. The credit of having introduced this notion must be given to Euler, who employed it in his last treatment of the problem, published in 1772.

Very closely connected with the foregoing principle is the device of making the Moon's mean longitude disappear from the equations employed in the treatment. Instead of using co-ordinates referred to fixed axes, there are employed those referred to two axes in the plane of the ecliptic having a velocity of rotation equal to the motion of the Moon's mean longitude. This contrivance is also due to Euler.

From the use of these two principles results a distinguishing mark in the treatment. Every treatment must begin with a stage called that of the first approximation, from which, by degrees, a superior precision is attained. All the Lunar theories mentioned, except the two particularly described, start from the Keplerian ellipse, or this modified by moving lines of apsides and nodes, as a first approximation. But in the last theory of Euler, and that of Professor Brown, the first approximation is the variational curve, already somewhat roughly derived by Newton in the *Principia*. It is the fashion now to call this a periodic solution, as the inequalities of motion go through all their varieties in a lunation; but we may arrive at a notion of the matter more simply in the following manner:—The motion of the Moon largely depends on the constant called the eccentricity, and this constant

might have the value zero; in like manner, the motion might take place in the ecliptic; the solar eccentricity might be zero; and, in fine, the Sun might be supposed at such a distance that its action in disturbing the relative position of the Moon might be regarded as always parallel to its action on the Earth. Let all these four possibilities be fulfilled; the result is a greatly simplified set of differential equations, easy of solution, although methods of approximation must still be employed.

Having obtained this solution, we are prepared to advance further. Let it be granted that the four mentioned constants, instead of having zero value, have values so small that their squares and products may be neglected. This supposition gives rise to a set of equations denominated by M. Poincaré as "equations to variation." They are of the class known as "linear," and are generally more easy to integrate than those called "non-linear." Their most important quality is their being, with the exception of the known parts, the same in all the stages of the approximation. It thus results that a considerable portion of the calculations made for the second stage of the approximations is still available for all following.

It may be stated that the series which satisfy the differential equations of a Lunar motion belong to the class now known as Lindstedt series, or those resulting from the addition of terms, each the product of two factors, one being a constant, the other a sine or a cosine of an angular argument, the latter being equivalent to the sum of positive or negative integral multiples of a definite number of elementary arguments. In the restricted form of the Lunar theory treated by Professor Brown the definite number is four.

In the elaboration of the integration it is not only necessary to determine the constant factors of the series, but the derivation of the motion of the elementary arguments. The latter have similar forms with the coefficients, and, like them, cannot be determined at one step; we must be content to derive the successive portions in their turn. In the Lunar theory the matter is somewhat alleviated by the circumstance that the motion of two of the four elementary arguments may be regarded as known at the outset; it is necessary only to derive the rates of motion of the mean anomaly, and the mean argument of the latitude. At particular stages it becomes necessary to notice that the previously-used motion of one or the other of the arguments needs correction by the addition of a new set of terms.

When the Lindstedt series are substituted in the equations to variation, the result is a group of linear algebraical equations in number precisely sufficient to determine the coefficients, or the corrections to coefficients, as well as the corrections to the motions of the arguments, if there be any. For solving the above-mentioned group of equations the most feasible method of treatment to adopt is still that of successive approximations. The most important quantity to be noticed in this solution is the determinant

of the equations. Not unfrequently this quantity is much smaller than the coefficients from which it is derived. When this is the case the coefficients involved must be computed to a correspondingly greater degree of exactitude. In two or three cases the matter is so pressing that three more decimals must be added to the values of the quantities involved. This is the most troublesome circumstance attending the elaboration of a Lunar theory, and variation of method does nothing towards the removal of it. It is very vexing for the investigator to find that he must return on his steps, and push certain quantities to a higher degree of precision. The matter is so complicated that it is impossible to prescribe *a priori* rules for procedure. Delaunay in his Lunar theory has noted, at the foot of the page, all the places where such a modification of process was necessary.

Such, in brief, is an outline of the method of elaborating a Lunar theory to-day. In his paper, to which reference has already been made, published in vol. liii. of our *Memoirs*, our medallist stated that he had then (1897) been engaged for six years in attempting to develop the ideas contained in Hill's "Researches in the Lunar Theory," by calculating the coefficients of terms with certain definite characteristics, to which I have already alluded; and he defines the "characteristic" of any part of a coefficient as being that part of its expression which consists of powers and products of the eccentricities, the inclination, and the ratio of the mean parallaxes. He goes on to say: "Dr Hill had obtained these which had the characteristic unity, that is, which were functions of the mean motions of the Sun and Moon only, and also that part of the motion of the perigee which was a function of the same quantities; Adams had done the same thing for the motion of the node. It remained, therefore, to obtain the general equations, to put them into forms suitable for calculation, and to show how the other parts of the motions of the perigee and node might be obtained."

I must now notice the degree of advance in our knowledge of the Lunar motions, attained through the labours of Professor Brown. When Professor Newcomb made his comparison of Hansen and Delaunay, we were in doubt as to the value of some of the coefficients in longitude to the extent of half a second of arc at least; also the motions of the perigee and node were uncertain to correspondingly larger quantities. On the other hand, the degree of accuracy aimed at by our medallist has been such that there should be included the coefficients of all periodic terms in longitude, latitude, and parallax which are greater than $0''\cdot01$ of arc, and that the results should be correct to this amount. That he has been able to carry out such a programme is assuredly a matter demanding our heartiest congratulations; in fact, with such a notable advance one is almost inclined to say that there is nothing left to be desired; however, this phrase has been so often upset in the past that it would be unsafe to employ it.

A natural question here arises: "What will be the effect

Professor Brown's researches on the accuracy of Lunar tables based upon them?" This is a question of great interest and importance, but in the present state of our knowledge it is one to which it is difficult to give a definite answer. It has, however, been my privilege to receive an expression of opinion from Dr G. W. Hill, who speaks with the highest authority, and this opinion I may be permitted to quote. He says: "Much as we rightly welcome the results of Professor Brown's devoted labours, we should be unwarranted in assuming that their employment in the Lunar tables would give rise to a marked improvement in the representation of observations. A slight one indeed might be expected; but it has been evident for some time that the Moon deviated from its calculated orbit more because it is subject to irregular forces, which we have not yet the means of estimating, than because the tables are affected by slight defects in the mathematical treatment of the forces which are already recognised. This circumstance in no sense diminishes the credit due to Professor Brown's work." This is a very weighty expression of opinion, and it indicates that there is yet ample work to be done by investigators of Lunar motions. I should add that such research has been most materially aided by the important work of our medallist, who, by giving accurate values to the known perturbations, has defined more clearly the further irregularities of which the explanation has yet to be ascertained.

I have perhaps, in this address, dwelt at somewhat undue length on certain matters of a more or less historical character, but my excuse is that the facts I have stated may serve to emphasise the difficulty and the onerous nature of the work in which our medallist has been engaged. That he has succeeded in making great advances in a field of research which has received the deepest attention from the leading mathematicians of the world for the past century or more is in itself a most eloquent testimony to his powers. It must be borne in mind that in a problem of this character, the solution of which depends upon so many diverse terms, the investigator has not only to acquire such knowledge of the various terms as will enable him to discard those of which the influence is unimportant, but he has also, in the case of those retained, to devise such methods of treating them as will enable them to be practically dealt with in a reasonable period. In both these respects our medallist has achieved admirable results. Early in his work of calculating inequalities whose characteristics are the first, second, and third powers of the ratio of the mean parallaxes of the Sun and Moon, and the same powers of the eccentricity of the Moon, Brown found that the forms of equations then available left much to be desired, and were apt to lead to errors in the practical calculations; and in a paper entitled "Investigations in the Lunar Theory" contributed to the *American Journal of Mathematics*, vol. xvii., he showed how these difficulties could be avoided and the labour of computation diminished.

The thoroughly practical character of Brown's method of work * is most striking. Speaking of the value of Euler's suggestions, he has himself said:—"The working value of a method of treatment is not really tested by the closeness with which the first or second approximation will make the further approximations converge quickly to the desired degree of accuracy; the real test is, perhaps, the ease with which the final approximation can be obtained. Here we have the essential difference between the present method and all other methods. The approximations of the latter proceed along powers of the disturbing force. Euler's idea was to approximate along powers of the other small constants present. This gives a most rapid convergence, and a degree of certainty in knowing the limits of error of the final results which no other method approaches."

In a letter which I have received from Professor Brown he modestly states that the only portions of his work presenting real difficulties were those arising from the *direct* and *indirect* actions of the planets. On approaching these problems he found the subject in a somewhat chaotic state, and it was necessary to clear the ground and get the theory into good shape. In doing this, the first requirement was to be able to compute the derivatives of the Moon's co-ordinates with respect to n = the Moon's mean motion. But in our medallist's theory the numerical value of n had been substituted, so that the derivatives could not be calculated directly from it. Delaunay's literal theory might have been used for the purpose, but, owing to slow convergence, Professor Brown did not consider it accurate enough. Under these circumstances he succeeded in finding a method for getting these derivatives accurately from his theory, in spite of the fact that the numerical value of n had been substituted. The idea which led up to this

* The records of our medallist's work have been largely contained in papers contributed to our Society. In May 1897 Professor Brown sent in Part I., chapters 1 to 4, of the paper entitled "Theory of the Motion of the Moon, containing a New Calculation of the Expressions for the Co-ordinates of the Moon in Terms of the Time," from which I have already quoted; and this was followed, in February 1899, by Part II.; in May 1900, by Part III.; and in January 1905, by the conclusion, Part IV. [Parts I. and II. of Professor Brown's paper are contained in vol. liii., Part III. in vol. liv., and Part IV. in vol. lvii. of the *Memoirs* of the R.A.S.] Besides this, he contributed a number of papers which have appeared in our *Monthly Notices*, namely, "On the Mean Motions of Lunar Perigee and Node," and "On the Theoretical Values of the Secular Accelerations in the Lunar Theory," abstracts of which appeared in March 1897; a paper entitled "Note [illegible] the Mean Motions of Lunar Perigee and Node," published June 1897 [illegible] "On the Verification of the Newtonian Law," which appeared May [illegible] two papers entitled respectively "On the Degree of Accuracy of the Lunar Theory and on the Final Values of the Mean Motions of the [illegible] and Node," and on "The Parallactic Inequality and the Solar Pa[illegible] published April 1904; one "On the Completion of the Solution [illegible] Main Problem in the New Lunar Theory," published December [illegible] one "On the Final Values of the Coefficients in the New Lunar [illegible] contained in *Monthly Notices* for January 1905.

method was first stated in a paper * contributed to the Cambridge Philosophical Transactions in 1899; and the method itself, which is believed to be quite new, was dealt with in a paper † contributed to the American Mathematical Society in February 1903. Since then the calculations have been performed, but are not yet published.

The calculation of the *indirect* inequalities gave considerable trouble, but Professor Brown was able ultimately to show in a paper ‡ contributed to the American Mathematical Society in February 1905 that it was not necessary to calculate the perturbations of the Earth by the planets in order to get the resulting effect on the Moon, but that it was possible to go straight to the disturbing function of the Earth by the planet. The calculations of the *direct* inequalities were completed last autumn, and Professor Brown hopes to publish them during the ensuing summer.

The precautions taken by our medallist to secure accuracy in the final results have been most refined. In accordance with the original programme, every coefficient in longitude, latitude, and parallax, which is so great as one-hundredth of a second of arc, has been computed, and is regarded as accurate to at least this amount, the results being really obtained to one-thousandth of a second. To avoid the occurrence of errors of computation, equations of verification have been computed at every step of the work, every page of the manuscript having, on the average, not less than two test equations computed. I am indebted to Mr Cowell for the remark that our medallist is the first Lunar theorist to use independent equations of verification, thus creating a higher degree of confidence in his results than could ever come from mere duplicate calculation. It was his device to form the equation for a small variation of this solution of Hill's equations. Says Mr Cowell:— "The numerical application of this device was rendered possible by calculating series for various complicated fractions of the coordinates in Hill's variation curve. The utility of the plan is obvious as soon as it is got into working order, and its conception implies rare insight on the part of our medallist. It lies at the root of his success in obtaining more accurate results, with less labour than his predecessors. He has also obtained theorems by which the higher parts of the motion of the perigee and the node may be calculated in advance of the corresponding group of periodic terms."

For the motions of the perigee and node, the final values obtained, and a comparison of these values with the results of

* "On the Solution of a Pair of Simultaneous Linear Differential Equations which occur in the Lunar Theory," *Cambridge Philosophical Transactions*, vol. xviii.

† "On the Formation of the Derivatives of the Lunar Co-ordinates with Respect to the Elements," *Transactions of the American Mathematical Society*, vol. iv.

‡ "On a General Method for Treating Transmitted Motions, and its Application to Indirect Perturbations," *Transactions of the American Mathematical Society*, vol. vi.

observations, were given in the paper published in the number of the *Monthly Notices* for April 1904,* to which reference has already been made. In giving these values our medallist pointed out that there was one constant of which the observed value was so far doubtful as to affect the results by as much as the tenth of a second, this constant being the ellipticity of the Earth. As there appeared to be two competing values for this constant, namely, $\frac{1}{292 \cdot 9}$ and $\frac{1}{298 \cdot 3}$, between which no definite choice could be made, Professor Brown decided to calculate the results for the two values. The final results, with the portions of which they are made up, are given in the subjoined table :—

Final Mean Values of the Annual Mean Motions of the Perigee and Node.

Epoch 1850·0.

			Perigee.		Node.
Solar Action	Char^c. I.		+148 524″·92		−69 287″·90
	,, e^2	−	519·31	−	616·09
	,, γ^2	−	1 739·85	+	260·59
	,, e'^2	+	156·27	−	25·46
	,, a^2	+	2·24	−	1·11
	,, e^4	+	·04	+	·07
	,, $e^2\gamma^2$	+	6·72	−	1·70
	,, γ^4	−	1·51	+	·05
	,, $e^2 e'^2$	−	·99	−	·57
	,, $\gamma^2 e'^2$	−	1·61	+	·08
	,, rem^g.		·00 ± ·04		·00 ± ·02
Terms in No. 2		−	·70	+	·20
Terms in No. 3		+	·01	−	·01
Planetary direct		+	2·66 } ± ·06	−	1·42 } ± ·03
Planetary indirect		−	·20 }	+	·06 }
Figure of Earth (α)		+	6·57	−	6·15
Figure of Earth (β)		+	6·41	−	6·00
Calculated sum (α)			+146 435·26 ± ·10		−69 679·38 ± ·05
Calculated sum (β)			+146 435·10 ± ·10		−69 679·23 ± ·05
Observed			+146 435·23		−69 679·45
C − O (α)		+	0·04 ± ·10	+	0·08 ± ·05
C − O (β)		−	0·12 ± ·10	+	0·23 ± ·0

Note.—The "Figure of Earth" values "(α)" correspond ellipticity of $\frac{1}{292 \cdot 9}$, and those "(β)" to an ellipticity of

* A slight correction to the values there given will be found in *Notices*, vol. lxv. p. 276. This correction has been made in the now given.

For the theoretical secular accelerations the values finally arrived at by our medallist (as given in his paper published in *Monthly Notices* for March 1897) are as follows:—

Theoretical Values of the Secular Acceleration per Century.

The Mean Motion	$+ \ 5''.91 \pm 0''.02$
The Perigee	-38.9 ± 0.1
The Node	$+ \ 6.56 \pm 0.02$

In devising the details of his research, our medallist arranged the work so that considerable proportions could be done by computers; but, as a matter of fact, only one—Mr Ira L. Sterner, of Haverford College, of whose ability and accuracy Professor Brown speaks in the highest terms—has been so employed. It may be interesting to quote here some details given by our medallist as to the time and labour expended on the work. He states: "From 1890 to 1895 certain classes of inequalities were calculated, but the work was only begun on a systematic plan, which involved a fresh computation of all inequalities previously found, at the beginning of 1895. Mr Sterner began work for me in the autumn of 1897, and finished it in the spring of 1904, though neither of us was by any means continuously engaged in calculation during that period. He spent on it—according to a carefully-kept record—nearly three thousand hours, and I estimate my share as some five or six thousand hours, so that the calculations have probably occupied altogether about eight or nine thousand hours. There were about 13,000 multiplications of series made, containing some 400,000 separate products; the whole of the work required the writing of between some four or five millions of digits and *plus* and *minus* signs."

Professor Brown has, as I have stated, completed his solution of the problem of three bodies for the case of the Sun-Earth-Moon, and has achieved an accuracy very far in excess of that of any of his predecessors; while he has done this by methods involving striking elegance and originality, and showing great powers of resource. He has, however, by no means finished his labours. As he himself pointed out, in announcing the completion of the main problem, much still remained to be done before it was advisable to proceed to the construction of tables. On this work our medallist is now engaged, and we may rest assured that he will continue to bring to bear upon it that energy and power of organised inquiry which have enabled him already to secure such brilliant results. We may, I think, further hope that in the present award he may find some encouragement in his labours.

I much regret that Professor Brown is not with us this evening to receive the medal personally, but a combination of circumstances—amongst them the serious illness of a relative—has rendered it impossible for him to cross the Atlantic at the present

time. This being so, I have been in communication with Sir Edward Grey, H.M. Secretary of State for Foreign Affairs, and I am glad to say that he has kindly arranged for the medal to be forwarded in the Foreign Office bag to His Majesty's Chargé d'Affaires at Washington, by whom it will in due course be transmitted to Professor Brown.

I will now ask Mr Lewis to receive the medal on Professor Brown's behalf, and to transmit it with our most sincere hope that he may long be spared in full health and strength to carry out the important researches to which he has devoted himself.

Before passing to other business, there is another matter connected with our medallist on which I should like to say a few words. I have not alluded to it in my address, because it has nothing to do with the award of the medal, but it will, I think, be of interest to the Fellows generally. As many present are aware, Professor Brown is an Englishman who has been long resident in America, and who has for the past sixteen years been connected with Haverford College. That association will, however, be broken in the ensuing summer, and next autumn Professor Brown proceeds to Yale University. It is exceedingly gratifying to know that his work on the Lunar theory, which he has been able to carry on at Haverford under most favourable conditions, will not be interrupted by this change. By a letter received from Professor Brown, I learn that not only have the Yale authorities recognised the importance of his work by arranging special facilities for its continuance, but they have also most generously undertaken to provide the funds required for both the preparation and the publication of the Lunar Tables which will form the natural outcome of our medallist's labours.

The Meeting then proceeded to the election of the Officers and Council for the ensuing year, when the following Fellows were elected: —

President.

H. F. Newall, Esq., M.A., F.R.S.

Vice-Presidents.

Sir W. H. M. Christie, K.C.B., M.A., D.Sc., F.R.S., Astronomer Royal.
Sir David Gill, K.C.B., LL.D., D.Sc., F.R.S.
Major P. A. MacMahon, D.Sc., F.R.S.
W. H. Maw, Esq.

Treasurer.

Major E. H. Hills, C.M.G.

Secretaries.

Thomas Lewis, Esq.
S. A. Saunder, Esq., M.A.

Foreign Secretary.

Sir William Huggins, K.C.B., O.M., LL.D., D.C.L., F.R.S.

Council.

Sir R. S. Ball, M.A., LL.D., F.R.S., Lowndean Professor of Astronomy and Geometry, Cambridge.
Bryan Cookson, Esq., M.A.
P. H. Cowell, Esq., M.A., F.R.S.
A. C. D. Crommelin, Esq., B.A.
F. W. Dyson, Esq., M.A., F.R.S., Astronomer Royal for Scotland.
Alfred Fowler, Esq., Assistant Professor of Physics, South Kensington.
J. W. L. Glaisher, Esq., M.A., Sc.D., F.R.S.
J. A. Hardcastle, Esq.
A. R. Hinks, Esq., M.A.
E. B. Knobel, Esq.
E. J. Spitta, Esq.
H. H. Turner, Esq., D.Sc., F.R.S., Savilian Professor of Astronomy, Oxford.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

Vol. LXVII. March 8, 1907. No. 5

H. F. Newall, Esq., M.A., F.R.S., President, in the Chair.

Edward George Bloomfield Barlow, Ditton Lodge, Stourwood Avenue, Bournemouth;
Lieut. F. G. Cooper, R.N.R., H.M.S. "Ocean," 131 Sutton Court, Chiswick, W.; and
Edward Power, F.S.A., F.G.S., 16 Southwell Gardens, S.W., and Watership, Newbury, Berks,

were balloted for and duly elected Fellows of the Society.

One hundred and eight presents were announced as having been received since the last meeting, including, amongst others:—

Carl Friedrich Gauss Werke, Band vii. (presented by Prof. M. Brendel); Report of the Italian Commission on Observations of the total solar Eclipse of 1905 at Alcala Chivert, Spain (presented by the Catania Observatory); J. A. C. Oudemans, Mutual Occultations and Eclipses of the Satellites of Jupiter (presented by Prof. Nijland); Prague Observatory, Astronomische Beobachtungen, 1892–99, nebst Zeichnungen und Studien der Mondoberfläche, [illegible] L. Weinek (presented by the Observatory); Müller und K[illegible] Photometrische Durchmusterung, Generalkatalog (presented [illegible] Potsdam Observatory).

Astrographic Chart of the heavens: 40 charts pres[illegible] the Royal Observatory, Greenwich, and 20 charts b[illegible] Fernando Observatory.

Baxendell's Observations of U Geminorum. Edited by
H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. These observations are published at once for the reason mentioned on p. 119 of this volume, viz.:—the University of Utrecht has offered a prize (open to Dutch astronomers) for a dissertation on this Variable, and there have been requests made for original observations.

Mr Joseph Baxendell has now put into my hands the MS. observations made by his father, which go back to 1836. Inquiry having been made as to this particular star, it was elicited that these valuable observations of variables were for the most part not ledgered, but remained in the original observation books. Of these there are—

(*a*) Three foolscap volumes, 1836–48, 1848–56, and 1856–60,
(*b*) Seven small notebooks, 1861–77,
(*c*) A quarto notebook, 1877–88,

of which (*a*) and (*b*) represent work at Manchester, and (*c*) work at Southport.

2. The material (*b*) and (*c*) has all been now copied out in ledger form (under each star), and the ledgers are deposited in a building different from that containing the original books. The material (*a*) is more difficult to transcribe in ledger form. It is scarcely possible to hand it to anyone who is not familiar with variable star records, and procedure is under consideration. I take this opportunity of saying that I should be very glad of skilled volunteer assistance, at any rate in dealing with the copied ledgers for different stars, and perhaps with these early records also. If any variable star observer has leisure for work of the kind and would communicate with me, I should gratefully accept assistance in making this mass of valuable material ready for publication as soon as possible. Unaided, my work at it must necessarily be slow.

3. The observations of the elder Baxendell (*b* 1815–*d* 1887) divide themselves into two periods:—

1836–1877 at Manchester. "With his friend Mr Robert Worthington, of Crumpsall Old Hall, he erected the Crumpsall Observatory, where the large 13-inch reflector (the speculum of which he had himself cast, ground, and polished) was mounted, beside a small 5-inch equatorial refractor" (*Monthly Notices*, xlviii. p. 157). The former instrument is designated R(13) below, and the latter A. In *M. N.*, xviii. p. 11, the focal length of A is given as 70 inches. Besides these a 12-inch reflector belonging to Mr Williamson was used, and is designated R(12); and other small instruments—a 30-inch achromatic, a 22-inch achromatic, and a Tully telescope. These are called 30(*a*), 22(*a*), and T respectively. There are also B ("Mr Bowman's $7\frac{1}{4}$-inch") and C, a comet seeker of about $2\frac{1}{4}$ inches aperture. Mr Baxendell tells me that 22(*a*) was an excellent $2\frac{3}{4}$-inch by Dancer, who also made A.

1877–1888 at Southport. "In 1871 he was appointed superintendent of a meteorological observatory in Hesketh Park, fitted up and presented by John Fernley, Esq., formerly of Manchester; and in 1877 he erected his own private astronomical observatory in Birkdale, Southport, and resumed his observations of variable stars, etc. with a 6-inch equatorial refractor by Cooke & Sons, assisted for some years (previous to 1888) by his son" (*loc. cit.*, p. 158).

4. A careful note in the quarto MS. book (*c*) above gives a more detailed description of the Southport Observatory, and the following paragraphs concern us:—

"The observatory, the equatorial and micrometer, the portable transit instrument, the sidereal chronometer, and sidereal watch belong to Thomas S. Bazley, Esq., of Hatherop Castle, Fairford, Gloucestershire, who has kindly granted me the use of them so long as I may be able or inclined to make astronomical observations, and has also borne the expense of their removal from Hatherop Castle, and of the re-erection of the observatory, and re-mounting of the equatorial and transit instrument; and I think it a duty to make this record of his unlooked-for kindness and liberality, for which I am most sincerely and deeply grateful.

"The object-glass of the equatorial is 6 inches in diameter, and has a focal length of 87½ inches. There are six negative eyepieces, a comet eyepiece, two reflection sun eyepieces, and three sunshades. The powers of the six negative eyepieces, as determined by Mr Bazley, are 48, 80, 125, 180, 260, and 360.

This instrument is denoted by E below. In 1870 and 1871, before this instrument was erected, he used Mr Gladstone's 7½-inch achromatic.

5. *Comparison Stars.*—The brighter comparison stars *a*, *b*, *c*, etc., are the same as those of Pogson and Knott, except that Knott's star *l* was at first denoted *x* by Baxendell. As, however, there is no doubt about the identification, and as he himself used *l* after 1864, I have (though with some misgivings) altered the notebook *x* into *l* in what follows. He made one or two determinations of the magnitudes of the brighter stars, but not many: probably he accepted these data from Pogson (or Knott?), and no discussion of his separate observations is necessary. But see the letter from Father Hagen, appended to this paper.

6. As regards faint stars, Baxendell made some n
observations in the early years. One instance does n
observations of the variable, but may be given
indication of what he could see. He makes the
(I have written ξ and η for x and y to avoid
another x and y):—

"1858 February 1. Pogson's star *c* is a d
there is a fourth star perhaps sufficiently near
quadruple; ξ about 13¼ mag.; η about 14 mag.
light blue colour."

"1858 February 8. Companions of Pogs
13¼ mag., η about 14 mag."

"1858 April 17. Pogson's star *c* quadruple; A 9·2 mag., B 12, C $13\frac{1}{2}$, D 14; C a light *blue* colour, very striking for so small a star. The magnitudes merely estimated."

There are no other references to these stars except that a diagram is appended on February 1, though without any scale.

7. Now there is some confusion about Pogson's *b* and *c* (see *M. N.*, lxvii. pp. 125–8), Knott using *c* and *b*, where Pogson used apparently *b* and *c*. Assuming Baxendell's identification to be the same as Knott's, *c* is the double star Σ1158; and the position angle and apparent magnitudes given in Mr Lewis's recent catalogue of the Struve stars accord well with the diagram in Baxendell's notebook for the pair marked *c*. This supplies the scale of the diagram approximately, and it can be accordingly read as follows (very roughly):—

			Mags.	
	Pos. Angle.	Dist.	Bax.	Σ.
A	0	0·0	9·2	8·8
Σ Companion or B	(333°)	(7″·5)	12	10·0
η or C	230	9″	$13\frac{1}{4}$	
ξ or D	320	60″	14	

But there is no mention of either ξ or η in *Memoirs R.A.S.*, vol. lvi. p. 226. Perhaps some double star observer may care to look at Σ1158 again. (See note by Mr Lewis at end.)

8. The faint stars in the neighbourhood of the variable, used in comparisons, are given in a diagram on 1858 February 1, which includes Knott's *f*, *g*, *h*, and *l*, so that there is no difficulty in reading it. In the neighbourhood of U there are no less than *five* other stars, two in ink, and three in pencil perhaps added afterwards. One of these is lettered as the variable U, but this identification can only be taken as provisional, as subsequent notes show. The best plan seems to be to give the positions of these stars as below (so that anyone can make an accurate diagram for himself in a few moments if he so desires), and the relevant notes just as they stand in the notebooks. For reading the diagram, the particulars given in Hagen's *Atlas Stell. Var.* have been used. A scale was constructed to fit them as nearly as possible, and the comparison of data with results shows the skill with which Baxendell made such diagrams.

TABLE I.

Knott's Letter.	Hagen's No.	Hagen's Δα	Hagen's Δδ	Baxendell's Δα	Baxendell's Δδ	Baxendell's Letter.
		m s	′	m s	′	
f	18	+0 2	+5·4	+0 0	+5·4	*f*
g	25	+0 18	−0·1	+0 18	+0·5	*g*
h	24	+0 6	+4·4	+0 6	+4·0	*h*
	23	+0 7	−3·8	+0 9	−3·8	*k*
l	39	−0 2	+2·1	−0 1	+2·2	*l*
k	33	+0 4	−2·9	+0 5	−1·5	*w*
				0 0	+0·3	U ; ink.
				+0 2	+0·1	Pencil.
				+0 2	−0·3	Ink.
				+0 4	−0·2	Pencil.
				+0 4	+1·0	Pencil.

9. The notes are as follows :—

1858 February 1. 9^h to $9\frac{3}{4}^h$. Mr Worthington's 13-inch reflector.

h 1·3 > l ·4 or ·5 > U Gem.
h ·7 > w ·7 or ·8 > U ·3 to ·5 > min. vis.

I believe the star which I have marked U will prove to be the variable, now on its march to another maximum; though very small it is distinctly defined, has no haziness about it, and has a dull yellow colour. The above estimations make U = 14·0 mag., w = 13·17, l = 13·62, and the vanishing mag. with 13 inches aperture = 14·4 to 14·5.

1858 February 4. 7 to $9\frac{1}{2}$ h. 13-inch reflector.
h ·7 or ·8 > w ·5 or ·6 > l. h 1·5 > l ·6 > U. w 1·0 or 1·2 > U. U ·4 > a small star y not noticed February 1. U is therefore 14·3 mag., or ·2 or ·3 less than on the 1st inst., and is perhaps therefore not the variable. y, which *is* perhaps the variable, is 14·7 mag.

1858 February 6. 7^h to 8^h. Mr Williamson's 12-inch reflector.
l about ·4 > *min. visible.* Supposed U and y occasionally seen by transient glimpses, and y is perhaps the brightest (say both under 14·2 mag.).

1858 February 7. 8^h to $9\frac{1}{2}^h$. 13-inch reflector.
h ·8 > w ·7 > l ·8 > supposed U (say u) ·2 > y f ·4 > k ·4
∴ u = 14·5 or 14·6 mag., and y = 14·8 mag.

1858 February 8. 8^h to $9\frac{1}{2}^h$. 13-inch reflector.
l ·6 > u ·3 > y a glimpse star near to and following y. u and y = 14·7 mag.

1858 February 17. 9^h to 10^h. Mr Williamson's 12-inch reflector.

h 1·2 > *w* ·3 > *l* ·6 > *u* ·2 or ·3 > *y*. *w* still appears to be decreasing. The light of *u* very changeable, sometimes appearing fully equal to *l*, and at other times quite disappearing. *h* 1·5 > *l*.

Say $u = 14·5$ $y = 14·7$ mag.

1858 February 18. $8\frac{1}{2}^h$ to $9\frac{1}{2}^h$. 13-inch reflector.

w ·4 or ·5 > *l* ·6 > *u* ·3 > *y*. *h* 1·0 > *w*.

Say $u = 14·5$. $y = 14·8$ mag.

u precedes *g* $19^s·0$ and on same parallel; or, if any difference, *u* south, but not to the extent of 5″; observation difficult, as *u* will not bear an illumination sufficient to render the spider lines distinctly visible.

1858 February 19. 9^h to 10^h. 13-inch reflector.

h 1· or 1·2 > *w* ·4 > *l* ·5 > *u* ·2 or ·3 > *y*.

1858 February 20. $8\frac{1}{2}^h$ to $9\frac{1}{2}^h$. 13-inch reflector.

w ·4 > *l* ·5 > *u* ·2 or ·3 > *y*, suspicion of a glimpse star near to and sf star *u*. Fluctuations in the brightness of *u*, which I have frequently observed lately, are very striking to-night.

1858 February 22. 8^h to 9^h. 13-inch reflector. *w* and *l* both seen occasionally pretty steadily; *u* seen two or three times by glimpses, but no change since last observed. Moon too near for a very satisfactory observation.

1858 February 25. 9^h to $9\frac{1}{2}^h$. 13-inch reflector. Owing to the strong moonlight, I can only say that U Gem. is below 13 mag., star *w* glimpsed occasionally.

1858 February 28. $8\frac{1}{2}^h$. 13-inch reflector. *w* and *l* both occasionally seen, but owing to the moonlight, etc.

1858 March 6. 8^h to 10^h. 13-inch reflector.

h ·1 > *g* ·9 or 1·0 > *w* ·3 > *l* ·4 or ·5 > *u* ·2 > *y*. Fluctuations in brightness of *u* noticed again to-night.

1858 March 13. $9\frac{1}{4}^h$. 7-inch reflector, p. 130.

w and *l* occasionally seen pretty steadily by glimpses, and even *u* two or three times seen in transient glimpses, or *u* and *y*, as in the best glimpses *u* had the *cometary* appearance of a very faint coarse double star. The difference of mag. between *w* and *l* cannot be more than ·2.

1858 March 16. 8^h to 10^h. 13-inch reflector, p. 196.

g = or ·1 > *h*: *f* 1·0 to 1·2 > *h*: *w* ·4 > *l* ·4 or ·5 > *u* ·2 > *y*. Occasionally a suspicion of a small companion sf *u* about 15″ distant.

1858 May 1. $9^h\ 25^m$. 13-inch reflector, p. 199.
$u\ \cdot2 > l$. When u first became visible in the twilight l was quite invisible. $9^h\ 40^m\ u\ \cdot2 > l$. 10^h (after interruption by clouds) $l =$ or $> u$: $10^h\ 10^m\ l\ \cdot2 > u$: $w\ \cdot6 > u$.

1858 May 3. $9\frac{1}{2}^h$ to $10\frac{1}{2}^h$. 13-inch reflector, p. 199.
$w\ \cdot4 > l\ \cdot4 > u\ \cdot3 > y$. No indications of any minute star near u, although carefully looked for.

1858 May 4. $9\frac{1}{2}^h$ to $10\frac{1}{2}^h$. 13-inch reflector, pp. 199 and 350.
$w\ \cdot4 > l\ \cdot4$ or $\cdot5 > u\ \cdot3$ or $\cdot4 > y$. Could not satisfy myself of the existence of any minute star near u.

1858 November 12. 12^h. 7-inch reflector, p. 130.
l glimpsed; two or three transient glimpses of u. ?U 14 mag.

1858 November 14. 12^h. 7-inch reflector, p. 40.
U Gem. has suddenly burst forth, and is now = Pogson's $b = 9{\cdot}3$ mag. With power 130 it is much less sharply defined than a, b, or c, and compared with them has somewhat of a nebulous appearance. Its colour is *white*. It appears to be exactly on the place of my star u.

1859 February 27. 9^h. 13-inch reflector, p. 199.
$f\ \cdot8 > \mathrm{U}\ \cdot2 > h$ U $12{\cdot}2$ mag. Neither with achromatic p. 223 nor reflector pp. 199 and 300 is U so well defined as the neighbouring stars; but I think it is not so hazy-looking as it was in November last. y only seen by glimpses, and no star seen or suspected near U.

1859 February 28. 13^h. 13-inch reflector, pp. 199 and 300.
$\mathrm{U}\ \cdot2 > w\ h\ \cdot8 > \mathrm{U}\ \therefore$ U $13{\cdot}1$ mag. No small star visible very near U, and from the position of U with respect to w, x, and y, I can have no doubt that U and Winnecke's star are identical. U *white* or *bluish white*; light steady and haziness still perceptible.

1859 April 22. $9\frac{3}{4}^h$. 13-inch reflector, p. 196.
$l\ \cdot4$ or $\cdot5 > u\ \cdot3$ or $\cdot4 > y$. From the position of u with respect to w, l, and y, I have no doubt it is the variable. It has, however, a ruddy appearance, and is, when best seen, very sharply defined.

1859 November 30. 13-inch reflector, pp. 81 and 196.
u and y both seen. U below $14{\cdot}2$ mag.

This last note is interpreted to mean, "if U is not the same as u, it is below $14{\cdot}2$ mag.," for it seems clear that Baxendell con sidered u to be the variable.

10. These are practically all the notes that help in any w identify the small stars. When, as on 1858 February 28, light or haze interfered, the note has been curtailed, and ge notes that do not help us have been omitted. The compar

these small stars are collected below in Table II. It will be seen that Baxendell observed sometimes to 0·05, taking his unit as approximately 0·1 magnitude, which by comparison with Hagen's magnitudes we see is very nearly correct. Adjusting the differences to fit the four stars g, h, w, l as nearly as possible, we have—

	$25=g$	$24=h$	$33=w$	$39=l$	u	y
Hagen	11·1	11·0	12·1	12·6	...	...
Baxendell . . .	11·05	11·10	12·05	12·45	12·90	13·20

And we may compare with these Baxendell's own absolute determinations for u and y, viz. $u=14·5$ (four nights), $y=14·8$ (four nights); showing that his adopted magnitudes are in excess, as is indicated above in the case of Σ1158.

TABLE II.

Baxendell's Comparisons of Faint Stars.

Date.		g	h	w	l	u	y
1858 Feb.	1	...	7	...	4½	...	
	4	...	7½	5½	6	4	
	7	...	8	7	8	2	
	8	...	...	...	6	3	
	17	...	12	3	6	2½	
	18	...	..	4½	6	3	
	19	...	11	4	5	2½	
	20	...	...	4	5	2½	
Mar.	6	−1	11	3	4½	2	
	13	−1	...	2	...	...	
	16	+½	...	4	4½	2	
Apr.	1	+1	10	2½	4½	4	
	4	+2	9	4	4	5	
	6	+1	...	4½	3	5½	
	10	+1	...	4½	3½	...	
	13	...	...	4½	2½	...	
	17	...	...	3	4	...	
May	3	...	...	4	4	3	
	4	...	...	4	4½	3½	
	5	..	...	4	4	...	
	6	...	...	4	4	...	
	10	...	...	4	3½	3½	
	11	...	...	4	4	...	
1859 Apr.	22	...	...	...	4½	3½	
Mean		+·05	+·95	+·40	+·45	+·30	

11. *The Observations of the Variable.*—The observations are collected in Table III. in a concise form. Baxendell used the symbol > in his notebooks; but as it can always be inferred from the order of the other symbols, it has been omitted. When he gives alternative readings such as ·5 or ·6, the mean has been taken. Finally the decimal points have been omitted. Thus his record

$$h \cdot 7 \text{ or } \cdot 8 > w \cdot 5 \text{ or } \cdot 6 > l$$

becomes

$$h\ 7\tfrac{1}{2}\ w\ 5\tfrac{1}{2}\ l.$$

Other contractions will be readily understood. Thus on 1859 May 20 the inference $< b + 15$ means that the faintest star visible was judged to be a magnitude and a half fainter than b.

The letter u has been retained where Baxendell used it, although he afterwards decided that it was the variable near minimum. It seems advisable to indicate as closely as possible the gradual exploring of the small stars by keeping the notebook records.

12. The early observations are difficult to give in tabular form, and are better simply transcribed as follows:—

1857 February 25. Mr W.'s 5 in. ach. p. 68. Several very small stars about the place of U Gem.; but all much less than the star to the south, about $9\frac{1}{2}$ mag.

1857 March 4. 10^h. With 5 in. achr. Stars not brighter than 11 mag. in the place of U Gem.

1857 March 8. 11^h. With 5 in. ach. p. 68. A star of about 11 mag. in the place of U Gem., and another near it nf 2′ or 3′ distant and about half a mag. less.

1857 March 16. About 8^h. 5 in. ach. p. 68. No star brighter than 11 mag. about the place of U Gem.

1857 March 21. $11\frac{1}{2}^h$. 30 in. ach. Nothing visible in the place of U Gem.

[1857 March 24 is the last date in the book preceding April 13.]

1857 April 13. 30 in. ach. A very minute star occasionally visible about the place of U Gem., and therefore about $10\frac{1}{2}$ mag.

[1857 April 14, 15, 16. Other stars observed, but U Gem. apparently not looked for.]

[1857 April 18. $9\frac{1}{2}^h$. 5 in. ach. A diagram is made of the stars f, h, g and three brighter (Hagen's 18, 24, 25, 22, 21, 19), and the remark made "small companion of f" (that is $h = 11{\cdot}0$) "almost a *min. visible*, and therefore about $12\frac{1}{2}$ mag." Baxendell's magnitudes are too large, as shown elsewhere.]

1857 April 19. 10^h. 30 in. ach. Nothing visible on t place of U. Gem.

[1857 April 20. 10^h. 5 in. ach. p. 68. Sketch of Ap referred to, and Hagen's 18 judged ·1 or ·2 brighter than 21.

13. There is nothing more before October, and the obser from this point may be given in tabular form.

TABLE III.

Date.	d	h h	Telescope and Power.	Observations and Inferences.
1857.				
Oct.	28		T 35	Not seen.
Nov.	18		T 35	,, $<11\cdot0$.
	23		,,	,,
	26	11½	,,	,,
1858.				
Jan.	20	9½	A 68	,,
	21	13	T 35	,,
	25	11½	T 40	,, $<b+10$.
	31	10	R(12)	,, $<13\cdot0$.
Feb.	1	9½	R(13)	*l* 4½ *u* : *w* 7½ *u* 4 limit.
	4	7–9½	R(13)	*l* 6 *u* : *w* 11 *u* 4 *y*.
	6	7–8	R(12)	*u* = *y*.
	7	8–9½	R(13)	*l* 8 *u* 2 *y*.
	8	8–9½	R(13)	*l* 6 *u* 3 *y*.
	17	9–10	R(12)	*l* 6 *u* 2½ *y*.
	18	8½–9½	R(13)	*l* 6 *u* 3 *y*.
	19	9–10	R(13)	*l* 5 *u* 2½ *y*.
	20	8½–9½	R(13)	*l* 5 *u* 2½ *y*.
	22	8–9	R(13)	Not seen $<l$.
	25	9–9½	R(13)	,, $<w$.
	26	8½	R(13)	,, $<w$.
	28	8½	R(13)	,, $<l$.
Mar.	3	9	T 35	,, $<g$.
	6	8–10	R(13)	*l* 4½ *u* 2 *y*.
	11	10	A 68	Not seen $<l$.
	13	9¼	R(7) 130	,, $<l$.
	14	8	A 68	,, $<w$.
	16	8–10	R(13)	*l* 4½ *u* 2 *y*.
	20	8–10	A 40	Not seen $<g$.
	24	8	R(13) 81	,, $<12\cdot6$.
	26	8¼	R(13) 81	,, $<g$.
Apr.	1	8–10	R(13) 196	*l* 4½ *u* 4 *y*.
	4	8½–9½	R(13) 196	*l* 4 *u* 5 *y*.
	6	8½–9½	R(13) 196	*l* 3 *u* 5½ *y*.
	9	9½	R(7) 40	Not seen.
	10	8½–10	R(13) 196	*l* 3½ *u* 4 limit.

Date.	d	h h	Telescope and Power.	Observ… Infe…
1858.				
Apr.	12	9½	A 40	Not se
	13	8½–9½	R(13) 196	*l* 2½ *u*.
	17	9–11	R(13) 199	*l* 4 *u* 3
	21	9¾	R(13) 196	Not see
	23	9½	R(13) 196	,,
	25	9	R(13) 196	Not see
May	1	9½	R(13) 199	See abo
	3	9½–10½	R(13) 350	*l* 4 *u* 3
	4	9½–10½	R(13) 199	*l* 4½ *u* 3
	5	9¾–10½	R(13) 199	*l* 4 *u*.
	6	10–10½	R(13) 199	*l* 4 *u*.
	10	10–10½	R(13) 199	*l* 3½ *u* 3
	11	10–11	R(13) 199	*l* 4 *u*.
	16	10–10½	R(13) 199	Not see
	18	10–10½	R(13) 199	,,
Sept.	20	15	R(13) 196	*u* = *l*.
Nov.	2	…	(22)*a*	Not se
	5	…	(22)*a*. 21	,,
	6	…	(22)*a*. 21	,,
	10	12½	R(7) 130	,,
	11	12½	R(7) 130	,,
	12	12½	R(7) 130	*u* glim
	15	12	R(7) 40	U = *b*.
	16	11	R(7) 40	*b* 1½ U.
	17	11–11¾	R(7) 40, 130	*b* 5½ U
	18	13	R(13) 81	*b* 12 U
	21	12½	R(13) 199	Not se
Dec.	2	9½	R(13) 199	,,
1859.				
Feb.	22	10½	A 68	*a* 2½ U
	23	12¾	(22)*a* 21	*c* 2 U :
	24	7	A 68	*e* = U.
	26	12⅓	(22)*a*	*e* 2 U.
	27	7	A 68	*f* 7 U 3
	27	9	R(13) 199	*f* 8 U 2
	28	13	R(13) 300	*h* 8 U 2

TABLE III.—*continued.*

Date.	Telescope and Power.	Observations and Inferences.
h m		
8½	R(13) 300	*y* seen.
9¾	R(13) 196	*x* 4½ *u* 3½ *y*.
10½	A 40	Not seen.
10	A 40	,, < 11·0.
10	A 40	,, < *b* + 15.
10½	A 40	,, < *b* + 15.
10½	A 40	,, < 10·7.
...	R(13) 196	*u* and *y* seen.
...	A 68	Not seen < 10.
...	R(13) 196	,, < *l*.
...	R(13) 196	*u* and *y* seen.
10	R(13) 199	*l* 5 U.
9	A 68	Not seen < *w* +.
9¼	A 68	,, < 13·8.
8	A 223	,, < 11·0.
8	A 223	,, < 13·0.
10	R(13) 199	U = 14·3.
9¼	A 39	Not seen < 10·3.
9	A 223	,, < 13·7.
10	R(13)	,, < 14·0.
8¾	A 68	,, < 13·0.
10	A 68	,, < 13·0.
9½	A 68	*a* 2 U.
9	A 68	*a* 3½ U 3 *c*.
9	A 68	*a* 2 U 4½ *c*.
9	A 68	*a* 6 U 1 *c* : U 1½ *b*.
9¼	A 68	*c* 1½ U : *b* ½ U.
9	A 68	*c* 1½ U : *b* ½ U.
9	A 68	*b* 2 U : *a* 10 U.
10½	...	*f* 4 U.
9½	R(13) 196	*g* 10 U :
		h 7½ U 2½ *w*.
10 50	C 39	Not seen < 10·0.
13 +	R(7) 39	,, < 12·3.
9 +	A 68	,, < 12·3.

Date.			Telescope and Power.	Observations and Inferences.
1861.	d	h m		
Jan.	6	9 5	A 68	Not seen < 13·0.
	15	10 10	A 68	,, < 13·3.
	28	9 35	A 68	,, < 11·0.
Feb.	6	12 25	C 39	,, < 12·0.
	10	9 5	A 68	,, < 13·3.
	24	7 40	A 68	,, < 11·0.
	28	10 25	C 39	,, < 11·0.
Mar.	8	10 15	C 39	,, < 10·5.
	13	8 35	R(13) 81	*l* 5 U.
	16	10 5	A 68	Not seen < 13·3.
	21	8 30	A 68	,, < 9·5.
	30	8 35	A 68	,, < 13·3.
Apr.	3	9 10	A 68	,, < 13·3.
	6	8 30	A 68	,, < 12·5.
	9	10 40	C 39	,, < 10·5.
	10	10 30	R(13) 196	*l* 4 U.
	11	8 50	A 68	Not seen < 13·3.
	14	9 30	R(13) 81	*l* 4 U.
	20	8 30	A 68	Not seen < 11·0.
	28	10 0	A 68	,, < 13·3.
May	4	10 0	A 68	*a* 2 U 3 *c*; U 3½ *b*.
	9	9 30	A 68	*b* 2 U ; *c* 2 U.
	13	9 30	A 68	Not seen < 10·5.
	14	10 25	A 68	U 4 *f*.
Oct.	28	11 40	,,	Not seen < 12·5.
Dec.	28	9 0	,,	*w* 3½ U = *l*.
1862.				
Jan.	5	9 15	,,	*a* 8 U : *b* 1 U 1 *c*.
	10	9 30	,,	? visible < 10·2
	11	8 15	,,	*f* =
	22	8 15	,,	
Mar.	15	...	,,	
	19	10 0	,,	
	22	9 0	,,	
	31	10 5	,,	
Apr.	8	10 30		
	10	8 25		

TABLE III.—*continued.*

Date.	h m	Telescope and Power.	Observations and Inferences.
1862. d			
Apr. 11	8 50	A 68	Not seen < 10·5.
12	...	,,	Suspected < 11·0.
13	...	,,	Not seen < 12·0.
23	9 15	,,	Glimpsed < 13·1.
28	10 0	,,	?glimpsed < 13·5.
29	10 30	,,	Not seen < 13·3.
30	9 45	,,	Not seen < 13·3.
May 3	9 45	R(13) 81	Barely glimpsed < 13·3.
5	10 30	A 68	Not seen < 12·5.
16	10 10	A 68	,, < 11·0.
Oct. 18	12 50	C 39	,, < 10·5.
Nov. 15	13 10	,,	,, < 10·0.
16	10 10	R(13) 81	,, < 13·5.
23	...	A 68	,, < 13·0.
Dec. 26	9 0	,,	*e* ½ U 6 *f*.
29	11 30	,,	Not seen < 12·2.
30	14 0	,,	,, < 12·2.
1863.			
Jan. 23	7 40	,,	,, < 13·3.
Feb. 8	7 50	,,	Glimpsed ? < 13·3.
12	11 0	,,	Not seen < 13·3.
14	9 0	C 39	,, < 12·0.
15	9 0	A 68	,, < 13·3.
17	...	B 88	,, < 14·0.
21	8 10	A 68	,, < 13·7.
Mar. 3	9 40	,,	,, < 11·0.
15	11 0	,,	,, < 13·3.
21	11 0	,,	,, < 13·0.
23	9 0	,,	,, < 13·3.
26	8 45	,,	,, < 13·0.
Apr. 1	9 30	,,	,, < 10·0.
11	8 40	,,	*b* 2 U 6½ *d* : U 14 *e c* 3 U.
13	9 0	,,	*e* 2½ U 1 *f* : *d* 8½ U.

Date.	h m	Telescope and Power.	Observati[ons and] Infere[nces.]
1863. d			
Apr. 15	9 0	A 68	Not seen <
Nov. 2	12 0	,,	,, <
4	12 20	C 39	,, <
5	12 40	,,	,, <
8	13 30	,,	,, <
Dec. 2	13 0	,,	,, <
5	10 15	A 68	,, <
17	8 30	,,	,, <
27	7 30	,,	*b* 4 U.
29	9 30	C 39	Not seen <
1864.			
Jan. 4	8 30	A 68	,, <
Feb. 12	...	,,	,, <
Mar. 15	9 10	,,	,, <
23	9 40	,,	,, <
28	9 40	,,	,, <
Apr. 11	9 0	,,	,, <
13	...	,,	,, <
18	...	,,	,, <
21	...	,,	,, <
25	...	,,	,, <
28	...	,,	Suspecte[d]
May 7	10 10	,,	Not seen <
Oct. 7	..	R(13)	Glimpsed
31	..	A 68	Not seen
Nov. 5	...	R(13)	Doubtful
28	...	A 68	Not seen
Dec. 21	...	,,	,,
30	7 40	,,	*b* 1 U = *c*
31	11 10	,,	*a* 6 U 3
1865.			
Jan. 4	8 20	,,	*a* 12 U =
6	...	,,	*c* 1½ U.
20	...	,,	Not seen
Mar. 30	...	,,	,,
Apr. 10	...	,,	,,
15	...	,,	,,

TABLE III.—*continued.*

Date.	d	h m	Telescope and Power.	Observations and Inferences.
5-				
	24	10 0	A 68	*f* 9 U 3 *h* : U 1 *g* 12·1.
	25	10 0	,,	*g* 4 U 12·6.
	26	10 0	,,	*k* 1 U 13·4.
	21	...	,,	Not seen < 13·3.
	14	...	,,	,, < 13·3.
	22	...	,,	,, < 12·5.
	26	...	,,	,, < 12·3.
6.				
	15	9 40	,,	,, < 13·3.
	16	10 25	,,	*h* 6½ U 5 *k*, *g* 4 U 12·9.
	18	13 30	C 39	*a* 6 U 2 *c* ; U 3 *b*.
	20	8 15	A 68	*a* 6½ U 2 *b* ; U 1½ *c*.
	22	7 10	,,	*c* 1 U, *b* 1½ U.
	23	11 20	,,	*a* 6 U 1½ *b* ; U 2 *c*.
	28	...	,,	Not seen < 10·3.
	29	...	,,	,, < 10·5.
	2	8 40	,,	*k* 4½ U ; *l* = U 13·7.
-	13	...	,,	Not seen < 13·5.
-	13	...	,,	,, < 13·3.
	21	9 50	,,	*f* 3 U 8 *h*.
	22	10 30	,,	*h* 4½ U.
·	9	...	,,	Not seen < 12½.
57.				
	2	...	,,	,, < 13·3.
	8	...	,,	Suspected < 13·3.
	11	...	,,	Not seen < 13·3.
	16	...	,,	,, < 10·2.
	28	...	,,	,, < 13·4.
	2	..	,,	,, < 13·2.
	20	...	,,	,, < 13·0.
	26	...	,,	,, < 13·0.
r.	21	...	,,	,, < 13·0.
y	5	...	C 39	,, < 10·5.
	14	...	A 68	,, < 12·0.

Date.	d	h m	Telescope and Power.	Observations and Inferences.
1867.				
Nov.	17	...	A 68	Not seen < 13·0.
Dec.	18	8 10	,,	U 2 *f*.
	22	8 50	,,	Not seen < 12·8.
1868.				
Apr.	11	...	,,	,, < 13·3.
	23	...	,,	,, < 13·2.
1869.				
Feb.	5	...	,,	,, < 12·8.
Mar.	2	...	,,	,, < 13·5.
Dec.	23	...	,,	,, < 12·5.
1870.				
Mar.	25	...	G 60	*l* 3½ U.
1871.				
Mar.	13	...	C 39	Not seen.
Apr.	7	...	G	,, < 14·0.
	10	...	,,	,, < 13·7.
1877.				
Dec.	12	...	E 80 & 180	Not seen < 13·5.
1878.				
Jan.	9	...	...	,, < 13·0.
Mar.	24	...	E 80	,, < 13·5.
	31	...	,,	,, < 13·5.
April	1	...	,,	,, < 13·5.
	5	...	,,	Glimpsed 13·7.
	6	...	,,	Barely glimpsed.
	8	...	,,	Not seen < 12·5.
	17	...	,,	,, < 13·0.
May	1	9 40	,,	*f* 6 U 4 *h* ; U 7 *g*.
	3	10 30	,,	Not seen < 13.
Nov.	2	11 50	,,	,, <
	4	5 15	,,	,,
	25	...	,,	,,
	28	...	,,	,,
Dec.	7	...	,,	,,
	23	...	,,	,,
	30	...	,,	,,

TABLE III.—*continued.*

Date.	d	h m	Telescope and Power.	Observations and Inferences.
1879.				
Jan.	13	7 40	E 80	*b* 2 U : *c* 1 U ; *a* 10 U.
		8 0	,,	*b* 1 U = *c*.
	15	6 50	,,	*b* 1½ U ; *c* 1 U *a* 10½ U.
		8 15	,,	*b* 1½ U ; *c* 1 U.
	16	8 20	,,	*b* 3 U ; *c* 2 U 13 *e*.
		8 35 } 9 45 }	...	{ *b* 2½ U ; *c* 1½ U ; *a* 14 U 10 *d*.
	19	7 10	...	U ½ *e* ; U 7½ *f*.
	22	8 30	E 80	*h* 7 U 3 *k*.
		8 40	E 125	*h* 8 U 4 *k*.
	23	9 45	E 80	*k* 3 U 2 *l*.
	25	10 30	...	Not seen < 13·0.
April	10	9 15	...	Not seen < *l* + 2.
Sept.	19	14 30	...	,, < *k* + 3.
Oct.	14	12 50	...	,, < 13·5.
	15	13 0	...	,, < *k* + 3.
	20	13 0	...	,, < *k*.
Nov.	1	12 0	...	,, < *h* + 7.
1880.				
Jan.	18	12 0	...	Glimpsed = 13·5
	19	11 0	...	Not seen < *k* + 3.
	22	12 15	...	,, < 13·0.
	28	7 0	...	*a* 9 U = *b* 1½ *c*.
	29	10 40	...	*b* = U *a* 8 U.
	31	12 20	...	*b* 2½ U 12½ *e* ; *c* 1 U 10½ *d*.
Feb.	1	9 30	...	*b* 1½ U ; *c* ½ U.
	3	7 10	...	*b* 4 U 5 *d* ; U 10 *e*.
	5	9 25	...	*d* 2 U 5 *e*.
	8	9 0	...	*h* ½ U : *g* 7½ U.
	14	...		Not seen < *k*.
Apr.	29	10 0		,, < 13·0.
	30	11 0	.	Glimpsed 13·7.
May	1	9 50	...	Not seen.
	3	10 45	...	Glimpsed : *k* 2½ U.

Date.	d	h m	Observations and Inferences.
1880.			
May	6	10 20	Not seen < 13·0.
	9	...	,, < *g*.
	10	10 40	,, < *k*.
	12	10 35	,, < 13·5.
Oct.	9	12 40	,, < *k*.
	10	13 20	,, < *k*.
Nov.	26	11 30	*k* 5 U ; *l* 2½ U.
	27	12 42	= 13·7.
Dec.	1	10 15	Not seen < *k*.
	24	9 20	*f* 10 U 1½ *g* ; U 4½ *h*.
	25	8 25	U 2 *k* ; U 5 *l*.
1881.			
Apr.	1	8 50	*f* 10 U 2 *h* 1 *g*.
		11 55	U 1½ *f* : U 12½ *h*.
		12 14	U 4 *f* ; U 3 *e* ; *d* 2 U
		12 35	U 5 *f* : *d* 1½ U 3 *e*.
	2	8 15	U 1 *b* : *a* 5 U.
	3	9 5	*a* 5½ U 1½ *b* : U ½ *e*.
	5	8 40	*b* 1 U ; *c* 2 U : *a* 7 U.
	6	10 40	*b* 1½ U ; *c* 2½ U ; *a* 7
	7	9 5	*b* 1 U : *c* 2 U.
	8	8 35	*b* 4 U : *c* 5 U *a* 10 U 1
	9	9 15	*b* 7 U 8 *d* : *a* 13 U : *c*
	17	9 10	Not seen < 13·8.
Oct.	26	...	,, < *k*.
Nov.	14	...	,, < 13·5.
	17	...	,, < *l*.
Dec.	27	...	,, < *k*.
1882.			
Mar.	6	8 15	*f* 5 U 4 *g* : U 8 *h*.
	8	11 20	Not seen < *h*.
	12	12 0	Glimpsed < *l*.
Oct.	28	12 0	Not seen < *k*.
1883.			
Jan.	30	7 45	*a* 5 U = *b*.
1884.			
Jan.	20	...	Not seen < *k*.

TABLE III.—*continued.*

Date.	d	h m	Observations and Inferences.
1884.			
Feb.	2	...	Not seen < k + 2.
May	18	10 30	c 12 U 5 e : d 3 U.
	19	10 20	d 6 U 4½ e : c 17 U.
Dec.	20	12 0	l 3 U.
1885.			
Jan.	5	10 30	e 4½ U 2½ f.
	6	7 35	f 12 U 2 h ; U 3 g.
	7	10 5	h 6 U 7 l : g 5 U 5 k.
	8	...	k 3 U = l.
Mar.	22	...	Not seen < k.
Apr.	2	...	,, < l.
	8	8 40	a 7 U = b ; U 1½ c.
Nov.	15	11 0	Not seen < 12·5.
Dec.	1	10 0	,, < 13·0.
1886.			
Jan.	4	...	,, < 13·5.
	5	10 50	,, < 13·7.
	29	...	,, < k.
	30	...	,, < l.

Date.	d	h m	Observations and Inferences.
1886.			
Jan.	31	...	Not seen < l.
Feb.	14	...	,, < g ; h 5 g.
	23	...	,,
Mar.	3	11 5	k 2 U 2 l.
	6	8 0	b 3½ U ; c 2 U 8 d.
	7	8 30	b 4 U ; c 2 U.
	8	7 55	b 2½ U ; c ½ U 8½ d.
	9	7 50	b 1½ U = c ; U 10 d.
	10	7 45	b 3 U ; c 1 U 10 d.
	11	7 35	b 4½ U : c 3½ U 8 d.
	12	7 50	b 6 U : c 5 U 7 d.
Nov.	30	10 5	b = U 1½ c.
Dec.	1	11 20	b = U 1½ c.
	4	9 5	c 4 U 7 d.
	4	10 15	c 3½ U 7 d ; g 5 h.
1887.			
Feb.	26	8 0	about = f.
	27	8 0	f 7 U 3 h.
	28	9 30	h 10 U 7 k : f 8 h 1 g.

[The only remaining entries in the book are on March 3 (three stars), June 7 (one star), June 10 (five stars). From 1879 April there are no special entries about the telescope, which may be assumed to be E throughout.]

The following letter from Father Hagen explains itself. He draws attention to an unlucky misprint in my paper on p. 128, viz. Pogson's number for *m* in the first column should be, not 113, but 115, as in the diagram. After some consideration I have assumed that Baxendell's *k* was not Knott's *k*, but this star *m* (Hagen's 23): and Baxendell must have [illegible] for Knott's *k* (Hagen's 33). The identification of these f[illegible] stars is [illegible] troublesome.

Letter from Father Hagen on the Comparison Stars for U Geminorum.

My Dear Professor Turner,—Your publication of Pogson's observations of *U Geminorum* in the *Monthly Notices*, vol. lxvii. pp. 119–131, will prove very useful to students interested in this line of astronomy. The publication might perhaps be enhanced in its value by a few statements regarding Pogson's comparison stars. In making a final discussion of Pogson's observations, one will naturally not be satisfied with the identification of the comparison stars from diagrams alone, or with the preliminary magnitudes assigned to them by Pogson and Knott.

The exact identification of all the stars of his chart was made by Pogson himself in a manuscript catalogue now preserved at the Harvard College Observatory. The catalogue was made within the years 1856 to 1860 at the Radcliffe and Hartwell Observatories. On five pages it gives the R.A. and Decl. of all the stars on the chart, reduced to 1860·0. A complete illustration of Pogson's catalogues may be seen in a publication of Georgetown College Observatory, entitled "Supplementary Notes," etc., pp. 28–31. There is no chart of this Variable among the manuscripts preserved at Harvard. It is, however, not necessary to consult these manuscripts, since all of Pogson's comparison stars of *U Geminorum* can be identified by means of chart 2815 of the *Atlas Stellarum Variabilium* (Series II.).

As to the magnitudes of the stars, those of Pogson and Knott have apparently a common scale, and seem to be based on the theoretical limit of visibility in their instruments. The table subjoined gives, in addition, the magnitudes of the fainter stars by Winnecke, and those of all the stars, computed from the grades of the *A.S.V.* The latter two sets of magnitudes have again a common scale, and are both based upon the B.D. system.

The first two columns of the following table explain themselves from page 128 of your article. The third gives the letters and magnitudes of Winnecke, and is taken from the *Astr. Nachr.*, vol. xlvii. No. 1120. The two columns headed A.S.V. contain the numbers and magnitudes of the atlas chart 2815. From the same atlas are taken the columns $\Delta\alpha$ and $\Delta\delta$, which give the positions of the comparison stars relative to the Variable in R.A. and Decl. To them are appended two columns *D*, with the mean results deduced from your readings of the two diagrams. They may serve to show, on the one hand, how accurate the diagrams are, and, on the other, that there is no doubt left in the identifications.*

* In vol. xvii. of the *Astrophysical Journal*, p. 282, I suggested that Pogson's star 8·9 might be No. 5 of the Atlas Chart, instead of No. 1, judging from the B.D. magnitude of this star, and not having seen Pogson's diagram.

Pogson's Comparison Stars for U Geminorum.

Pogson.	Knott.	Winnecke.	A.S.V.		Δα.	D.	Δδ.	D.	Bax. [H.H.T.]
M	M	M		M	m s	s	′	′	
a 8·8	*a* 8·6		1	8·6	+0 4	7	−19·8	19·5	
b 9·2	*c* 9·2		3	8·8	+1 26	28	− 6·9	7·0	
c 9·4	*b* 9·3		4	9·0	+1 19	20	− 9·6	9·0	
d 10·3	*d* 10·3		9	9·6	−1 10	11	+14·5	14·5	
n 10·2			11	9·6	−0 46	46	+16·7	16	
e 10·8	*e* 10·6		16	10·0	−0 27	27	− 9·9	9·5	
f 11·3	*f* 11·2	*f* 10·9	18	10·3	+0 2	1	+ 5·4	6·0	*f*
m 11·5		*g* 10·7	23	10·8	+0 7	4	− 3·8	5	*k*
h 12·3	*h* 12·3	*e* 11·3	24	11·0	+0 6	6	+ 4·4	4·5	*h*
g 11·9	*g* 12·3	*d* 11·4	25	11·1	+0 18	18	− 0·1	+0·5	*g*
l 13·0	*k* 13·3	*b* 12·2	33	12·1	+0 4	5	− 2·9	1·5	*w*
k? 13·7	*l* 13·7	*a* 12·6	39	12·6	−0 2	5	+ 2·1	2	*l*

One word I wish to add about the remarkable fluctuation of light in this Variable as seen by Pogson on March 26, 1856. Attention has been called to this record, in substantially the same words, by Mr J. Baxendell in the *Astronomical Journal*, vol. xxii., 1902, p. 127. In the year following I had occasion to illustrate Pogson's observation of 1856 by four other instances of a similar nature, and to suggest that instantaneous fluctuations in the light of stars, when recorded by good authority, should not be rejected as unconfirmed (see *Astrophysical Journal*, vol. xvii., 1903, pp. 281–285).—Very faithfully yours, J. G. Hagen, S.J.

Specola Vaticana:
January 27, 1907.

Note by Mr Lewis.

The evening of 1907 March 11, on which I received the proof of Professor Turner's paper, being fairly good, the 28-inch refractor was set on Σ 1158 and measures made of the faint stars near. The results are :—

	Magnitudes.	Position.	Distance.
AB,	8·5 and 9·8	332°·0	7″·74 = Σ 1158
AC,	8·5 ,, 12·0	256·0	18·70
AD,	8·5 ,, 12·0	304·7	65·30
AE,	8·5 ,, 13·5	154·2	58·35

The measure of AB is a mean of Mr Eddington's and my own. The measures of C, D, E are by myself.—T. Lewis.

On the Classification of Long-period Variable Stars, and a possible Physical Interpretation. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

Summary.

§ 1. Reference to previous analysis of light-curves of long-period variables, and classification by A, the coefficient of sin θ in a harmonic analysis, counting θ from maximum.

§§ 2–6. Inclusion in the series of 12 curves determined by Mr J. A. Parkhurst, and formation of a set of seven typical light-curves.

§§ 7–13. The maximum calculated from the harmonic analysis does not agree with that assigned by the observer. Is the latter systematically wrong, owing to the fact that the method commonly adopted of bisecting chords is really no guide?

§§ 14–17. General sketch of physical hypothesis. If the variation in light is due to faculæ or flocculi which arise in high latitudes and approach the equator throughout most of the period, and then rather suddenly return to high latitudes, the aspect presented by the star would modify the light-curve, owing to the greater importance of the faculæ near the centre of the disc. If one of the star's poles were towards the observer, the faculæ in high latitudes would be most obvious, the equatorial faculæ being subject to foreshortening and absorption. The minimum would then follow the maximum early. Conversely, if the star's equator were towards us, it would be late.

§§ 18–21. Consideration of the actual case of the Sun, which does not give satisfactory results.

§§ 22–24. Reasons why the case of the stars may be entirely different from that of the Sun, owing to the smallness of the variation in the latter case.

§§ 25–29. Investigation of the average factor for foreshortening for different aspects of the star.

§§ 30–33. Inquiry whether foreshortening alone can explain the range in type of light-curves. The mean latitude of the faculæ must apparently change from nearly 80° to about 5°, which seems too large a range.

§§ 34–35. If we add the effect of absorption near the limb, these limits will be reduced. In default of any information as to the amount of such absorption, we can neither affirm nor deny the adequacy of the hypothesis. But a factor $\cos^2 \zeta$ for the combined effect of absorption and foreshortening, instead of $\cos \zeta$ for foreshortening alone (where ζ is the distance from the centre of the visible disc), would be quite sufficient to make the range in latitude comparable with that on the Sun.

§§ 36–37. Collection of data given by Chandler for $M - m$, the

interval between a maximum and the preceding minimum, and comparison with the data from complete light-curves; tabulation of cases near the ends of the series, which on the above hypothesis should represent stars with (*a*) their poles turned approximately towards us, and (*b*) with their equators turned towards us.

§ 38. Remark that while class (*b*) occur in all galactic latitudes, class (*a*) are absent from the galactic poles. This suggests that the axes of long-period variables are roughly parallel to the galaxy.

§ 39. Recapitulation of the chief points of the hypothesis, and remarks on a limitation not before noticed.

§§ 40–42. Brief consideration of possible inversions of the hypothesis.

(*a*) If the faculæ quite die out at minimum.

(*b*) If the dark spots and not the bright faculæ are the main source of variation.

Neither of these seems so suitable as the original hypothesis.

§ 43. Chandler's periodic terms cannot be due to a periodic change in aspect of the star. Possibly they may arise from the coexistence of several periodicities which vary in amplitude, as Schuster has found for the Sun.

1. In *Memoirs R.A.S.*, vol. lv. p. xcviii (or see also *Monthly Notices*, vol. lxiv. p. 547), the mean light-curves for 19 long-period variables observed at the Rousdon Observatory in the years 1885–1900 are reduced to the same scale of period and of range in magnitude, and analysed harmonically so as to be closely represented by the formula

$$M + A \sin \theta + B \cos \theta + C \sin 2\theta + D \cos 2\theta + E \sin 3\theta + F \cos 3\theta$$

When the results are arranged according to the values of the coefficient A, the other coefficients are found to be in approximate sequence also.

2. When Mr Parkhurst's "Researches in Stellar Photometry, 1894–1906,"* was received, containing 12 new light curves, from observations made chiefly at the Yerkes Observatory, it was natural to examine whether his curves fell into the same sequence, and the answer was sufficiently satisfactory. He has indicated in each case the epoch of maximum, and given diagrams of the smooth curves. From these curves readings were taken at twelve eq distant points, beginning with his assigned maximum. The re were then analysed harmonically, just as in the case of the R variables, and the results are given in the following t second decimal place must not be regarded too serious the reason that it is not always easy to make precis the curves, owing to the steep gradients which so

* Carnegie Institution, October 190

TABLE I.

(Corresponding to Table LIV., *Mem. R.A.S.*, lv. p. xcix.)

Name.	Chandler's Number.	6A	6B	6C	6D	6E	6F	Const.	Divisor.
S Cygni	7220	+0·24	−2·83	−·15	+·02	−·13	−·21	10·2	4·8
W Androm.	787	−0·36	−3·04	−·11	−·40	+·07	−·09	7·5	5·5
R U Herculis	5798	−0·54	−2·53	−·29	−·34	−·34	−·36	8·5	5·3
R Comæ	4315	−0·64	−2·84	+·20	−·56	+·08	+·02	8·8	4·9
Y Cassiop.	8629	−0·90	−2·73	−·09	−·90	+·16	−·28	9·7	4·0
V Androm.	267	−0·91	−2·67	+·25	−·14	−·12	−·12	9·3	4·2
R V Herculis	6100	−1·04	−2·73	+·53	−·03	+·08	−·08	10·5	3·9
S Lyræ	6894	−1·10	−2·55	+·64	−·28	+·02	+·13	10·2	4·6
T Androm.	103	−1·11	−2·73	−·02	−·32	+·02	−·16	8·5	4·3
[The Sun]	...	−1·49	−2·32	+·11	−·70	+·19	−·19	...	...
V Delphini	7458	−1·54	−2·24	+·06	−·64	+·04	−·16	9·7	7·0
S X Cygni	7269	−1·57	−2·58	−·14	−·20	−·02	−·36	9·1	4·2
Z Cassiop.	8518	−1·80	−2·18	+·04	−·54	−·02	−·30	10·6	4·8

3. In the series have been included the values obtained for the Sun (*Monthly Notices*, lxiv. p. 549) on the hypothesis that Wolf's spot numbers can be taken as an indication of light-variability. The spots themselves would imply loss of light; but this is probably more than compensated by the increased brilliance in the accompanying faculæ or flocculi. It is not unreasonable to compare spottedness directly with the light-curves, for the stellar magnitude of the Sun would be proportional to

$$-0{\cdot}4 \log (P+F)$$

where P is the brightness of the ordinary disc, and F the small increment due to faculæ which may be taken as roughly proportional to the observed spottedness.

$$\text{Now} \quad \log (P+F) = \log P + \log (1+F/P)$$
$$= \log P + kF \quad \text{approximately}$$

where k is a constant. Thus F appears in a form appropriate for comparison with stellar magnitudes, since in all cases we subtract a constant (corresponding to log P in this case) and divide by a factor (the factor k). The peculiarity of the Sun is that the subtracted constant is so large. This makes no difference at present, but it is important later (§§ 22–24).

4. A point calling for special remark is that the Sun is now included in the series instead of being outside it as before. The extreme value of A for the Rousdon variables was −1·26; but Mr Parkhurst's series contains three stars with negative values of A greater than this numerically. There is thus no longer any need to extrapolate for the Sun.

To show the accordance between the former series and that provided by Mr Parkhurst, the values of A, B, C, etc. for the Rousdon variables might have been repeated along with the above

values in Table I. But it was considered a better plan to go back to the light-curves and give the actual readings for the twelve standard points, as in Table II. The Rousdon curves are simply reproduced from Table LIII. of p. xcviii, *Mem. R.A.S.*, vol. lv.

TABLE II.

Light-Curves according to Value of A.

Name.	6A	Max.	2	3	4	5	6	7	8	9	10	11	12	
T Cassiop.	+1·23	0	16	40	63	91	99	78	48	33	32	24	10	
T Cephei	+0·75	0	16	39	63	87	102	98	78	52	38	34	12	
S Herculis	+ ·50	0	10	32	54	84	99	94	76	60	47	23	7	
S Boötis	+ ·46	0	7	22	47	76	94	100	87	57	31	14	4	
R Aurigæ	+ ·38	0	13	33	57	78	96	101	79	52	44	39	17	
S Urs. Maj.	+ ·35	0	6	19	44	67	85	98	86	53	25	14	5	
S Cygni	+ ·24	0	14	31	56	77	94	100	92	65	43	31	12	P
R Lyncis	+ ·07	0	8	26	45	66	87	100	90	62	39	27	8	
R Camelop.	+ ·07	0	6	24	46	66	86	100	88	66	40	21	7	
S Cephei	·00	0	9	27	51	75	93	98	86	64	57	35	10	
R Draco.	− ·08	0	7	27	49	75	92	101	97	80	54	24	5	
χ Cygni	− ·31	0	9	26	48	68	87	100	100	80	57	33	9	
W Androm.	− ·36	0	10	29	56	80	95	100	98	85	71	45	13	P
R U Herculis	− ·54	0	9	32	60	74	87	98	100	79	66	58	36	P
S Cassiop.	− ·63	0	10	25	45	66	87	97	97	87	65	44	16	
R Comæ	− ·64	0	6	27	53	69	82	92	100	98	84	47	10	P
R Cygni	− ·77	0	11	26	42	57	73	92	102	92	64	38	11	
T Urs. Maj.	− ·80	0	8	21	41	61	77	93	101	94	69	35	8	
T Draco.	− ·88	0	8	21	38	54	71	87	98	89	65	38	12	
Y Cassiop.	− ·90	0	12	35	62	78	90	98	100	100	92	73	18	P
V Androm.	− ·91	0	5	21	38	55	74	93	100	88	64	40	17	P
R V Herculis	−1·04	0	8	18	28	46	69	95	100	90	67	31	8	P
S Lyræ	−1·10	0	7	20	31	46	63	81	96	100	65	33	7	P
T Androm.	−1·11	0	7	21	44	63	84	98	100	96	79	[illegible]	[illegible]	
R Urs. Maj.	−1·11	0	8	22	39	59	78	92	100	98	[illegible]	[illegible]	[illegible]	
S Coronæ	−1·15	0	5	17	32	47	62	77	91	[illegible]	[illegible]	[illegible]	[illegible]	
R Cassiop.	−1·23	0	13	30	42	47	71	93	101	[illegible]	[illegible]	[illegible]	[illegible]	
U Orionis	−1·26	0	8	24	43	63	78	92	100	[illegible]	[illegible]	[illegible]	[illegible]	
[The Sun]	−1·49	0	9	23	38	54	71	84	94	[illegible]	[illegible]	[illegible]	[illegible]	
V Delphini	−1·54	0	9	23	37	54	69	83	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
S X Cygni	−1·57	0	5	17	31	50	79	95	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
Z Cassiop.	−1·80	0	8	21	33	48	67	85	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	

The unit for the table is 0·01, in order to avoid the use of decimal points. The curves due to Mr Parkhurst are indicated by the letter P in the last column.

5. Glancing down the columns, there are considerable irregularities, which may be too great to be ignored; but the accidental errors of light-estimations are large, errors of half a magnitude being far from impossible. [The average range in magnitude of these variables is about 4; and $0·5/4 = ·13$; so that half a magnitude means 13 units in the table.]

If we may regard the errors as chiefly accidental, we may estimate mean values by grouping the stars as follows:—

TABLE III.

Estimated Mean Ordinates for Seven Groups.

Group.	6A.	Max.	2	3	4	5	6	7	8	9	10	11	12
I.	+1·0	0	13	38	61	87	100	94	70	40	30	17	10
II.	+0·5	0	12	32	54	78	97	98	83	56	39	26	11
III.	0·0	0	11	28	48	70	90	100	93	72	52	34	12
IV.	−0·5	0	10	26	44	63	82	98	100	86	65	42	14
V.	−1·0	0	9	24	40	57	76	93	100	94	77	53	20
VI.	−1·5	0	8	22	36	52	70	87	98	98	89	64	28
VII.	−2·0	0	7	20	32	48	65	80	94	99	98	75	38

6. We may now submit these mean curves to analysis and obtain the following coefficients.

TABLE IV.

Coefficients of Harmonic Terms for Curves of Table III.

Group.	6A	6B	6C	6D	6E	6F	$6A_4$	$6B_4$
I.	+1·07	−2·57	−0·46	+0·08	+0·02	−0·22	−0·01	+0·14
II.	+0·46	−2·62	−0·25	+0·10	0·00	−0·22	+0·03	+0·07
III.	−0·12	−2·79	−0·02	0·00	0·00	−0·20	+0·02	+0·04
IV.	−0·66	−2·75	+0·18	−0·16	0·00	−0·17	+0·07	+0·04
V.	−1·12	−2·58	+0·18	−0·36	+0·02	−0·19	+0·05	+0·06
VI.	−1·53	−2·33	+0·11	−0·54	+0·05	−0·23	+0·04	+0·08
VII.	−1·88	−2·05	−0·05	−0·69	+0·06	−0·38	+0·04	+0·13

The coefficients of the fourth harmonic, $A_4 \sin 4\theta + B_4 \cos 4\theta$, have also been calculated as a general check. By retaining the factor 6 (which occurs in the arithmetic), the range in magnitude is represented as 6, a larger range than usually occurs. Hence the fourth harmonic seldom makes a difference as large as 0·1 magnitude, and we may not unreasonably neglect it.

7. A curious point now arises. The angle θ in the expression

$$y = A \sin \theta + B \cos \theta + C \sin 2\theta + D \cos 2\theta + E \sin 3\theta + F \cos 3\theta$$

has been reckoned from a maximum, and hence we should have y a maximum when $\theta = 0$; the condition for which is

$$A + 2C + 3E = 0$$

But this condition is not fulfilled: in group VII., for instance, we have

$$6(A + 2C + 3E) = -1{\cdot}88 - 0{\cdot}10 + 0{\cdot}18 = -1{\cdot}80$$

It was at first supposed that there was some mistake in calculation; and to test this, the curve was recalculated (C) from its formula, excluding the fourth harmonic, for comparison with the assumed 12 ordinates (O) as below

	Max.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
(C)	3	4	22	34	45	66	84	94	98	99	77	35
(O)	0	7	20	32	48	65	80	94	99	98	75	38
$(o-c)$	−3	+3	−2	−2	+3	−1	−4	0	+1	−1	−2	+3

There seems no reason to doubt the computations; but the observed and computed curves clearly do not fit closely at maximum. They intersect three times, and it would require a harmonic of a high order to obtain a good fit.

8. There is no reason why there should not be a harmonic of a high order; but neither is there any good reason why there should be. The necessity for it may be avoided by a simple supposition, but as yet we cannot say whether that supposition is true or false;

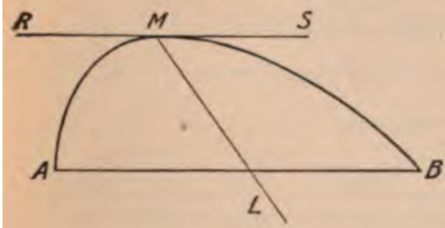

Fig. 1.

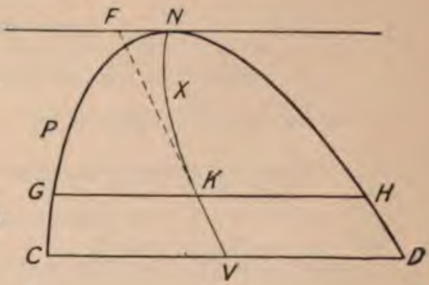

Fig. 2.

that can only be determined by a much more accurate knowledge of the shape of light-curves *close to maximum*. The supposition is that observers may have made a systematic error in assigning the epoch of maximum in cases where the rise and fall are not equally sharp. It is a common practice to assign this epoch by drawing the curve which bisects the chords joining equal magnitudes on the rise and fall; and it is now submitted that this practice cannot be defended, and may even be quite wrong. The reason will be clear from the above diagrams.

In fig. 1 is a portion of a parabola touching R M S at the point M. The locus of the middle points of parallel chords such as A B is, as we know, the straight line L M cutting the curve at the required point M. But it is quite easy to draw a curve, as in fig. 2, where the locus of these middle points is a curved line K X N, and where the portion V K produced would carry us towards a point F away from the true maximum. To realise this we have only to suppose the curve V K N drawn first along with one side C P N of the curve. We can then construct the other side by producing G K and similar lines to equal distances on the other side of V K N, when we shall get a smooth curve N H D.

9. We might have drawn V K X N much more curved, and still it might be made a locus of middle points of chords. The argument is of some importance, and may be put into an analytical form for comparison with our formulæ. We have adopted the formula

$$y = \mathrm{A}\sin\theta + \mathrm{B}\cos\theta + \mathrm{C}\sin 2\theta + \mathrm{D}\cos 2\theta + \mathrm{E}\sin 3\theta + \mathrm{F}\cos 3\theta \quad (1)$$

and near $\theta = 0$ this may be put into the form

$$y = \text{const} + (\mathrm{A} + 2\mathrm{C} + 3\mathrm{E})\theta - \tfrac{1}{2}(\mathrm{B} + 4\mathrm{D} + 9\mathrm{F})\theta^2 - \tfrac{1}{6}(\mathrm{A} + 8\mathrm{C} + 27\mathrm{E})\theta^3 + \text{etc.} \quad (2)$$

If the origin is a point of maximum, the constant is zero, and further

$$\mathrm{A} + 2\mathrm{C} + 3\mathrm{E} = 0 \; . \quad . \quad . \quad . \quad (3)$$

Thus the curve near the origin takes the form

$$y = b\theta^2 - a\theta^3 \; . \quad . \quad . \quad . \quad (4)$$

where $a = \tfrac{1}{6}(\mathrm{A} + 8\mathrm{C} + 27\mathrm{E}) = \mathrm{C} + 4\mathrm{E} \quad . \quad . \quad (5)$

in virtue of the relation (3).

We may put this in the alternative form

$$b^2\theta^2 - ay\theta - by = 0 \quad . \quad . \quad . \quad (6)$$

which has the advantage of giving only two values of θ for any assigned value of y, excluding a third value of θ given by equation (4), at a finite distance from the origin and irrelevant. Now if these two values of θ be θ_1 and θ_2, the co-ordinates of the middle point of the chord joining them are (x, y), where

$$x = \tfrac{1}{2}(\theta_1 + \theta_2) = ay/b^2 \quad . \quad . \quad . \quad (7)$$

and the equation to the locus of middle points of chords near the origin is thus

$$y = \frac{b^2}{a}x = \frac{(\mathrm{B} + 4\mathrm{D} + 9\mathrm{F} + \ldots)^2}{4(\mathrm{C} + 4\mathrm{E} + \ldots)}x \quad . \quad . \quad (8)$$

If there were only the first harmonic, all the coefficients C, D, E, F except A and B would be zero, and the coefficient of x in equation (8) would be infinite, so that the locus of middle points would cut the axis of x at right angles. If we take in the next harmonic, then since C and D are small compared with B, the coefficient of x is still very large, and the direction of the locus is *nearly* at right angles. But when we take in E and F, and more and more as we take in harmonics of higher order, they are multiplied by factors which increase rapidly and indefinitely; so that the sign and the magnitude of the coefficient of x in equation (8) are ultimately controlled entirely by harmonics of a high order, however small their coefficients may be. We cannot therefore assert *anything* about the ultimate direction of the locus unless we know from independent considerations what is the character of these harmonics of high order. The locus of middle points is not exactly a bad guide to the maximum (or minimum); it is simply no guide at all. The epochs should not be assigned in this way; * and if they have been so assigned in the past, they may be systematically wrong.

10. On the other hand, there is no way of ascertaining the error except by closer and more continued observations near maximum. The amount of possible error may be illustrated by determining the maxima in another way, viz. from the assumed formulæ. It is at least equally good (or bad) with the former, and a comparison of the two methods will show the kind of error to which either is liable.

11. It will suffice to give an approximate correction. In the neighbourhood of the origin we have

$$y=(A+2C+3E)\theta-\tfrac{1}{2}(B+4D+9F)\theta^2$$
$$=\alpha\theta-\tfrac{1}{2}\beta\theta^2$$

Let y be a maximum when $\theta=\epsilon$. Then

$$\alpha-\beta\epsilon=0$$

The values of α, β, and ϵ are given below in Table V. The deduced values of ϵ do not fit in very well with the hypothesis of a systematic error in estimating, for they are not symmetrical on opposite sides of group III.; in other words a greater error is made when the rise is sharp than when the fall is sharp, the reason for which is not apparent. But the phenomenon is of a progressive nature.

* Further consideration has led to the view that the characteristics of the curves must be determined in a totally different way, by calculating areas and centres of gravity and moments. But this discussion is reserved for a future paper.

TABLE V.

Calculation of Correction to Epoch of Maximum.

Group.	6α	6β	Ratio.	ϵ
I.	+0·21	−4·23	−0·050	−2°·9
II.	−0·04	−4·20	+0·009	+0·5
III.	−0·16	−4·59	+0·035	+2·0
IV.	−0·30	−4·92	+0·061	+3·5
V.	−0·70	−5·73	+0·122	+7·0
VI.	−1·16	−6·56	+0·177	+10·1
VII.	−1·80	−8·23	+0·219	+12·5
The Sun	−0·68	−6·83	+0·100	+5·7

12. When we alter the epoch of maximum by ϵ, the terms in 2θ are altered by 2ϵ and those in 3θ by 3ϵ. The new values of the coefficients are given in Table VI., which, on the hypothesis now made, should replace Table IV. The old value of 6A is inserted in the second column to show the change.

TABLE VI.

Corrected Coefficients of the Harmonic Terms.

Group.	6A	$6A_0$	$6B_0$	$6C_0$	$6D_0$	$6E_0$	$6F_0$
I.	(+1·07)	+0·96	−2·62	−0·45	+0·13	−0·01	−0·22
II.	(+0·46)	+0·49	−2·62	−0·25	+0·09	+0·01	−0·22
III.	(−0·12)	−0·02	−2·79	−0·02	0·00	+0·02	−0·20
IV.	(−0·66)	−0·49	−2·79	+0·20	−0·14	+0·03	−0·17
V.	(−1·12)	−0·80	−2·69	+0·26	−0·31	+0·08	−0·17
VI.	(−1·53)	−1·09	−2·57	+0·28	−0·47	+0·16	−0·17
The Sun	(−1·49)	−1·25	−2·44	+0·24	−0·71	+0·25	−0·13
VII.	(−1·88)	−1·39	−2·41	+0·28	−0·64	+0·28	−0·26

13. What is true of the groups is, of course, true of the determinations for individual stars; and if we return to Table II. and calculate the epochs of maximum from the coefficients given in Table I. (or for the Rousdon stars in *Memoirs R.A.S.*, vol. lv. p. xcix), we shall find different values for A. Those which follow the general law above noticed call for no further remark; but there are some which go the other way, so that their places in the series are considerably changed. The most notable cases are—

	ϵ	Old A	New A_0
S Urs. Maj.	+10°	+0·35	+0·84
R U Herculis	+17°	−0·54	+0·30
R V Herculis	− 4°	−1·04	−1·24
S Lyræ	− 5°	−1·10	−1·37
R Urs. Maj.	− 5°	−1·11	−1·38
S Coronæ	−10°	−1·15	−1·66

It was found, however, that nothing essentially new was added by making the individual corrections; and as it is by no means certain that they are justifiable, the corrected table need not be given.

14. The question now arises—Is there any physical reason for this gradation or sequence in the light-curves? The facts suggest a light-curve of essentially uniform type, modified by some secondary cause. The place of the Sun in the series suggests the consideration of solar phenomena as a possible type for all these variables. Is there, then, any way in which these phenomena would be slightly modified in the case of stars scattered at random?

15. One such source of modification is certainly the *difference in aspect*. We view the Sun from the direction of his equator, whereas the stars must present all aspects to us from pole to equator. What would happen to the Sun's light-curve (as represented by the spot-curve) if we viewed the Sun from the direction of one of its poles?

16. The general consequence is easily seen. Spots in high latitudes would always be near the centre of the apparent disc, while those near the equator would be considerably foreshortened. [The same would be the case with faculæ: it need not cause any misunderstanding if we speak of the more familiar spots throughout.] They would also suffer absorption near the limb; but for the present we will omit this fact, and think only of the foreshortening.

Beginning with the epoch of maximum, the subsequent spots, being in lower latitudes, would be more foreshortened and diminished in value, which would make the fall to minimum more rapid. Directly a few small spots appeared in high latitudes, their enhanced value would start the rise to maximum. But as the spots increased in number and size their latitude would diminish; they would leave the centre of the disc and become foreshortened. This would to some extent compensate the increase in number, and render the rise to maximum less steep.

17. The general effect above described is exactly that which would carry the Sun towards the other end of the series of light-curves. At present the fall is slow and the rise rapid; if we viewed the same phenomena from the direction of the Sun's pole, the fall would be more rapid and the rise slower. There remain the questions—What is the amount of the change? and how is the general character of the curve affected?

18. We will first consider the actual case of the Sun, though, for reasons which will presently appear, it is not a satisfactory example of the stars. The Greenwich Observations give the following values for the "mean distance from equator of all spots" in the years 1883–1904, arranged in two periods of eleven years.

1st Cycle	13°·1	11°·3	11°·8	10°·4	8°·4	7°·4	11°·6	22°·0	20°·3	18°·4
2nd Cycle	14·2	13·5	14·3	8·0	10·5	9·5	7·7	10·4	17·6	19·0
Mean	13·6	12·4	13·1	9·2	9·4	8·4	9·7	16·2	19·0	[illegible]

For the same years the "corrected areas" are given as follows:—

1st Cycle	1155	1079	811	381	179	89	78	99	569	1214	1464
2nd Cycle	1282	974	543	514	375	111	75	29	62	340	488
Mean .	1219	1027	677	448	277	100	77	64	316	777	976

The factor for foreshortening when the spots are supposed to be viewed from the pole is sin λ, which is very nearly proportional to λ, and may be so taken for the present purpose. Multiplying the two rows of means we get to 3 significant figures,

166 127 89 41 26 8 7 10 60 149 152

To treat this as we have treated the light-curves we subtract 166 and divide by 159, reversing the signs so that maximum is represented by zero and minimum by + 1·00, obtaining therefore

00 24 48 79 88 99 100 98 67 11 9

19. To compare with this we must take the "projected areas" as we see them from the direction of the equator. As given in the Greenwich volumes for the same years these are

1st Cycle	1595	1478	1122	527	243	125	103	133	745	1596	1983
2nd Cycle	1728	1330	745	695	532	159	101	41	86	434	653
Mean .	1662	1404	933	611	388	142	102	87	416	1015	1318

and treating these in the same way we get (after subtracting 1662 and dividing by 1575)

0 16 46 67 81 96 99 100 79 41 22

20. There are only eleven terms in each case instead of twelve. We might draw a curve and read off twelve points for analysis as before; but it seems better to take actual observations untouched, and the above numbers have been analysed on the assumption that the period is exactly eleven years. The results are as below:—

	6A	6B	6C	6D	6E	6F
From Pole . .	+0·50	−3·06	+0·53	−0·26	−0·14	+0·28
From Equator .	−0·28	−2·82	+0·24	−0·47	−0·10	−0·04

21. There is thus a change of sign in A, and at first sight it would appear that we have obtained an effect of the kind required. But no attempt was made to assign the maximum accurately, and this is still to be done by the method given above, the values of $A + 2C + 3E$ being different from zero in both cases. The value for the second curve is small, and we find $\epsilon = + 1^\circ\!\cdot 2$. For the first, $A + 2C + 3E$ is so large that the approximate method for finding ϵ will not work, and we must find it by trial. The value $\epsilon = 27^\circ$ is near the truth; and on substituting it the value of 6A is changed from $+0·50$ to $-0·95$, which *is on the other side* of the

value for equator view. Indeed a rough sketch of the two curves without any analysis shows that they are not very different in type; and that although the spot minimum of the curve has been shifted, the maximum has been shifted even *more* in the same direction; so that the curve, instead of being changed from a "late" to an "early" type, is "later" than ever. The fact is that the peculiarity of the Sun already referred to in § 3 makes it for this purpose an unsuitable representative of variable stars.

22. Let us consider what are the conditions determining the shift of a maximum or minimum. In the neighbourhood of such a point, for which $\theta = \theta_0$ say, the ordinates are of the form

$$y_1 = \mathrm{K}(\theta - \theta_0)^2 + \text{higher powers}$$

If to these we add ordinates of the form

$$y_2 = \mathrm{G}(\theta - \theta_0) + \ . \ . \ . \ .$$

we shift the position of the maximum or minimum. The condition for it is

$$0 = \frac{dy_1}{d\theta} + \frac{dy_2}{d\theta} = 2\mathrm{K}(\theta - \theta_0) + \mathrm{G}$$

and the shift is thus G/2K provided G is small compared with K. The quantity G is the gradient of the second curve, and K represents the curvature of the first. The shift can be increased then by increasing the gradient of the second curve, *or* by diminishing the curvature of the first. Finally, if curvature and gradient be both reversed in direction, the shift remains unaltered in direction.

23. To apply this to the example just given, we must first take logarithms of the quantities used in order to substitute the addition of two ordinates for their multiplication. Let S be the spotted area and λ the spot latitude: we formed the product

$$x = \mathrm{S}.\lambda$$

Taking logarithms we get

$$y = \log x = \log \mathrm{S} + \log \lambda = y_1 + y_2 \quad \text{say}$$

and we are concerned with the alteration of the maxima and minima of y_1 by the gradient of y_2. Now the gradient of y_2 is small near a maximum of y_1, but sharp near a minimum, when the spots quickly cross into high latitudes. Thus in § 18 we see that the mean latitude changes near minimum from $9^\circ\!\cdot\!7$ to $19^\circ\!\cdot\!0$ in two years, the change in the reverse direction requiring seven or eight years. The gradient at minimum being thus much sharper, we might expect the shift of the minimum to be much greater than that of the maximum; and in the case of the stars we shall presently see that this probably happens. But in the particular case of the Sun the sharper gradient in y_2 is more than compensated by the greater curvature in y_1. This does not appear

in the spot numbers themselves; but when we take logarithms we get the following quantities, omitting a constant, viz.—

1·31 1·23 1·05 0·87 0·67 0·22 0·12 ·03 0·73 1·11 1·21

which represent a curve with a curvature at minimum much sharper than at maximum, which is not the case with the magnitude curves of long-period variables. This difference between the Sun and the stars is important for our purpose. It is the sun-spot numbers themselves, and not their logarithms, that resemble the magnitude curves of variables; and it was remarked above (§ 3) that they provide the proper analogy when the variation in brightness is small.

24. Another way of regarding the matter is to think of actual brightness curves for the stars. A rise of 5 magnitudes, for instance, means that the brightness at maximum is 100 times that at minimum. If we drew the curve with the ordinate at maximum 5 inches high, the ordinates near minimum would be very small. The curve would be very flat near minimum, and shoot up into a sudden high peak near maximum with, of course, a sharp curvature. For the Sun this high peak is evanescent; or, if we magnify it to make it noticeable, the zero line of our light-curve recedes to a very great distance.

25. In considering the effects of foreshortening for the stars, we may deal with either the light-curves or the magnitude-curves. In the former case we multiply the ordinates by factors; in the latter we add other ordinates to them, representing the logarithms of the factors. But we must remember in either case that it is the faculæ only which are supposed foreshortened. Let P be the total brightness of the photosphere, F of the faculæ, and B of the two together, and let s be the factor for foreshortening. Then

$$B = sF + P$$

If we have two different aspects of the star for which the values of s are s_1 and s_2 respectively, then

$$\log B_1 - \log B_2 = \log (s_1F + P) - \log (s_2F + P)$$

Now, near maximum P is very small compared with sF, and we may neglect it. Thus

$$\log B_1 - \log B_2 = \log s_1 - \log s_2$$

26. Consider now the case of minimum. If the faculæ die out altogether, so that $F = 0$, then

$$\log B_1 - \log B_2 = 0$$

The minimum for both cases is then the same, as is indeed obvious.

If the faculæ do not entirely die out,

$$\log B_1 - \log B_2 = \log (s_1 + p) - \log (s_2 + p)$$

where p is written for P/F, and is positive. This difference is always less than $\log s_1 - \log s_2$, but approaches it as p diminishes, *i.e.* as the faculæ at minimum are brighter compared with the photosphere. It is not by any means impossible that if the immense increase in brilliancy of these stars is due to faculæ, then even at minimum there may be faculæ whose brightness largely exceeds that of the photosphere, so that we might write without sensible error

$$\log B_1 - \log B_2 = \log s_1 - \log s_2$$

throughout the whole light curve. For simplicity we shall consider this case first: the requisite modification for other cases can be considered later (§ 40). But we may again remark how the case of the Sun differs from that of the stars. For the Sun p is always a large quantity even at maximum; and a different formula is appropriate, viz.—

$$B_1 - B_2 = k(s_1 - s_2)F$$

27. We must now consider the factor s. When the Sun's or star's axis is tilted, the visibility of a spot is affected in two ways.

(*a.*) *Length of visible path.*—If the pole is towards us we see the whole path of a circumpolar spot; if the equator is towards us, only half of it. But if the spots are symmetrically distributed in both hemispheres, then, taking a pair of equal latitudes together, we always see just half the length of path of the spots in them. When a pole is towards us, we see the whole path in one hemisphere, none in the other; when the equator is towards us, we see just half of each. Hence the mean length of visible path is not altered by tilt of the axis.

(*b.*) *Foreshortening.*—Let λ be the latitude of a spot, ϵ the distance of the pole (P) of the star from the centre of the disc (Z, fig. 3). Then, denoting the angle SPH by Z, the angular distance ζ of the spot S from the centre of the disc Z is given by $\cos\zeta = \cos\epsilon \sin\lambda + \sin\epsilon \cos\lambda \cos Z$. The foreshortening factor is just this $\cos\zeta$, and the mean factor for the spot may be taken as

$$\int \cos\zeta \, dZ = [Z \cos\epsilon \sin\lambda + \sin\epsilon \cos\lambda \sin Z]$$

the expression in brackets being taken within limits, say from $-a$ to $+a$. Now if we take a pair of latitudes ($+\lambda$ and $-\lambda$) together, the limits for $-\lambda$ will be $-(\pi - a)$ to $+(\pi - a)$; and the total effectiveness will thus be

$$s = 2(2a - \pi) \cos\epsilon \sin\lambda \qquad \lambda \sin a$$

a being given by the equation

$$\cos a = -$$

It will be more convenient to pu use the angle β. The equatio[illegible] ome

$$\sin\beta =$$

and $$s = 4\beta \cos \epsilon . \sin \lambda + 4 \cos \beta \sin \epsilon . \cos \lambda$$

in which β is to be put equal to $\frac{\pi}{2}$ for all values of λ greater than ϵ.

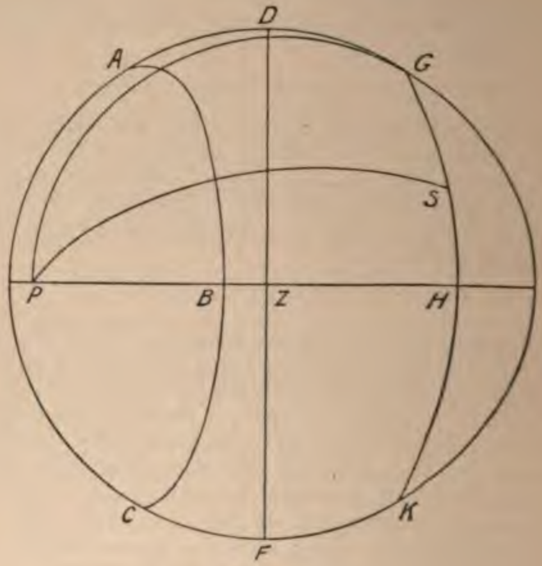

FIG. 3.

28. The extreme cases are simple :—

Equatorial View. Here $\epsilon = 90^\circ, \beta = 0^\circ$, and

$$s = 4 \cos \lambda$$
$$\log s = \log 4 + \log \cos \lambda$$

The log 4 may be disregarded as a constant addition to the light-curve. The term log cos λ can theoretically have any value from 0 to $-\infty$; but λ in practice is to be taken as a *mean* latitude, and is not likely to exceed (say) 70° ; for which log cos $\lambda = 9{\cdot}53$. The range in log s is thus not more than $0{\cdot}5$. Note that in this case log s *decreases* as λ *increases.*

Polar View. Here $\epsilon = 0^\circ, \beta = 90^\circ$, and

$$s = 2\pi \sin \lambda$$
$$\log s = \log 2\pi + \log \sin \lambda$$

The range in log sin λ is again theoretically infinite ; but the mean latitude of the spots is not likely to approach nearer the equator than say 5°, for which log sin $\lambda = 8{\cdot}94$. The range in log s is therefore little greater than unity, and it *increases* as λ increases.

29. *Intermediate Cases.* For other values of ϵ the formula is more complex, but the change in log s is intermediate between the above extreme cases. The general character of it can be readily seen as follows :—

$$\text{Put} \quad \beta \cot \gamma = \tan \epsilon \cos \beta$$
$$\text{then} \quad s = 4 \cos \epsilon . \beta \operatorname{cosec} \gamma . \cos (\lambda - \gamma)$$
$$\text{When} \quad \lambda = 0, \beta = 0 \text{ and } \gamma = 0$$

When λ is small

$$\beta = \lambda \cot \epsilon, \ \gamma = \lambda \cot^2 \epsilon, \ s = 4 \sin \epsilon \cos k\lambda, \quad \text{where} \quad k = \cot^2 \epsilon - 1$$

$$\text{And when} \quad \lambda = \epsilon, \beta = \frac{\pi}{2} \text{ and } \gamma = \frac{\pi}{2}$$

and these latter values of β and γ are to be retained for all values of λ greater than ϵ.

Hence when λ is near 90°

$$s = 2\pi \cos \epsilon \sin \lambda$$

The case when $\epsilon = 45°$ will perhaps sufficiently illustrate the nature of the variation in log s. The values of β and γ are given below in degrees, but this need cause no misunderstanding.

Table VIII.

Reduction Factors when $\epsilon = 45°$.

λ °	β °	γ °	$\gamma - \lambda$ °	$\log \cos (\gamma - \lambda)$	$\log \beta \operatorname{cosec} \gamma$	$\log s =$ Sum.
0	0·0	0·0	0·0	0·00	0·00	0·00
10	10·1	10·1	0·1	0·00	0·00	0·00
20	21·3	21·9	1·9	0·00	0·00	0·00
30	35·2	37·0	7·0	0·00	0·01	0·01
35	44·4	47·3	14·3	9·99	0·02	0·01
40	57·0	61·3	21·3	9·97	0·06	0·03
45	90·0	90·0	45·0	9·85	0·20	0·05
50	90·0	90·0	40·0	9·88	0·20	0·08
60	90·0	90·0	30·0	9·94	0·20	0·14
70	90·0	90·0	20·0	9·97	0·20	0·17
80	90·0	90·0	10·0	9·99	0·20	0·19
90	90·0	90·0	0·0	0·00	0·20	0·20

It will be seen how rapidly β and γ change near the $\lambda = \epsilon$, introducing rapid changes in log cos $(\gamma - \lambda)$ and in cosec γ. But these are in opposite directions and practica pensate one another, so that the variation in log s (sho last column with a constant omitted) is small. This is might expect in the case midway between $\epsilon = 90°$, wh decreases, and $\epsilon = 0°$, when it increases as λ increases.

30. We return to the extreme cases, and to the inquiry whether the available range is sufficient to convert one extreme type of magnitude-curve into the other. The conversion is made by the equation

$$\begin{aligned}\log B_1 - \log B_2 &= \log s_1 - \log s_2 + \text{const.}\\ &= \log\tan\lambda + \text{const.}\\ &= \log\tan\lambda - \log\tan\lambda_0\end{aligned}$$

where λ_0 is the latitude common to the two curves.

31. Referring now to Table III., let B_1 refer to group VII. and B_2 to group I.; and let us inquire what values of λ will convert one of these curves into the other.

First superpose the maxima.

The table is arranged on this plan, the assumed maxima falling under one another. Subtracting the twelve ordinates we get for I.–VII.

$$\cdot 00 + \cdot 06 + \cdot 18 + \cdot 29 + \cdot 39 + \cdot 35 + \cdot 14 - \cdot 24 - \cdot 59 - \cdot 68 - \cdot 58 - \cdot 28$$

These, however, are not differences of logarithms to base 10. We must multiply by about 4 to restore the range in magnitude, and by 0·4 the base of the magnitude scale, both factors together making 1·6. We must also reverse the signs, since increasing magnitudes mean decreasing brightness. Finally we must add the constant log tan λ_0; and since λ_0 is unknown, three suppositions are made, viz. $\lambda_0 = 10^\circ$, $\lambda_0 = 20^\circ$, $\lambda_0 = 30^\circ$, as follows :—

$\lambda_0 = 10^\circ$, $\log\tan\lambda_0 = 9{\cdot}25$

log tan λ =	9·25	9·15	8·96	8·78	8·62	8·69	9·03	9·63	10·20	10·34	10·18	9·70
λ =	10°·0	8°·0	5°·2	3°·5	2°·6	2°·8	6°·1	23°·1	58°·0	65°·4	56°·6	26°·5

$\lambda_0 = 20^\circ$, $\log\tan\lambda_0 = 9{\cdot}56$

log tan λ =	9·56	9·46	9·27	9·09	8·93	9·00	9·34	9·94	10·51	10·65	10·49	10·01
λ =	20°·0	16°·0	10°·6	7°·0	4°·9	5°·8	12°·6	41°·0	73°·0	77°·6	72°·0	46°·0

$\lambda_0 = 30^\circ$, $\log\tan\lambda_0 = 9{\cdot}76$

log tan λ =	9·76	9·66	9·47	9·29	9·13	9·20	9·54	0·14	0·71	0·85	0·69	0·21
λ =	30°·0	24°·6	16°·4	11°·0	7°·6	9°·0	19°·1	54°·1	79°·0	82°·0	78°·5	58°·4

32. The smallest assumed value of λ_0 brings the spots at minimum too close to the equator, while the largest takes them too close to the pole. In every case the range required is too large to be probable, though not impossible. There are, however, some considerations which reduce the range. We have superposed the observed maxima of Table III., but the time of maximum will certainly be altered by a change of aspect, as well as the time of minimum. Further, there is the uncertainty about the real time of maximum. Both these can be taken account of by tracing the

curves on paper and superposing them experimentally, and then reading off new points on one of them to correspond with the twelve points already tabulated for the other.

33. *Superposing the curves empirically*, then, in the neighbourhood of maximum, we get new readings for group VII. as follows, group I. remaining as before :—

	Max.											
VII.	·04	·16	·28	·42	·60	·74	·90	·98	·99	·83	·48	·18
I.	·00	·13	·38	·61	·87	1·00	·94	·70	·40	·30	·17	·10
I.–VII.	–·04	–·03	+·10	+·19	+·27	+·26	+·04	–·28	–·59	–·53	–·31	–·08

The range is now from +·27 to –·59 instead of from +·39 to –·68 before. Multiplying by 1·6 to convert into logarithms to base 10 we find the values of λ when $\lambda_0 = 20^\circ$ to be now

22°·6 22°·2 14°·0 10°·3 7°·6 7°·9 17°·6 45°·8 73°·0 68°·8 49°·0 26°·0

The requisite range in latitude is thus reduced by 4°·6 near the pole (from 77°·6 to 73°·0) and 2°·7 near the equator (from 4°·9 to 7°·6), and the improbability therefore sensibly diminished.

34. A much greater diminution can be made if we take into account the effect of absorption of the light near the star's limb, in addition to the simple foreshortening. In the absence of any information as to its amount, it is impossible to go beyond a hypothetical illustration. But, as an illustration merely, if the reduction factor for foreshortening and absorption combined were $\cos^2 \zeta$ instead of the previous $\cos \zeta$ for foreshortening alone, then we must replace $\log \tan \lambda$ in the above calculations by $2 \log \tan \lambda$; in other words, we may halve the differences 1·6 (I.–VII.) before finding $\log \tan \lambda$ from them.

For the case $\lambda_0 = 20^\circ$ the values of λ would now be (for the empirical superposition),

21°·3 21°·0 16°·8 15°·1 12°·6 12°·6 18°·8 31°·7 47°·0 44°·5 33°·0 23°·0

But it is interesting to take another value of λ_0, viz. that which would be applicable in the case of the Sun. Putting $\lambda_0 = 11^\circ·3$, the values of λ would be

12°·0 11°·8 9°·4 8°·0 7°·0 7°·0 10°·6 18°·6 30°·5 28°·0 19°·5 13°·0

and we may compare with these the values given in the Greenwich volumes for 1882 and the following eleven years for the "mean distance from equator of all spots," viz.—

17°·9 13°·1 11°·3 11°·8 10°·4 8°·4 7°·4 11°·6 22°·0 20°·3 18°·4 14°·5

Thus a range not much greater than that of the spots on the Sun would, on this hypothesis as to the reduction factor, suffice for converting one of the extreme cases in Table II. or Table III. into the other extreme.

35. There are a few stars outside these limits, as we shall see in § 36, § 37, so that a greater range is required. But it is tolerably clear that, in default of information as to the magnitude of absorption, we can neither affirm nor deny the sufficiency of this cause to explain the whole range of variation in type of light-curve.

36. As regards this range, one point remains to be considered. How far do the thirty-one stars of Table I. cover the range in type? Are there other stars much "earlier" or "later" than these? The best information available is that given in Chandler's Third Catalogue and the revised edition, where the quantity $M - m$, being the number of days by which maximum follows minimum, is tabulated alongside the period. If $2\ (M - m)$ exceeds the period, the star is an "early" star like T Cassiopeiæ: if it is less than the period, the star is "late" like the Sun. We may take $\alpha = \{2\ (M - m) - P\}/P$ as a characteristic of the star. Now by calculation we find the minimum of group I. in Table VI. falls at $180° - 18°$, so that $\alpha = +\cdot 10$: and the minimum of group VII. falls at $180° + 45°$, so that $\alpha = -0\cdot 25$. But these refer to intervals between *calculated* maxima and minima, which differ systematically from those assigned by the observer, as already pointed out. If we took the observer's maximum we should increase α numerically in both cases; and if we assume that the same will be true also at minimum, we increase α still further. It would not be unreasonable to double the numerical range for α, and consider that in Chandler's lists we may expect a range of

$$+\cdot 20 \text{ to } -\cdot 50$$

corresponding to the range of $+\cdot 10$ to $-\cdot 25$ adopted in comparing with the hypothesis.

37. In the first instance Chandler's Third Catalogue was taken, and the value of

$$\alpha = \{2\ (M - m) - P\}/P$$

formed for all variables of period greater than 100 days. The results were as follows:—

TABLE IX.

α	No of Stars.	α	No. of Stars.
$> +\cdot 20$	3	$-\cdot 21$ to $-\cdot 30$	12
$+\cdot 20$ to $+\cdot 10$	5	$-\cdot 31$ to $-\cdot 40$	4
$+\cdot 09$ to $\cdot 00$	12	$-\cdot 41$ to $-\cdot 50$	0
$-\cdot 01$ to $-\cdot 10$	38	$< -\cdot 50$	1
$-\cdot 11$ to $-\cdot 20$	28	Total	103

Thus four cases out of one hundred and three fell outside the limits above assigned. But on afterwards referring to Chandler's "Revision of Elements" (*Astronomical Journal*, 1904, Jan. 8), it was found that two cases out of the four had been altered so as to fall within the limits, as the particulars given below will show.

TABLE X.

Name.	No.	3rd Cat. M − m	3rd Cat. P	a	Revision. M − m	Revision. P	a
S Leonis	3994	125	190d	+·32	93	189	−·02
W Scorpii	5795	146	222	+·32	130	221	+·18
V Cephei	8591	220	360	+·22	220	360	+·22
S Tauri	1582	70	376	−·63	70	380	−·63

Further, as regards S Tauri, it is to be remarked that Chandler gives twenty-one maxima but only two minima as the basis of elements, and possibly further observation may alter the value of $M - m$: and V Cephei only varies a magnitude at most, so that the epochs of maximum and minimum are not very easy to determine. The elimination of two exceptional cases out of four in the "Revision" of elements is not without significance, and we may perhaps expect further modifications in the future. On the other hand, one exceptional star has been introduced in the Revision (X Ophiuchi, see below), a star with small range in magnitude, and subject to the same criticisms as V Cephei.

TABLE XI.

Twenty Stars with Large Values of a.

a	Name.	No.	R.A. h m	Dec. °	Gal. Lat.
+·25	X Ophiuchi	6682	18 34	+9	12
+·22	V Cephei	8591	23 52	+83	23
+·19	R S Libræ	5511	15 18	−23	25
+·18	W Scorpii	5795	16 6	−20	18
+·17	V Ophiuchi	5887	16 21	−12	20
+·16	S Carinæ	3637	10 6	−61	12
+·13	S Arietis	715	1 59	+12	42
+·12	T Cassiop.	107	0 18	+55	5
+·11	T Capricorni	7659	21 17	−16	46
+·10	S Cephei	7779	21 36	+78	23
+·08	T Cephei	7609	21 8	+68	18
+·08	V Tauri	1717	4 46	+17	13
+·07	X Capricorni	7577	21 3	−22	44
+·07	W Cygni	7754	21 32	+45	13
+·07	U Boötis	5338	14 50	+18	56
+·05	V Cygni	7428	20 38	+48	10
+·05	R Camelop.	5190	14 25	+84	35
+·03	R Aurigæ	1855	5 9	+53	15
+·03	W Cassiop.	294	0 49	+58	5
+·02	R Sagittarii	6905	19 11	−19	16

TABLE XII.

Twenty Stars with Small Values of α.

α	Name.	No.	R.A.	Dec.	Gal. Lat.
			h m	°	°
− ·63	S Tauri	1582	4 24	+10	20
− ·42	R Androm.	112	0 18	+38	20
− ·35	R Gemin.	2528	7 1	+23	20
− ·34	S Coronæ	5504	15 17	+32	55
− ·34	R Comæ	4315	12 0	+19	75
− ·31	R Cancri	2946	8 11	+12	25
− ·27	R Urs. Maj.	3825	10 38	+69	48
− ·27	V Lyræ	6871	19 5	+30	5
− ·27	R Cygni	7045	19 34	+50	15
− ·27	U Cassiop.	243	0 41	+48	10
− ·26	R Centauri	5095	14 9	−59	5
− ·25	o Ceti	806	2 14	−3	51
− ·23	R Can. Min.	2539	7 3	+10	12
− ·22	Y Virginis	4492	12 29	−4	54
− ·22	S Hydræ	3170	8 48	+3	28
− ·22	R Aquilæ	6849	19 2	+8	0
− ·20	U Orionis	2100	5 50	+20	0
− ·20	V Bootis	5194	14 26	+39	65
− ·19	S Piscium	434	1 12	+8	50
− ·19	T Arietis	976	2 43	+17	32

In Tables XI. and XII. are collected the stars with extreme values of α at both ends of the series. Further observations of the light-curves of some of these would be specially welcome.

38. If the present hypothesis, which classifies the stars according to the orientation of their axes, be correct, it is natural to inquire whether any peculiarity of distribution in space is suggested. In the last column are given the Galactic Latitudes of the stars. It will be noticed that while the stars with large negative values of α occur in all Galactic Latitudes, those with positive values do not occur near the Galactic Poles. In terms of the hypothesis above sketched, this would be the case if the axes of rotation of these stars were roughly parallel to the plane of the Milky Way. We should then see stars equatorially in all directions, but we should not have a polar view of those lying near the Galactic Poles.

It may be remarked that the axis of the Sun is inclined about 30° to the plane of the Milky Way. It is not difficult to suggest physical reasons for such a general configuration, in view of the recent discoveries of two stellar systems in relative motion; but the material is so small that it would be laying too much stress on the figures to enter on such speculations. At the same time they are sufficiently suggestive to warrant further investigation.

39. We have now followed out the consequences of the assumption which first suggested itself, leaving, however, one important point for further consideration (§ 26), to which we must

return. But before doing so it will be convenient to recapitulate the main features of the hypothesis above dealt with, and to point out a limitation not hitherto noticed. The chief features are as follows:—

(*a*) The travelling of the star's activity in latitude will modify the apparent variation according to the aspect presented to the observer.

(*b*) The travelling is represented by a moderate gradient G near maximum, and a sharper gradient g near minimum, when the activity leaves the equator and breaks out in high latitudes.

(*c*) Both maximum and minimum are displaced in consequence. The amount of displacement depends on the gradient and the curvature near the maximum (or minimum). Near maximum the log (activity) is represented by a curve,

$$y_1 = \mathrm{K}(\theta - \theta_0)^2$$

and the log (foreshortening) by a curve

$$y_2 = \mathrm{G}(\theta - \theta_0)$$

and the shift of maximum is $\mathrm{G}/2\mathrm{K}$. Using small letters for minimum, the shift is $g/2k$.

(*d*) Now K and k are opposite in sign but nearly equal numerically. G and g are opposite in sign but g is greater. Hence maximum and minimum will be displaced in the same direction, but the minimum more than the maximum. Relatively to the maximum the minimum is displaced by

$$g/2k - \mathrm{G}/2\mathrm{K} = (g - \mathrm{G})/2\mathrm{K} \quad \text{approximately.}$$

And the effective displacement thus depends roughly on the excess of g over G.

(*e*) This excess depends on the fact that the spots travel from a high latitude λ_1 to a low latitude λ_2 in a time t_1, and back over the same distance in a much shorter time t_2. Let $t_1 = nt_2$, then roughly we shall have $g = n\mathrm{G}$, and effective displacement $= (n-1)\mathrm{G}/2\mathrm{K}$
$= (n-1) \times$ displacement of maximum.
To get a large range in effective displacement, therefore, $(n-1)$ should be large, and therefore t_2 small.

(*f*) But now comes the limitation not hitherto noticed. The period t_2 is a superior limit for the displacement of minimum; for outside t_2 the gradient would be no longer g but G, which is opposite in sign. We thus require t_2 to be *large*, which conflicts with paragraph (*e*).

The value of $\alpha = \{2(\mathrm{M} - m) - \mathrm{P}\}/\mathrm{P}$ according to Chandler's Catalogues ranges at least from $+ {\cdot}22$ to $- {\cdot}40$: that is, the minimum may fall anywhere between ·39P and ·70P after maximum.

$$\text{Thus} \quad t_2 \not< {\cdot}31\mathrm{P} \qquad t_1 \not> {\cdot}69\mathrm{P}$$
$$\therefore\ n \not> 2{\cdot}2 \qquad n - 1 \not> 1{\cdot}2$$

If we accept the view that the observer's assignment

and minima are systematically erroneous, in the manner indicated in §§ 7–13, the range for a may be reduced. If we reduce it to one half (which is probably more than we are entitled to do), we get

$$t_2 \nless \cdot 16P \qquad t_1 \ngtr \cdot 84P$$
$$\therefore n \ngtr 5{\cdot}3 \qquad n-1 \ngtr 4{\cdot}3$$

It is really straining the possibilities to give n so large a value as 4. For one thing, we are allowing nothing for the change from gradient G to gradient g; and for another, we can scarcely admit so great a systematic error in the estimates of maximum and minimum. Probably n cannot exceed 3, and thus the displacement of minimum will be three times that at maximum, and the effective displacement about twice that at maximum. It follows that too favourable a case was taken both in § 31 and in § 33. In the former the maxima were superposed, which makes the curves I. and VII. fit more easily than when both maximum and minimum are displaced. In the latter, the fit was adjusted to be even easier.

It is true that these and other difficulties in stretching the hypothesis to suit the facts can all be removed by assuming a large enough effect for absorption of light near the limb in addition to the foreshortening, but there is a natural reluctance to draw upon an unknown factor of this kind.

40. We now return to § 26, where it was remarked that if the faculæ die out altogether at minimum, the epoch of minimum will not be altered at all. The alteration in type of curve will then be entirely due to the displacement of maximum. This is a complete inversion of our hypothesis: the "early" stars will now be those regarded from the equator, and the "late" stars those regarded from one of their poles. Since the Sun is regarded from the equator and is a "late" star, there is at first sight an inconsistency here; but it has been already pointed out how the case of the Sun differs from that of the stars, owing to the rudimentary nature of the variation. So far as actual calculation goes, this new hypothesis fits the facts for the Sun better than the old; for it was found in § 21 that on regarding the Sun from one of his poles, his curve became "later" than ever, owing to the great displacement of maximum.

41. We can quickly run through the work again and make the necessary modifications. Returning to Table III., we now inquire what additions to the curve I. will transform it into curve VII. when the *minima* are superposed. The following superposition is sufficiently exact to bring out the main features:—

I.	0	13	38	61	87	100	94	70	40	30	17	10
VII.	32	48	65	80	94	99	98	75	38	0	7	20
I.–VII.	−32	−33	−27	−19	−7	+1	−4	−5	+2	+30	+10	−10

The difference becomes nearly zero near minimum, owing to the faculæ dying out and leaving only the photosphere. The differences

from zero need not concern us at present; we are chiefly concerned with the variation near maximum, and we see that the differences are smaller than in § 31, ranging only from $+30$ to -33, instead of from $+39$ to -68. Thus log tan λ changes by $1{\cdot}6 \times {\cdot}63 = 1{\cdot}0$ instead of by $1{\cdot}6 \times 1{\cdot}07 = 1{\cdot}7$; and λ need only change from 73° to 18° instead of (say) from 78° to 5°. But we are met by the difficulty that this change from 73° to 18° must occur in about *one quarter of the period near maximum*, which is too improbable. If we now make the same assumption about absorption as before, viz. that it converts the foreshortening factor $\cos\theta$ into $\cos^2\theta$, or log tan λ into 2 log tan λ, then the range in λ becomes 45° to 18°; or 30° to 10° would do. Even this seems a large change in latitude to take place in one-quarter of the period. It is not impossible, but it seems unlikely. The balance of probabilities seems to lie with the former hypothesis.

42. We can also invert these hypotheses in another way, by assuming that the maximum is a time of few spots and the minimum of many. But we should then find it difficult to get large changes in the epochs. At minimum the photosphere must be practically covered with spots, so that the mean latitude could not vary much; and at maximum there would be so few that their situation could make little difference.

Thus at present the first hypothesis seems preferable.

43. One other point may be briefly mentioned. When this idea of the variation in light-curve with aspect of the star first presented itself, it was thought that it might lead to an explanation of Chandler's periodic terms. He gives, for instance, the epochs of maximum of R Andromedæ as

$$2400141 + 410{\cdot}3\,E + 30 \sin(12^\circ E + 90^\circ)$$

the mean period being $410{\cdot}3$ days, but successive maxima being gradually displaced until they fall thirty days later than at first, and then back until they are thirty days earlier, and so on periodically. If the axis of rotation of the star were not fixed, but had a free period of conical oscillation, its aspect to us would change, and such variations in the epoch of maximum would follow. But we should apparently have to assume very large oscillations in some well-authenticated cases; and there are some, such as S Serpentis, for which Chandler gives

$$368{\cdot}5\,E + 116 \sin(4^\circ E + 6^\circ)$$

which the foregoing work shows that we could not explain at all. The origin of these periodic terms is more likely to lie in t[he] existence of several periodicities of spot variation with var[ying] amplitudes, such as Professor Schuster has recently discover[ed in] the case of the Sun.

On the Jupiter Evection Term. By P. H. Cowell, M.A., F.R.S.

In *Monthly Notices*, vol. lxiv. p. 417, and again in vol. lxv. p. 145, I have shown that the observed value of the coefficient of the Jupiter Evection term in the Moon's longitude was $0''\cdot89 + 0''\cdot23 = +1''\cdot12$.

In an interesting paper in the *Astronomical Journal*, No. 592, Professor Newcomb states that he has calculated the theoretical coefficient of this term as $1''\cdot15$, whereas Hill and Radau had previously agreed in giving $0''\cdot89$.

The removal of the only serious discordance between observed and theoretical coefficients for short terms of a period clearly differing from the periods of possible errors of observation strengthens the case stated in the *Monthly Notices*, vol. lxvi. p. 306, for Le Verrier's mass of Venus against the recent reduction by 2 per cent.

Observations of Minor Planets from Photographs taken with the 30-inch Reflector of the Thompson Equatorial at the Royal Observatory, Greenwich, during the year 1905.

(*Communicated by the Astronomer Royal.*)

The following positions of minor planets were obtained from photographs taken with the 30-inch Reflector during the year 1905.

The plates were measured with the astrographic micrometer. Four reference stars were, as a rule, measured with the planet, their positions being derived when possible from the Catalogues of the Astronomische Gesellschaft.

The positions given are not corrected for Parallax.

log Parallax Correction = log Parallax Factor − log Δ.

Date and G.M.T. 1905.		Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Dec.
d	h m s	h m s	° ′ ″		
		(407) Arachne.			
Jan. 10	10 30 43	5 51 53·74	+27 14 9·3	−7·556	+0·554
		(78) Diana.			
Jan. 17	11 42 14	6 23 31·99	+35 8 29·1	+9·105	+0·410
		(71) Niobe.			
Jan. 17	11 25 46	7 44 5·90	+35 56 50·1	−8·785	+0·372

Date and G.M.T. 1905.		Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Log. Parallax Factor. Dec.
d	h m s	h m s	° ′ ″		
		(270) Anahita.			
Jan. 17	11 58 24	8 43 15·76	+15 5 32·2	−8·978	+0·719
		(374) Burgundia.			
Mar. 9	10 50 3	9 58 16·04	−0 34 10·9	−∞	+0·839
		(148) Gallia.			
Feb. 25	12 29 16	10 20 12·02	+16 14 45·7	+8·697	+0·704
Mar. 2	10 30 5	10 16 17·94	+17 11 47·3	−9·037	+0·698
3	10 51 44	10 15 30·60	+17 23 7·7	−8·824	+0·692
		(92) Undina.			
Feb. 25	12 4 23	11 45 1·98	+15 6 36·7	−9·113	+0·722
Mar. 2	10 45 0	11 41 46·68	+15 37 31·9	−9·330	+0·731
3	11 16 48	11 41 4·70	+15 43 48·2	−9·206	+0·720
22	9 46 25	11 27 30·85	+17 22 50·4	−9·218	+0·704
30	9 19 27	11 22 9·74	+17 50 55·2	−9·176	+0·697
		(487) Venetia.			
Mar. 30	10 11 41	11 52 23·75	+15 51 57·3	−9·058	+0·713
Apr. 6	10 22 35	11 47 13·59	+16 19 36·8	−8·635	+0·703
		(350) Ornamenta.			
Apr. 6	10 50 45	11 48 42·65	+37 35 59·9	−∞	+0·316
8	10 11 58	11 47 19·90	37 32 3·4	−8·765	+0·326
12	11 23 18	11 44 42·39	37 20 46·8	+9·076	+0·348
		(42) Isis.			
Mar. 22	10 13 9	12 10 56·26	+13 13 28·9	−9·271	+0·747
27	10 8 24	12 6 13·42	+13 40 45·4	−9·198	+0·739
30	10 33 27	12 3 24·23	+13 55 16·5	−8·982	+0·730
Apr. 5	10 56 56	11 57 57·41	+14 19 12·7	−8·062	+0·723
		(28) Bellona.			
Mar. 22	10 22 59	12 20 39·52	+8 0 10·9	−9·264	[illegible]
27	10 23 28	12 16 52·64	+8 40 53·4	−9·174	[illegible]
30	10 50 22	12 14 37·46	+9 3 40·2	−8·937	[illegible]
Apr. 6	9 55 29	12 9 41·22	+9 49 57·2	−9·08	[illegible]

Date and G.M.T. 1905. d	h m s	Apparent R.A. h m s	Apparent Dec. ° ′ ″	Log. Parallax Factor. R.A.	Dec.
		(163) Erigone.			
May 5	10 5 42	12 52 32·41	+0 12 13·1	+7·982	+0·834
8	10 56 51	12 51 6·68	+0 20 43·5	+9·042	+0·835
		(334) Chicago.			
May 5	10 53 54	13 58 6·56	−5 37 36·8	−8·285	+0·867
		(26) Proserpina.			
May 5	11 21 37	14 31 33·43	−14 29 8·0	−8·448	+0·903
8	11 44 35	14 28 49·23	−14 21 29·2	+8·518	+0·903
9	11 7 42	14 27 57·02	−14 19 2·8	−8·296	+0·903
10	10 44 38	14 27 4·98	−14 16 37·0	−8·693	+0·902
22	10 58 57	14 17 34·27	−13 51 6·5	+8·827	+0·900
		(386) Siegena.			
June 23	10 36 6	15 42 6·50	+7 31 19·4	+8·980	+0·785
26	12 2 16	15 40 42·39	+7 22 5·6	+9·373	+0·796
27	11 39 4	15 40 17·57	+7 18 46·5	+9·326	+0·794
		(470) Kilia.			
June 28	10 34 23	16 49 24·46	−9 41 11·3	+8·210	+0·885
		(8) Flora.			
June 14	11 51 44	17 51 37·82	−19 9 15·0	−8·711	+0·917
14	12 3 17	17 51 37·21	−19 9 15·8	−8·506	+0·920
19	11 37 25	17 45 59·09	−19 19 14·0	−8·507	+0·918
19	11 50 27	17 45 58·59	−19 19 17·4	−7·962	+0·918
22	11 18 9	17 42 34·77	−19 25 30·8	−8·580	+0·918
		(19) Fortuna.			
June 23	11 31 16	18 26 4·32	−21 1 20·8	−8·920	+0·920
26	12 27 11	18 22 55·60	−21 2 1·4	+8·575	+0·922
July 7	10 45 15	18 11 39·00	−21 4 32·8	−8·648	+0·922
		(478) Tergeste.			
July 8	10 41 51	18 26 58·15	−9 45 53·6	−8·810	+0·885

Date and G.M.T. 1905. d	h m s	Apparent R.A. h m s	Apparent Dec. ° ′ ″	Log. Parallax Factor. R.A.	Dec.
			(46) Hestia.		
June 23	11 4 54	18 50 12·05	−18 58 12·3	−9·214	+0·908
July 7	11 7 31	18 36 36·14	−19 12 8·4	−8·674	+0·917
8	11 6 43	18 35 37·64	−19 13 19·4	−8·608	+0·918
25	10 19 33	18 20 59·57	−19 36 26·0	+8·229	+0·919
			(313) Chaldæa.		
July 7	11 34 45	19 5 7·92	−5 16 15·9	−8·667	+0·865
19	11 59 11	18 53 55·98	−6 0 58·0	+8·934	+0·868
25	11 11 53	18 48 54·29	−6 29 32·3	+8·749	+0·871
			(433) Eros.		
July 19	13 18 52	21 25 5·15	−13 49 49·5	−8·471	+0·901
Aug. 23	9 31 50	20 24 9·19	−12 33 0·5	−8·874	+0·895
			(176) Idunna.		
Sept. 7	11 26 34	21 30 4·67	+12 16 26·9	+9·001	+0·746
			(248) Lameia.		
Sept. 7	12 0 35	21 53 7·36	−5 44 6·8	+9·062	+0·866
			(84) Clio.		
Sept. 8	12 16 24	22 37 56·92	−2 27 38·5	+8·882	+0·850

The anonymous planet discovered on photographs taken on 1903 Aug. 6, 31 and Sept. 1 has received the designation 1903 LX[a], and a provisional circular orbit has been calculated for it by Professor Kreutz (*Astron. Nachr.*, No. 4154).

Royal Observatory, Greenwich:
1907 *March* 7.

A New Nebula. By the Rev. T. E. Espin, M.A.

On January 18, while looking for new double stars, I came across a bright nebula, which, as far as I am aware, is an unrecorded object. It was estimated to be about 5″ in diameter and elongated north. On January 27 the nebula was seen for a few moments between clouds. On February 1 it was well seen, a conspicuous object, and equal to a 10 magnitude star. The elongation was very marked, and sometimes it looked like two nebulæ. The measures gave—

Major Axis	6″·90
Minor Axis	6·35

The major axis was found to be roughly at position 10°·5. A small star was noted S *p.* On February 11 the position was determined from B.D + 33°,746. It was found to precede this star by $7^s·80$, and to be 2′ 25″ south of it. On February 18 what appeared to be a star or nucleus was seen north and measured. It was also seen on February 22 and February 23, and measures made of it with great difficulty. The later observations seem to suggest a planetary nebula, with a small star at the northern edge. The following are my measures of the two stars near the nebula:—

Neb. and star A:—

1907·1314	P 14°·9	D 4″·10		Mag. 13·0
1424	13·3	4·75		
1451	13·3	3·90		
Mean 1907·140	13·8	4·25	3 nts.	

Neb. and star B:—

1907·0849	244·6	16·75		Mag. 12·0
1314	244·3	17·05		
1424	245·3	15·90		
1451	238·4	17·05		
Mean 1907·126	243·1	16·69	4 nts.	

The place of B.D + 33°·746 for 1855·0 is—

$\alpha = 3^h\, 47^m\, 14^s·89$ $\delta = 33°\, 29'\, 14''·7$ (*Bonn Obs.*, vol. vi.)

Professor Burnham observed the nebula with the 40-in. of the Yerkes Observatory on February 20 in moonlight and a sky not clear, and measured the star B as follows:—

P 238°·1 D 17″·86

and also found for B.D + 33°·746 and neb.

P 213°·9 D 177″·87 single distance.

Observations of Occultations. By Rev. Leonard A. Williams, B.A.

(*Communicated by A. C. D. Crommelin.*)

First Series.

At Stoke Wake Rectory,
Blandford, Dorset.

Longitude, 2° 20′ 5″ W.

Latitude, 50 51 20 N.

Height above sea, 440 feet.

Star and Magnitude.		Disappearance G.M.T.	d	h m s
A Leonis	4·6	1905 May	12	8 44 27
γ Tauri	3·9	Sept.	19	10 35 40
Solar Eclipse—				
First con.		Aug.	29	23 45 19
Last con.			30	2 13 45

Second Series.

At Farningham Vicarage,
Dartford, Kent.

Longtitude, 0° 13′ 25″ E.

Latitude, 51 22 45 N.

Height above sea, 120 feet.

Star and Magnitude.		Disappearance G.M.T.	d	h m s
μ Ceti	4·4	1905 Dec.	8	5 34 26
χ Leonis	4·7	1906 April	6	7 3 27
ο Sagittarii	3·9	Nov.	19	5 29 30
ξ^2 Ceti	4·3	1907 Jan.	21	9 5 9
ν Geminorum	4·1		26	6 39 55

Method of determining time.—At Stoke Wake: Equal altitudes with sextant, and artificial horizon (mercury). At Farningham: Meridian observations with transit instrument.

Chronometer.—Deck watch in box, No. 08354, by Frodsham. Rate per diem for Jan. 1907, $-·9^s$.

Results.—The times are not *guaranteed* to the exact second; but they are assumed, with good reason, to be correct to 1·5 seconds.

1907 *January* 30.

Appendix to the above Paper. By A. C. D. Crommelin.

I have reduced Mr Williams' observations in the same manner as the Greenwich observations of occultations. The R.A., N.P.D., and parallax of Moon have been interpolated with second differences from the Nautical Almanac, and the semidiameter found by the equation log semidiameter = log parallax + 9·43542, this being the constant to reduce to Struve's semidiameter 15′ 32″·65, which is now used at Greenwich in the reduction of occultations. The R.A. and N.P.D. of the stars are taken with the Nautical Almanac values of their mean places and star corrections.

Then assuming—

True Time of Phenomenon = Observed Time + t^s
,, R.A. of Moon = Naut. Alm. value + x'' + t × mot$^{n.}$ in 1^s
,, N.P.D. . . . = ,, + y + t × mot$^{n.}$
,, R.A. of Star = ,, + e
,, N.P.D. . . . = ,, + f
,, Parallax of Moon = ,, × $\left(1 + \frac{}{1}\right.$
,, Semidiameter ,, = Naut. Alm. Par$^{x.}$ × ·272535

I have found the following equations of condition; in the case of occultations observed at Greenwich, the Greenwich value of Semid. – Distance is given for comparison :—

Date.	Semid.—calculated Distance. Williams.	Greenwich.	$= e \times$	$+ f \times$	$+ x \times$	$+ y \times$	$+ l \times$	$+ m \times$	$+ n \times$
1905 May 12	$-7''.40$		$= +.77$	$+.62$	$-.77$	$-.61$	$-.43$	$-.60$	$-.89$
Sep. 19	-0.91		$= +.11$	$-.99$	$-.11$	$+.99$	$-.19$	$+2.17$	$-.96$
Dec. 8	-4.18		$= +.78$	$-.61$	$-.78$	$+.61$	$-.42$	$+.04$	$-.88$
1906 Ap. 6	-6.36	-7.81	$= +.51$	$+.86$	$-.51$	$-.86$	$-.37$	-2.97	$-.97$
Nov. 19	-2.33	-5.12	$= +.92$	$+.18$	$-.92$	$-.18$	$-.46$	$+.69$	$-.99$
1907 Jan. 21	-1.56		$= +.99$	$+.05$	$-.99$	$-.05$	$-.36$	$+1.24$	$-.90$
26	-3.20	-4.68	$= +.87$	$+.37$	$-.87$	$-.37$	$-.35$	-2.18	$-.89$

[The following were received too late for insertion in the Annual Report of the Council.]

Report of the Melbourne Observatory. (*Director Mr P. Baracchi.*)

The principal astronomical work done at this observatory during the year 1906 was limited, as in previous years and for the same reasons, to meridian observations and stellar photography, including the measurement of plates of the Sydney and Melbourne Zones, in regard to which a separate report is appended.

Meridian Observations.—These were made with the 8-inch Transit Circle, and were as follows:

	Observations in R.A.	Observations in N.P.D.
Clock Stars	505	...
Azimuth Stars	269	117
List Stars	1276	1289
Total .	2050	1406

The list stars were selected from the Melbourne plates of the Astrographic Catalogue, to serve as fundamental points of reference for the reduction of these plates.

The total number of this class of stars now completely observed not less than three times is 5545.

The reductions, including the preparation of the annual catalogue for 1905, are well advanced.

A general catalogue for the epoch 1900, including all stars observed since 1894, is in course of preparation.

No authority has yet been obtained for printing the general catalogue for the epoch 1890, the MS. of which was prepared some years ago.

Stellar Photography.—The Melbourne portion of the photographic catalogue and chart of the heavens has been further advanced as follows:—

	Passed as satisfactory.	Rejected.
Chart Plates with triple exposure of 30^m each .	45	3
Catalogue Plates (duplicate series)	19	...
Test Plates on South Polar region	14	...
Test Plates on Oxford Type Charts . . .	5	...
Plates for trials, adjustment of centre, etc. . .	15	...

Prolonged ill-health of the observers is partly accountable for the small progress shown by the above return.

The astrophotographic work now stands thus:—

The first series of 1149 plates, covering the entire region, $-64°$ to the South Pole, twice, and the series of chart plates with single exposure of one hour, the centres being at even degrees of declination from $-66°$ to $-90°$, were completed some time ago.

A duplicate catalogue series is now being made, and of this, 335 plates have been taken and passed as satisfactory.

In the chart series with triple exposures of 30^m each, the centres being at odd degrees of declination, 519 plates have been taken and passed as satisfactory; 65 plates require to be taken again to conclude this part of the work.

It is intended to extend the series of triple exposure chart plates to the regions with centres at even degrees of declination.

The following routine duties and other miscellaneous work were carried out for local requirements, as in former years:—

The time service; the weather service, comprising the control of some 980 country stations; the rating of chronometers for the shipping, and the testing of nautical meteorological and surveying instruments; the operations of the Bureau of Standard Weights and Measures; the continuous registration of the variations of sea-level, atmospheric elements, seismic disturbances, and the elements of terrestrial magnetism, including absolute magnetic measurements and the measurement and reduction of hourly ordinates on the magnetic curves of past years, for the purpose of clearing up, completing, and preparing for publication the results of a long series of magnetic records, extending back to the year 1868.

Joint Report of the Directors of the Observatories of Sydne[y] Melbourne on the Measurement of the Plates of the Ast[ro]graphic Catalogue.

The measurement of catalogue plates obtained [at the] observatories was continued by the Bureau established [for the] purpose at the Melbourne Observatory in 1898, and si[nce main]tained at the joint expense of the States of New South V[Wales and] Victoria.

The work of the year 1906 was carried out by the usual temporary staff of six young ladies, assisted by a permanent officer of the Melbourne staff, using the same measuring instruments, and following in every respect the same methods as described in reports of former years.

The numbers of rectilinear co-ordinates measured during the year in the direct and reverse positions of the plates are as follows:—

149 Sydney Plates, containing 75,162 stars.
181 Melbourne Plates, containing 45,069 stars.

The total aggregate numbers of plates, etc. measured to 31st December 1906 are as follows:—

562 Sydney Plates, containing 325,978 stars.
836 Melbourne Plates, containing 268,714 stars.

ERRATUM IN ANNUAL REPORT.

On p. 298 insert the name of Prof. E. E. Barnard in list of Donors to the Library.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXVII. APRIL 12, 1907. No. 6

H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Arthur Neville Brown, M.A., Ludgrove, New Barnet, Herts;
Harry Cooper, 19 Cromer Road, Eastville, Bristol;
Phanindralal Gangooly, M.A., University, Calcutta, India; and
William Newsam McClean, 42 Durdham Park, Bristol,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Major-General Henry Herbert Lee (late R.E.), The Mount, Dinas Powis, near Cardiff, South Wales (proposed by T. E. Heath);
Augustus Edward Hough Love, D.Sc., F.R.S., Sedleian Professor of Natural Philosophy, Oxford (proposed by H. H. Turner);
Rev. Reginald Wm. Bickerton Moore, M.A. Oxon., Vicar of St James', Bath, 11 Devonshire Buildings, Bath (proposed by Rev. D. Higham Sparling); and
Herbert Gerard Tompkins, Examiner of Local Fund Accounts, N.W.P. and Oudh, Lahore, India (proposed by S. A. Saunder).

Anders Donner, Director of the Observatory, Helsingfors, Russia, was proposed by the Council as an Associate of the Society.

Eighty-eight presents were announced as having been received since the last meeting, including, amongst others :—

Besançon Observatory, Bulletin Astronomique, Bulletin Chronométrique, Bulletin Météorologique, etc., presented by the Observatory.

Astrographic Chart of the heavens; 20 charts presented by the Royal Observatory, Greenwich; 47 charts from the Paris, Algiers, and Toulouse Observatories, presented by the French Minister of Public Instruction.

Determinations of Personal Equation depending on Magnitude, made with the Transit Circles and the Heliometer at the Royal Observatory, Cape of Good Hope. By Sir David Gill, K.C.B., F.R.S., and S. S. Hough, F.R.S., H.M. Astronomer at the Cape.

The observations here discussed were planned at the time when a discussion had arisen between Mr Hinks and Dr Cohn * as to the existence of personality depending on magnitude in transit observations made by the Repsold travelling wire method.

The question at issue is so important in connection with the future of meridian observation, that it appeared desirable to devote a considerable amount of time and labour to its settlement. The position of a bright star relative to two symmetrically situated faint stars of equal magnitude can be determined by means of heliometer observations with great precision and absolute freedom from personality. With the modern heliometer the images of the brighter star can be reduced by means of screens to similarity with the images of the fainter stars. Even if we grant that there may be a personality depending on a residual apparent difference of magnitude between the image of the star whose brightness is reduced and that of the other star under measurement, it is impossible to imagine that the observer can be affected by any difference of personality in the measurement of distance of two pairs of stars where one component of one pair precedes and the other follows the star whose magnitude has been reduced by the screen, because the two conditions of measurement are exactly similar.

Farther, if the measures of both distances are made simultaneously (*i.e.* in the order a, b, b, a), the instantaneous scale-value must be the same for both. Then, if we suppose the position-angles also to be measured, we can introduce both the scale-value and the index error of the position-circle as unknown quantities, and determine the R.A. and Dec. of the brighter star relative to the fainter stars—free from all personality. And if the differences of declination are either very small or accurately determined, we

* *Monthly Notices*, lxvi. p. 481, and Ast. Nach., No. 4119.

determine directly the R.A. of the bright star relative to the mean R.A. of the faint stars, free from personal and other systematic error.

We have in our experience at the Cape many instances of this freedom from personality in heliometer observations, of which a very good instance will be found in the *Annals of the Cape Observatory*, vol. viii. part 2, pp 73B–81B, in connection with the determination of the parallax of β Crucis by Gill and Finlay. There we have observations of the distances of two stars of $6\frac{1}{2}$ and 7 magnitude in almost exactly opposite directions from β Crucis, and situated at distances of $2812''$ and $3434''$ respectively from it. Gill measured these distances on seventeen nights, and Finlay on fifteen nights. In forming the equations of condition for determining the parallax, the difference of the two distances for 1890 was assumed to be $622''\cdot050 + x_g$ for Gill, and $622''\cdot050 + x_f$ for Finlay. The simultaneous solution of the equations gave

$$x_g = -0''\cdot001 \pm 0''\cdot019$$
$$x_f = -0\ \cdot021 \pm 0\ \cdot023$$

and the probable error of the single observation $\pm 0''\cdot077$.

But if we regarded $x_g = x_f$, the probable error of the single observation became $\pm 0''\cdot074$.

The latter solution is thus the better of the two, proving that there is no sensible systematic difference in the determination of the position of β Crucis by the two observers.

About the time that the after-mentioned operations were commenced, the completion of the current Catalogue with the old Transit Circle left the observers free to make simultaneous observations with the latter, and a verification of their personal equation depending on magnitude, derived some two years previously by the screen method, was thus rendered as convenient as it was desirable.

The groups of stars selected consisted of three stars approximately on the arc of a great circle and, as nearly as could be chosen, equi-spaced. The middle star was on the average of about $3\frac{1}{2}$ magnitude, and the preceding and following stars on the average about $8\frac{1}{2}$ magnitude. The difference of R.A. was always large enough to allow of all three stars being observed without undue haste on the same night, while the distances were restricted by the limitation that they must fall within the range which could be measured by the heliometer, viz. about 2°.

The transit observations involved the transit of each of the three stars in succession, so that their differences of R.A. could be derived independently of clock error or other instrumental adjustments.

The heliometer measurements consisted in the measurement of distance and position-angle of the bright star from each of th fainter ones of the group. From these observations the positic of the bright star relatively to the two fainter ones can be deriv

independently of the instantaneous scale-value or index correction to the position-circle or other equatorial adjustments of the heliometer.

The adopted magnitudes of the stars have been taken when possible from the Harvard Photometry. For stars not contained therein, the magnitudes are derived from estimates made by the Transit Circle observers, using, as standards, stars whose magnitudes have been determined at Harvard.

The following is a list of the groups observed, numbered consecutively in order of R.A. The initials *a*, *b* are used to distinguish the preceding and following pairs of the group. It should be noticed that in some cases the same bright star has been used in more than one group. For simplicity of reduction, however, such groups have been treated as if they were independent.

TABLE I.

List of Magnitude Personal Equation Stars.

Group No.	Name. B.D. No.	Mag.	R.A. 1906·0.	Dec. 1906·0.
			h m s	° ′ ″
1 *a*	$-3°$, 3882	8·7	16 4 41·01	− 3 17 56·17
	δ Ophiuchi	3·1	16 9 25·10	− 3 27 9·56
b	$-3°$, 3915	8·2	16 12 48·75	− 3 48 14·44
2 *a*	$-25°$, 5787	8·7	16 18 2·23	−26 3 4·54
	α Scorpii	1·3	16 23 38·51	−26 13 25·74
b	$-26°$, 5678	8·5	16 29 28·55	−26 16 58·14
3 *a*	$-10°$, 4383	8·3	16 40 10·37	−10 29 26·91
	20 Ophiuchi	4·7	16 44 37·97	−10 37 1·73
b	$-10°$, 4403	8·3	16 48 22·13	−10 35 56·57
4 *a*	$-15°$, 4439	8·7	16 58 40·38	−16 3 37·95
	7 Ophiuchi	2·6	17 4 59·14	−15 36 32·12
b	$-15°$, 4502	6·7	17 10 53·66	−15 7 6·94
5 *a*	$+12°$, 3234	6·7	17 25 59·98	+11 59 45·75
	α Ophiuchi	2·1	17 30 34·24	+12 37 40·72
b	$+13°$, 3421	6·3	17 34 38·73	+13 22 51·73
6 *a*	$-9°$, 4616	8·9	17 46 49·71	− 9 57 13·63
	ν Ophiuchi	3·5	17 53 51·07	− 9 45 44·99
b	$-9°$, 4646	8·6	18 0 0·97	− 9 35 0·23
7 *a*	$-3°$, 4255	9·5	18 8 38·83	− 3 0 44·65
	7 Serpentis	3·5	18 16 26·72	− 2 55 24·93
b	$-2°$, 4638	8·3	18 24 20·45	− 2 50 0·69

TABLE I.—*continued.*

Group No.	Name. B.D. No.	Mag.	R.A. 1906·0. h m s	Dec. ° ′ ″
8 *a*	−4°, 4557	9·2	18 37 23·90	− 4 54 0·99
	6 Scuti	4·5	18 42 11·22	− 4 50 56·03
b	−4°, 4603	9·2	18 47 50·28	− 4 50 38·87
9 *a*	−5°, 4835	9·1	18 55 25·09	− 5 30 7·85
	λ Aquilæ	3·5	19 1 15·63	− 5 1 25·81
b	−4°, 4719	8·6	19 7 23·99	− 4 37 11·97
10 *a*	−5°, 4845	8·3	18 57 50·51	− 5 40 17·73
	λ Aquilæ	3·5	19 1 15·63	− 5 1 25·81
b	−4°, 4712	8·7	19 5 15·79	− 4 12 10·57
11 *a*	+2°, 3856	7·8	19 15 55·70	+ 2 45 43·22
	δ Aquilæ	3·4	19 20 45·54	+ 2 55 36·96
b	+3°, 4043	6·4	19 25 50·94	+ 3 14 51·72
12 *a*	+3°, 3990	8·8	19 17 42·36	+ 3 9 33·36
	δ Aquilæ	3·4	19 20 45·54	+ 2 55 36·96
b	+2°, 3892	5·9	19 23 37·64	+ 2 44 18·32
13 *a*	+8°, 4198	8·4	19 39 50·81	+ 8 51 55·66
	α Aquilæ	0·8	19 46 11·83	+ 8 37 10·64
b	+8°, 4275	8·0	19 52 21·75	+ 8 11 48·34
14 *a*	+8°, 4227	9·0	19 44 39·25	+ 8 22 10·96
	α Aquilæ	0·8	19 46 11·83	+ 8 37 10·64
b	+8°, 4247	8·2	19 47 57·91	+ 8 53 20·02
15 *a*	−0°, 3911	8·6	20 2 1·01	− 0 24 11·00
	θ Aquilæ	3·4	20 6 27·31	− 1 6 2·36
b	−1°, 3935	7·5	20 11 34·11	− 1 47 16·90
16 *a*	−1°, 3902	8·0	20 4 31·69	− 1 32 57·82
	θ Aquilæ	3·4	20 6 27·31	− 1 6 2·36
b	−0°, 3942	7·5	20 8 14·67	0 36 47·98
17 *a*	−18°, 5663	8·5	20 18 10·75	18 31 1·02
	π Capricorni	5·1	20 21 56·51	18 31 12·58
b	−18°, 5705	8·3	20 26 2·67	18 24 0·52
18 *a*	−5°, 5349	7·9	20 37 31·64	5 38 0·18
	3 Aquarii	4·5	20 42 46·70	5 22 20·10
b	−5°, 5402	8·5	20 47 21·78	5 3 22·24

TABLE I.—*continued.*

Group No.	Name. B.D. No.	Mag.	R.A. 1906·0. h m s	Dec. ° ′ ″
19 *a*	− 5°, 5368	7·7	20 38 58·33	− 5 55 44·88
	3 Aquarii	4·5	20 42 46·70	− 5 22 20·10
b	− 5°, 5395	8·1	20 45 45·03	− 5 8 30·84
20 *a*	− 17°, 6140	8·3	20 55 10·77	− 17 14 39·95
	θ Capricorni	4·1	21 0 39·87	− 17 36 24·39
b	− 18°, 5886	8·3	21 8 31·41	− 17 57 9·39
21 *a*	− 17°, 6167	7·6	20 59 34·81	− 17 32 13·51
	θ Capricorni	4·1	21 0 39·87	− 17 36 24·39
b	− 18°, 5862	6·0	21 2 27·97	− 17 49 58·95
22 *a*	− 6°, 5757	8·3	21 21 37·93	− 6 24 29·01
	β Aquarii	3·0	21 26 36·68	− 5 59 6·11
b	− 5°, 5592	7·7	21 31 36·79	− 5 38 22·65
23 *a*	− 6°, 5761	8·9	21 23 5·00	− 6 1 43·73
	β Aquarii	3·0	21 26 36·68	− 5 59 6·11
b	− 6°, 5782	8·8	21 28 57·14	− 5 53 47·51
24 *a*	+ 9°, 4871	8·9	21 35 27·07	+ 9 33 42·93
	ε Pegasi	2·7	21 39 34·15	+ 9 26 37·45
b	+ 9°, 4899	8·7	21 43 23·01	+ 9 24 36·95
25 *a*	+ 5°, 4947	7·7	22 0 58·56	+ 5 30 32·09
	θ Pegasi	3·7	22 5 27·48	+ 5 44 6·65
b	+ 5°, 4982	8·5	22 10 51·38	+ 6 10 29·60
26 *a*	− 11°, 5823	8·0	22 19 31·52	− 11 37 58·19
	σ Aquarii	4·8	22 25 40·44	− 11 9 32·79
b	− 11°, 5875	8·9	22 31 41·51	− 10 46 31·29
27 *a*	− 14°, 6337	8·6	22 39 59·06	− 14 7 52·55
	τ Aquarii	4·4	22 44 36·92	− 14 5 21·01
b	− 14°, 6367	8·9	22 50 25·35	− 14 14 28·93
28 *a*	+ 15°, 4737	9·0	22 54 12·54	+ 15 16 43·73
	α Pegasi	2·6	23 0 4·66	+ 14 41 57·77
b	+ 13°, 5059	7·8	23 5 16·14	+ 13 55 7·89
29 *a*	− 20°, 6568	8·7	23 11 40·04	− 20 28 29·33
	b′ Aquarii	4·3	23 18 2·08	− 20 36 49·97
b	− 20°, 6606	8·8	23 25 9·70	− 20 41 22·09

TABLE I.—*continued.*

Group No.	Name. B.D. No.	Mag.	R.A. 1906·0. h m s	Dec. ° ′ ″
30 *a*	−15°, 6462	8·5	23 32 32·46	−14 44 13·17
	ω^2 Aquarii	4·5	23 37 50·88	−15 3 53·03
b	−15°, 6500	8·9	23 43 44·24	−15 22 50·19
31 *a*	+5°, 5230	8·4	23 48 19·79	+ 6 10 34·47
	ω Piscium	4·0	23 54 29·03	+ 6 20 34·66
b	+6°, 5242	7·8	0 2 15·15	+ 6 21 10·58
32 *a*	−9°, 23	8·5	0 7 33·83	− 9 13 56·19
	i Ceti	3·7	0 14 38·33	− 9 20 41·89
b	−9°, 79	8·0	0 22 33·65	− 9 10 40·25
33 *a*	−18°, 98	8·8	0 31 33·79	−18 7 41·30
	β Ceti	2·3	0 38 52·31	−18 30 8·60
b	−19°, 133	8·6	0 45 59·89	−19 1 2·22

The results of the observations are contained in the following Tables II. and III.

The transit observations have been made by seven different observers, two using the old Transit Circle and recording the times of transit over fixed wires in the usual manner by means of a chronograph, and the remaining five observing with the new reversible Transit Circle by means of a travelling wire guided by hand, and automatically recording on the chronograph the instants when the wire reached certain fixed positions. The results have been referred to the equinox 1906·0 by the application of the usual star-corrections. Table II. contains the observed differences of R.A. derived by the different observers after application of these corrections, the suffixes indicating the number of separate observations involved in each result.

The heliometer measures have been made by two observers, Messrs Whittingdale and Baldwin. The results quoted in Table III. are the mean results from all the measures of each pair. They have been corrected for refraction, but, except in a few cases, no corrections for aberration or nutation have been applied. these quantities will be practically eliminated simultaneou the instrumental scale-constant and index correction t circle

TABLE II.

Observed Differences of Right Ascensions.

Epoch 1906·0.

Instrument.	8-inch Transit.		Reversible Transit Circle.			
Observer.	Power.	Pead.	Wilkin.	Jeffries.	Mullis.	Wood.
	m s	m s	m s	m s	m s	m s
1 *a*	...	4 44·075$_2$	4 44·135$_1$	...	...	...
b	...	3 23·692$_2$	3 23·597$_1$	...	...	...
2 *a*	...	5 36·185$_2$	5 36·230$_2$	...	5 36·225$_1$	...
b	...	5 50·015$_2$	5 49·970$_2$	...	5 49·935$_1$	...
3 *a*	...	4 27·514$_3$	4 27·564$_1$	...	...	4 27·524$_1$
b	...	3 44·178$_3$	3 44·141$_1$	...	...	3 44·151$_1$
4 *a*	6 18·694$_2$	6 18·647$_3$	6 18·679$_2$	...	6 18·709$_2$	...
b	5 54·470$_2$	5 54·557$_3$	5 54·512$_2$	...	5 54·472$_2$	...
5 *a*	4 34·146$_2$	4 34·168$_4$	4 34·201$_2$	...	4 34·279$_1$	4 34·259$_1$
b	4 4·520$_2$	4 4·508$_4$	4 4·485$_2$	...	4 4·455$_1$	4 4·455$_1$
6 *a*	7 1·217$_1$	7 1·197$_3$	7 1·270$_2$	7 1·247$_1$	7 1·242$_1$	7 1·310$_1$
b	6 9·909$_1$	6 9·945$_3$	6 9·873$_2$	6 9·899$_1$	6 9·917$_1$	6 9·848$_1$
7 *a*	...	7 47·742$_5$	7 47·811$_2$	7 47·762$_1$	7 47·829$_1$	...
b	...	7 53·716$_5$	7 53·691$_2$	7 53·722$_1$	7 53·670$_1$	...
8 *a*	4 47·277$_1$	4 47·196$_4$	4 47·281$_2$	...	4 47·263$_1$	...
b	5 39·054$_1$	5 39·088$_4$	5 39·048$_2$	...	5 39·010$_1$	...
9 *a*	5 50·477$_3$	5 50·447$_6$	5 50·507$_4$	5 50·526$_2$	5 50·564$_1$	...
b	6 8·389$_3$	6 8·416$_6$	6 8·356$_4$	6 8·351$_2$	6 8·313$_1$	...
10 *a*	3 25·144$_4$	3 25·122$_3$	...	...	...	...
b	4 0·112$_4$	4 0·167$_3$	...	...	...	..
11 *a*	4 49·789$_4$	4 49·739$_7$	4 49·819$_3$	4 49·823$_2$	4 49·795$_3$	4 49·846$_1$
b	5 5·322$_4$	5 5·403$_7$	5 5·356$_3$	5 5·333$_2$	5 5·365$_3$	5 5·356$_1$
12 *a*	3 3·194$_2$	3 3·162$_5$	...	...	...	...
b	2 52·048$_2$	2 52·099$_5$	...	...	...	...
13 *a*	6 20·946$_6$	6 20·913$_5$	6 20·956$_1$	6 20·946$_1$	6 21·017$_2$	6 21·014$_2$
b	6 9·867$_6$	6 9·870$_5$	6 9·876$_1$	6 9·785$_1$	6 9·776$_2$	6 9·803$_2$
14 *a*	1 32·548$_4$	1 32·537$_5$	...	...	...	...
b	1 46·042$_4$	1 46·082$_5$	...	...	...	...
15 *a*	4 26·223$_4$	4 26·251$_4$	4 26·284$_1$	4 26·251$_3$	4 26·236$_2$	4 26·279$_1$
b	5 6·768$_4$	5 6·759$_4$	5 6·761$_1$	5 6·775$_3$	5 6·743$_2$	5 6·758$_1$
16 *a*	1 55·632$_4$	1 55·639$_4$	...	..	...	...
b	1 47·332$_4$	1 47·352$_4$	...	...	...	...

TABLE II.—*continued.*

...trument.	8-inch Transit.		Reversible Transit Circle.				
...server.	Power.	Pead.	Wilkin.	Jeffries.	Mullis.	Wood.	Jackson.
	m s	m s	m s	m s	m s	m s	m s
a	3 45·821$_4$	3 45·787$_3$	3 45·816$_1$	3 45·761$_2$	3 45·812$_2$	3 45·777$_2$	3 45·781$_3$
b	4 6·091$_4$	4 6·104$_3$	4 6·086$_1$	4 6·141$_2$	4 6·102$_2$	4 6·148$_2$	4 6·141$_3$
a	5 15·029$_4$	5 15·023$_3$	...	5 15·023$_4$	5 15·047$_3$	5 15·058$_3$	5 15·045$_2$
b	4 35·123$_4$	4 35·122$_3$	...	4 35·097$_4$	4 35·059$_3$	4 35·076$_3$	4 35·087$_2$
a	3 48·363$_4$	3 48·357$_3$	...	...	...	...	...
b	2 58·309$_4$	2 58·338$_3$	...	...	...	...	...
a	6 29·125$_1$	6 29·129$_1$	6 29·078$_1$	6 29·183$_2$	6 29·111$_3$	6 29·116$_5$	6 29·149$_2$
b	7 51·533$_1$	7 51·524$_1$	7 51·522$_1$	7 51·506$_2$	7 51·510$_3$	7 51·517$_5$	7 51·482$_2$
a	1 5·031$_4$	1 5·023$_3$	...	...	...	...	...
b	1 48·093$_4$	1 48·087$_3$	...	...	...	...	...
a	4 58·727$_5$	4 58·755$_1$	4 58·725$_1$	4 58·752$_4$	4 58·745$_2$	4 58·746$_6$	4 58·757$_3$
b	5 0·103$_5$	5 0·077$_1$	5 0·127$_1$	5 0·124$_4$	5 0·063$_2$	5 0·124$_6$	5 0·110$_3$
a	3 31·661$_5$	3 31·578$_1$	...	...	...	...	...
b	2 20·476$_5$	2 20·472$_1$	...	...	...	...	...
a	4 7·040$_4$	4 6·987$_1$	4 7·017$_1$	4 7·024$_3$	4 7·077$_2$	4 7·086$_3$	4 7·044$_2$
b	3 48·891$_4$	3 48·918$_1$	3 48·858$_1$	3 48·872$_3$	3 48·843$_2$	3 48·844$_3$	3 48·865$_2$
a	4 28·907$_3$	4 28·878$_1$	4 28·858$_2$	4 28·907$_3$	4 28·870$_3$	4 28·886$_4$	4 28·907$_4$
b	5 23·885$_3$	5 23·973$_1$	5 23·913$_2$	5 23·897$_3$	5 23·925$_3$	5 23·918$_4$	5 23·894$_4$
a	6 8·872$_2$	...	6 8·880$_1$	6 8·901$_2$	6 8·903$_2$	6 8·918$_3$	6 8·923$_3$
b	6 1·094$_2$	...	6 1·012$_1$	6 1·079$_2$	6 1·040$_2$	6 1·037$_3$	6 1·078$_3$
a	4 37·844$_1$	4 37·796$_1$	4 37·875$_1$	4 37·846$_1$	4 37·824$_1$	4 37·875$_2$	4 37·849$_3$
b	5 48·458$_1$	5 48·531$_1$	5 48·443$_1$	5 48·437$_1$	5 48·468$_1$	5 48·434$_2$	5 48·390$_3$
a	5 52·140$_2$	...	5 52·109$_2$	5 52·164$_1$	5 52·072$_2$	5 52·123$_3$	...
b	5 11·474$_2$	...	5 11·473$_2$	5 11·478$_1$	5 11·506$_2$	5 11·453$_3$	...
a	6 22·070$_1$	6 21·909$_1$	6 22·031$_2$	6 21·984$_1$	6 22·049$_3$	6 22·021$_3$	6 22·004$_3$
b	7 7·631$_1$	7 7·670$_1$	7 7·618$_2$	7 7·630$_1$	7 7·603$_3$	7 7·589$_3$	7 7·637$_3$
a	5 18·536$_1$	5 18·395$_1$	5 18·458$_2$	5 18·393$_1$	5 18·426$_3$	5 18·480$_3$	5 18·395$_3$
b	5 53·125$_1$	5 53·394$_1$	5 53·313$_2$	5 53·375$_1$	5 53·339$_3$	5 53·309$_3$	5 53·345$_2$
a	6 9·257$_1$	6 9·106$_1$	6 9·236$_2$	6 9·266$_1$	6 9·234$_3$	6 9·258$_3$	6 9·241$_3$
b	7 46·215$_1$	7 46·213$_1$	7 46·099$_2$	7 46·099$_1$	7 46·191$_3$	7 46·094$_3$	7 46·112$_3$
a	7 4·385$_1$	...	7 4·528$_1$	...	7 4·506$_3$	7 4·490$_3$	7 [illegible]
b	7 55·441$_1$	...	7 55·280$_1$	...	7 55·295$_3$	7 55·310$_3$	7 [illegible]
a	...	...	7 18·515$_2$	7 18·576$_1$	7 18·537$_3$	7 18·559$_3$	7 [illegible]
b	...	...	7 7·585$_2$	7 7·586$_1$	7 7·642$_3$	7 7·572$_3$	7 [illegible]

Table III.

Table of Instrumental Distances and Position-Angles observed with the Heliometer.

Group.	No. of Observation. D.	No. of Observation. P.A.	a. Distance.	a. Position-Angle.	b. Distance.	b. Position-Angle.
			″	° ′ ″	″	° ′ ″
1	3	2	4289·419	277 59 48	3299·816	293 7 31
2	3	1	4569·911	278 23 13	4711·909	273 9 33
3	2	1	3971·132	277 9 18	3304·070	269 26 39
4	3	2	5702·217	254 0 19	5422·158	251 34 51
5	2	2	4617·319	241 3 32	4483·717	233 23 13
6	3	1	6262·879	264 14 24	5506·352	263 49 53
7	3	1	7015·268	267 58 24	7102·951	267 57 26
8	3	1	4297·166	268 7 12	5067·526	271 44 11
9	3	2	5510·700	252 22 22	5693·633	255 46 51
10	3	3	3849·927	233 18 15	4650·005	231 6 24
11	4	2	4381·633	262 47 1	4716·047	256 23 16
12	3	2	2868·977	287 32 12	2665·012	285 18 48
13	3	2	5716·171	279 28 13	5693·492	286 4 18
14	3	3	1642·636	237 21 33	1845·856	238 55 22
15	3	3	4717·547	302 44 44	5222·509	298 51 26
16	3	3	2369·918	227 36 35	2380·705	223 6 48
17	3	1	3211·763	270 46 36	3528·059	263 31 31
18	3	2	4796·276	259 16 9	4263·023	255 4 48
19	3	3	3954·919	240 6 58	2789·282	253 14 37
20	3	2	5719·491	283 45 15	6848·518	281 2 45
21	3	3	963·525	285 42 17	1745·713	298 23 20
22	3	2	4708·532	251 41 55	4843·676	248 12 41
23	3	1	3161·216	267 42 56	2120·031	261 56 20
24	3	1	3679·683	277 12 26	3388·578	272 36 47
25	3	2	4095·903	259 6 58	5085·403	252 26 28
26	3	2	5686·703	253 7 15	5493·288	256 0 56
27	3	1	4045·352	268 24 42	5097·784	276 46 35
28	3	3	5511·906	292 48 46	5327·954	302 24 6
29	3	1	5389·438	275 53 44	6008·341	273 10 11
30	3	2	4764·892	284 54 45	5239·097	283 7 28
31	3	1	5538·700	264 21 44	6949·113	270 16 35
32	3	1	6297·759	274 15 11	7062·612	265 40 51
33	3	2	6389·479	282 45 16	6349·276	287 32 53

Reduction of Heliometer Observations.

The approximate places of the stars for the epoch 1906·0 are quoted in Table I. With these approximate places, tabular distances and position-angles were computed. On comparing these tabular distances with the distances as derived from the heliometer measures we obtain relations between the corrections to the co-ordinates of the stars of the following form,—

$$S_a\sigma + (\Delta a_2 - \Delta a_1)\cos\delta_a \sin p_a + (\Delta\delta_2 - \Delta\delta_1)\cos p_a = O_a - C_a$$
$$S_b\sigma + (\Delta a_3 - \Delta a_2)\cos\delta_b \sin p_b + (\Delta\delta_3 - \Delta\delta_2)\cos p_b = O_b - C_b$$

where the suffixes 1, 2, 3 refer to the stars in increasing order of R.A., the suffixes a, b to the pairs 1–2, 2–3 respectively. S denotes the distance, σ a constant depending on the instrumental scale-value, δ the mean declination, and p the position-angle of a pair, and O, C respectively the observed and computed distances.

On eliminating σ we derive

$$(\Delta a_2 - \Delta a_1)\frac{\cos\delta_a \sin p_a}{S_a} - (\Delta a_3 - \Delta a_2)\frac{\cos\delta_b \sin p_b}{S_b}$$
$$+ (\Delta\delta_2 - \Delta\delta_1)\frac{\cos p_a}{S_a} - (\Delta\delta_3 - \Delta\delta_1)\frac{\cos p_b}{S_b} = \frac{O_a - C_a}{S_a} - \frac{O_b - C_b}{S_b} \quad . \; . \; . \; \text{I.}$$

In like manner, from the observations of position-angle introducing a constant quantity π to denote the mean index correction to the position-circle during the several observations of a group, we form equations of condition as follows,—

$$S_a\pi + (\Delta a_2 - \Delta a_1)\cos\delta_a \cos p_a - (\Delta\delta_2 - \Delta\delta_1)\sin p_a = (O_a' - C_a')\sin S_a$$
$$S_b\pi + (\Delta a_3 - \Delta a_2)\cos\delta_b \cos p_b - (\Delta\delta_3 - \Delta\delta_2)\sin p_b = (O_b' - C_b')\sin S_b$$

and on elimination of π, S_a, S_b being expressed in seconds of arc

$$(\Delta a_2 - \Delta a_1)\frac{\cos\delta_a \cos p_a}{S_a} - (\Delta a_3 - \Delta a_2)\frac{\cos\delta_b \cos p_b}{S_b}$$
$$- (\Delta\delta_2 - \Delta\delta_1)\frac{\sin p_a}{S_a} + (\Delta\delta_3 - \Delta\delta_2)\frac{\sin p_b}{S_b} = (O_a' - C_a' - O_b' + C_b')\sin 1'' \quad . \; . \; . \; \text{II.}$$

On eliminating $\Delta\delta_2$ from the equations I. and II. we obtain a linear relation between Δa_1, Δa_2, Δa_3, $\Delta\delta_1 - \Delta\delta_3$. The algebraic elimination is cumbersome, but the numerical elimination is easily performed in special cases. This relation takes the form

$$a(\Delta a_2 - \Delta a_1) - \beta(\Delta a_3 - \Delta a_2) = \gamma(\Delta\delta_1 - \Delta\delta_3) + n$$

where, if the stars selected are well chosen, we have approxi[illegible] $a = \beta$, while γ is small in comparison with either of th[illegible] forming these equations such a factor has been introduc[illegible] reduce the coefficient of Δa_2 to unity, *i.e.* so that $a + \beta =$ [illegible]

The following are the equations of condition resulting in this manner from the heliometer measures:—

Equations of Condition resulting from Heliometer Measures.

Group.				
1	$0.433(\Delta\alpha_2 - \Delta\alpha_1)$	$-0.566(\Delta\alpha_3 - \Delta\alpha_2)$	$= +0.065(\Delta\delta_1 - \Delta\delta_3)$	$+0.29$
2	0.508	−0.492	= −0.025	+0.53
3	0.455	−0.546	= −0.035	+0.33
4	0.487	−0.514	= −0.011	+0.25
5	0.493	−0.506	= −0.035	+0.17
6	0.468	−0.532	= −0.002	−0.43
7	0.503	−0.497	= 000	+0.42
8	0.541	−0.459	= +0.005	−0.03
9	0.507	−0.492	= +0.015	−0.04
10	0.547	−0.453	= −0.009	+0.63
11	0.519	−0.481	= −0.028	+0.44
12	0.482	−0.518	= −0.009	+0.70
13	0.499	−0.501	= +0.029	+0.65
14	0.529	−0.470	= +0.007	+1.07
15	0.527	−0.475	= −0.017	+0.23
16	0.501	−0.499	= −0.020	+0.11
17	0.524	−0.476	= −0.034	+0.85
18	0.470	−0.530	= −0.018	−0.10
19	0.412	−0.587	= +0.056	+0.96
20	0.545	−0.454	= −0.012	+0.49
21	0.647	−0.353	= +0.053	+0.01
22	0.507	−0.493	= −0.015	+0.01
23	0.401	−0.599	= −0.024	−0.44
24	0.480	−0.521	= −0.020	−0.09
25	0.553	−0.446	= −0.029	−0.30
26	0.491	−0.509	= +0.013	+0.29
27	0.559	−0.441	= +0.037	0.00
28	0.491	−0.509	= +0.043	+0.12
29	0.527	−0.473	= −0.013	−0.03
30	0.524	−0.475	= −0.008	+0.09
31	0.556	−0.443	= +0.026	+0.27
32	0.529	−0.471	= −0.038	+0.32
33	0.499	−0.501	= +0.020	+0.68

The quantities $\Delta\delta_1$, $\Delta\delta_3$ are retained in the right-hand members, as in a few cases the finally adopted values of the declinations differ from those quoted in Table I., which were derived from a few early observations only. The following are the corrections derived from the inclusion of additional observations of the declination with the Transit Circles :—

Group.	$\Delta\delta_1$	$\Delta\delta_3$
3	$-0''.38$	...
18	$-\ .39$	-0.51
19	$-\ .06$	-1.05
20	$-\ .16$	$-\ .11$
21	$-\ .50$	$-\ .30$
28	...	$+\ .30$
32	$-\ .13$	$-\ .68$
33	...	$-\ .26$

In order to subject the magnitude personality to an analytical treatment, it was now assumed that each transit observation is affected by an error which may be expressed analytically by the formula

$$\alpha(m-4)+\beta(m-4)^2$$

where m denotes the magnitude of the star and α, β constants for the observer. The adopted magnitudes are those quoted in Table I.

Thus for any pair of stars of R.A. a_1, a_2, say of magnitudes m_1, m_2, the observed difference of R.A. may be equated to

$$a_2-a_1+\alpha(m_2-m_1)+\beta(m_2-m_1)(m_2+m_1-8)\ ;$$

or if we denote this quantity by O and the tabular difference of R.A. as dependent on Table I. by C we find

$$\Delta a_2-\Delta a_1=\mathrm{O}-\mathrm{C}-\left[\alpha(m_2-m_1)+\beta(m_2-m_1)(m_2+m_1-8)\right]$$

On substituting these expressions in the left-hand members of the equations of condition resulting from the heliometer measures, we derive a series of linear equations for the determination of the quantities of α, β which may be expressed in the following tabular form. The suffixes attached to the absolute terms represent the number of meridian observations involved in each.

Group.	Coefficients of		Absolute terms for						
	α	β	Power.	Pead.	Wilkin.	Jeffries.	Mullis.	Wood.	Jackson.
			"	"	"	"	"	"	"
1	5·31	18·73	...	$+0.78_2$	-0.43_1	...	...	...	...
2	7·30	13·92	...	$+1.07_2$	$+0.39_2$	...	$+0.18_1$	...	...
3	3·61	18·02	...	$+1.08_3$	$+0.44_1$	...	...	$+0.78_1$	-0.41_1
4	5·07	12·50	$+0.34_2$	$+1.37_3$	$+0.78_2$	...	$+0.25_2$	...	$+0.08$
5	4·89	3·08	$+1.16_2$	$+0.91_4$	$+0.49_3$	...	-0.31_1	-0.16_1	-0.40
6	5·30	22·74	$+0.65_1$	$+1.08_3$	-0.02_2	$+0.36_1$	$+0.54_1$	-0.49_1	-0.49
7	5·41	24·14	...	$+1.28_5$	$+0.57_2$	$+1.18_1$	$+0.29_1$	...	-0.24_1
8	4·70	26·80	$+0.28_1$	$+1.17_4$	$+0.20_2$	...	$+0.07_1$	...	-0.53_1
9	5·40	23·25	$+0.66_4$	$+1.08_6$	$+0.18_4$	0.00_2	-0.56_1	...	$+0.04_2$
10	5·03	19·84	$+0.10_4$	$+0.66_3$	...	...	...	...	...
11	3·72	9·92	$+0.27_4$	$+1.25_7$	$+0.29_3$	$+0.09_2$	$+0.54_3$	$+0.07_1$	$+0.04_3$
12	3·85	12·45	$+0.20_2$	$+0.82_5$	...	...	...	...	...
13	7·30	7·35	$+0.80_6$	$+1.07_5$	$+0.80_1$	$+0.19_1$	-0.41_2	-0.19_2	$+0.81_1$
14	7·82	12·06	$+1.05_4$	$+1.42_5$	...	...	...	...	...
15	4·74	16·50	$+0.61_4$	$+0.33_4$	$+0.08_1$	$+0.44_3$	$+0.34_2$	$+0.10_1$	-0.19_3
16	4·35	13·55	-0.19_4	-0.10_4	...	...	...	...	...
17	3·30	18·34	-0.13_4	$+0.24_3$	-0.12_1	$+0.70_8$	$+0.03_2$	$+0.62_3$	$+0.55_3$
18	3·67	17·42	$+0.66_4$	$+0.69_3$	...	$+0.50_4$	$+0.02_3$	$+0.08_3$	$+0.27_1$
19	3·39	15·10	$+0.72_4$	$+1.02_3$	...	...	...	...	...
20	4·19	18·48	$+0.23_1$	$+0.14_1$	$+0.55_1$	-0.43_2	$+0.20_3$	$+0.20_5$	-0.30_1
21	2·93	9·82	$+0.24_4$	$+0.29_3$	...	...	...	...	...
22	5·01	15·13	$+0.13_5$	-0.27_1	$+0.33_1$	$+0.10_4$	-0.29_2	$+0.15_6$	-0.03_3
23	5·83	22·40	-0.18_5	$+0.28_1$	...	...	...	...	...
24	6·20	21·68	$+0.44_4$	$+1.03_1$	$+0.35_1$	$+0.40_3$	-0.21_2	-0.26_3	$+0.21_2$
25	4·35	16·31	-0.29_3	$+0.54_1$	$+0.30_2$	-0.21_3	$+0.28_3$	$+0.10_4$	-0.23_4
26	3·71	19·67	$+0.75_2$	...	$+0.06_1$	$+0.42_2$	$+0.11_2$	-0.03_3	$+0.25_2$
27	4·38	22·56	$+0.25_1$	$+1.14_1$	-0.11_1	$+0.10_1$	$+0.49_1$	-0.17_2	-0.23_3
28	5·84	17·60	-0.09_2	...	$+0.13_2$	-0.23_1	$+0.66_2$	-0.11_3	...
29	4·45	22·47	-0.19_1	$+1.33_1$	$+0.03_2$	$+0.48_1$	-0.22_3	-0.10_3	$+0.37_1$
30	4·14	21·71	-1.89_1	$+1.13_1$	-0.05_2	$+1.01_1$	$+0.49_3$	-0.15_3	$+0.78_1$
31	4·18	17·26	$+0.76_1$	$+2.01_1$	$+0.16_2$	-0.09_1	$+0.79_3$	-0.05_3	$+0.21_1$
32	4·57	18·18	$+2.08_1$	...	-0.20_1	...	$+0.07_3$	$+0.31_3$	$+0.24_2$
33	6·40	19·55	...	...	$+0.76_2$	$+0.31_1$	$+1.02_3$	$+0.33_3$	$+0.40_1$

For the purpose of combining these equations, it was assumed that the errors in the absolute terms resulting from the heliometer measures were insignificant compared with those resulting from the transit observations, and the equations were accordingly weighted according to the number of separate observations of each group

obtained by the meridian circle observers. A least square solution then led to the following results:—

Observer.	α.		β.		Probable error of	
	In arc.	In time.	In arc.	In time.	α.	β.
	"	s	"	s	s	s
Power .	+0·134	+0·0089	−0·020	−0·0013	±0·0020	±0·0006
Pead .	+ ·148	+ ·0099	+ ·011	+ ·0007	± ·0016	± ·0005
Wilkin .	+ ·117	+ ·0078	− ·018	− ·0012	± ·0012	± ·0003
Jeffries .	− ·053	− ·0035	+ ·028	+ ·0019	± ·0027	± ·0007
Mullis .	+ ·011	+ ·0007	+ ·010	+ ·0007	± ·0023	± ·0006
Wood .	− ·015	− ·0010	+ ·006	+ ·0004	± ·0016	± ·0004
Jackson .	− ·003	− ·0002	+ ·006	+ ·0004	± ·0025	± ·0006

The resulting corrections to the observations to make all correspond with those of stars of the fourth magnitude are as follows:—

Correction for Personal Equation in R.A. depending on Magnitude.

Mag.	Power.	Pead.	Wilkin.	Jeffries.	Mullis.	Wood.	Jackson.
	s	s	s	s	s	s	s
0	+0·057	+0·029	+0·050	−0·044	−0·007	−0·010	−0·007
1	38	24	34	27	− 4	7	− 4
2	23	17	21	14	− 1	4	− 2
3	+ ·010	+ ·009	+ ·009	− ·005	000	− ·001	− ·001
4	...	...	...	...	...	...	...
5	− ·008	− ·011	− ·007	+ ·002	− ·002	− ·001	000
6	− 13	23	11	000	− 4	000	− ·001
7	− 16	36	12	− ·006	− 8	− ·001	− ·003
8	− 15	51	12	− 16	− 13	− ·002	− ·006
9	− 14	− ·068	− ·009	− ·029	− 20	− ·005	− ·009

The quantities thus derived in the case of the observers who used the travelling wire method appear to be quite insignificant except at the extreme limits of magnitude involved in the table. The strength of the determination at these limits depends, however, not so much on the actual weight derived directly from the observations, as on the weight artificially extended to it by the assumption that the magnitude personality may be expressed by the formula $\alpha m+\beta m^2$.

The use of such a formula can only be justified as a convenient means of interpolation over the range of magnitude covered by stars actually observed. If we limit ourselves to the range within which limits the majority of the stars are contair may safely conclude that the observations dealt witl evidence of the existence of sensible personality d magnitude. Beyond these limits the quantities ar than might reasonably be expected to arise from t

empirical 'extrapolation' formula, which will have the effect of magnifying the accidental errors of observation at the extremities of the table.

The results for the observers with the old Transit Circle from observations made partly through screens in 1900 and 1904 are as follows:—

Mag.	Power.	Pead.
	s	s
0	+0·008	+0·037
1	7	28
2	5	18
3	+0·003	+ ·009
4	...	...
5	− ·004	− ·009
6	− 9	− ·018
7	− 15	− ·026
8	− 21	− ·034
9	− 28	− ·042

The agreement with the present results is as close as could be expected, except in the case of the brighter stars for Power and the fainter stars for Pead. The discordances are doubtless due to the inadequacy of the material employed to correctly determine the magnitude personality at these extreme limits of magnitude, rather than to any real change in the observer's habits during the interval between the two sets of observations.

The plan of these observations was prepared by Sir David Gill, and the observations were well advanced before his departure from the Cape. The observations were completed and the computations made and prepared for press in their present form under the direction of Mr S. S. Hough.

On the Value of the Solar Parallax resulting from the Greenwich Photographs of Eros, 1900–1901.

(*Communicated by the Astronomer Royal.*)

The discussion of the photographs of Eros taken at Greenwich during the opposition of 1900–1901 has now been completed, and a value of the solar parallax has been deduced as resulting from the photographs taken at Greenwich. The plan of the Eros Commission contemplated, ultimately, the combined discussion of the photographs taken at all the co-operating observatories, but there is obviously great difficulty in treating such a mass of heterogeneous material, and it seems desirable, in the first instance, to discuss separately the observations of the individual observatories, and deduce a value of the solar parallax in each case.

With this view the results obtained from the Greenwich photographs are here given. There were in all 197 photographs taken with the Astrographic 13-inch refractor and 153 taken with the Thompson 26-inch refractor between 1900 October 1 and 1901 February 25, but for the determination of the solar parallax the discussion has been confined to the period from 1900 October 14 to 1901 January 18 (151 photographs with the Astrographic and 103 with the Thompson), the material before and after these dates not being suitable for determination of the solar parallax, though useful for the position of Eros.

It may be recalled here that, as explained in a paper on the "New Greenwich micrometer for measurement of photographs of Eros" (*Monthly Notices*, vol. lxiv. p. 633), ten to twelve "reference" stars and six "comparison" stars were measured with Eros on each Astrographic photograph, and the results from these two sets of stars to which Eros was referred have been discussed separately. The "reference" stars selected from M. Loewy's list within 55′ of the plate centre are all brighter than ninth magnitude and therefore brighter than Eros. The "comparison" stars, on the other hand, are of approximately the same brightness as the planet and within 25′ of the plate centre, so as to be well within the field of the Thompson plates for which they served as points of reference.

The determination of the movement of Eros in the interval between the groups of photographs compared, which is essential for the deduction of the parallax, was first undertaken. The measured positions of Eros were compared with tabular places from M. Millosevich's ephemerides (Astrographic Conference, Circular No. 9), using the value 8″·800 for the solar parallax, which was a sufficient approximation for this purpose. The change of error of the ephemeris during the interval between the groups of observations to be compared for parallax was generally considerable (amounting at times to ″·10 per day), but it could be determined with great accuracy. The means of groups of measured positions of Eros were taken so as to give the error of the ephemeris at intervals of about five days; from these the error of the ephemeris was represented by a smooth curve, which represented the observations very satisfactorily.

The correction to the adopted parallax was derived by comparing observations at different hour-angles. It was not usually desirable to make comparisons between groups of observations separated by an interval of more than one day, owing to the accidental errors introduced by erroneous places of the st which Eros was referred. The same reference and co stars were used throughout each night, so that in ded parallax from comparisons between photographs tak evening and the following morning errors in the sta eliminated; but in all other comparisons some of th are different for the two groups of plates, and th interval the weaker the connection between the star-p

From a discussion of the *Right Ascensions* the following results have been derived :—

	Solar Parallax.	*Theoretical Weight.*
Astrographic photographs, Reference Stars,	$8''.793 \pm ''.005$	39·7
,, ,, Comparison Stars,	8·809± ·0052	39·7
Thompson photographs, Comparison Stars,	8·800± ·0063	31·9
Combined discussion of Astrographic and Thompson photographs, Comparison Stars,	8·800 ± ·0044	71·1

The unit of weight used here is the weight of a comparison between one morning and one evening photograph under practically the most favourable conditions, viz. when Eros undergoes a parallactic displacement of 25″ between the two plates.

The difference between the results derived from the Reference and Comparison Stars is of a systematic character. It is found that if plates taken at a large hour-angle are compared with those taken near the meridian (on the same night and with the telescope on the same side of the pier), the comparison stars are displaced relatively to the reference stars, always in the same direction and in the mean by about ″·07 in R.A. It is not clear whether this displacement depends on the comparison stars being fainter than the reference stars, or their being nearer to the centre of the plate. In either case we must suppose that Eros behaves like the comparison stars rather than like the reference stars. For that reason only the results from the comparison stars are used in obtaining the final value of the parallax. It should be noticed that no diminution of the accidental error would be obtained by taking the mean of the results of the comparison and reference stars; for the two determinations are not independent as regards accidental error, depending, as they do, on the same measures of the image of Eros.

The close agreement between the results derived from the Astrographic and Thompson photographs (Comparison stars), and the fact that the same star-places were used in the two cases, seemed to justify a combined discussion of their results, regarding the two sets of places as homogeneous. There was a considerable gain in thus treating them together, as the two instruments had supplemented one another considerably. In this discussion, comparisons extending over more than a day were rigorously excluded. The result given above must be regarded as the principal conclusion of the whole investigation. The results of the separate comparisons are given in Table I., at the end of the paper.

Without much loss of material, the discussion could be rearranged, so that comparisons were only made between the evening and following morning observations. A very considerable reduction in the probable error (inferred from the discordances) took place (notwithstanding the loss of material), owing to the elimination of erroneous star-places. A comparison was also made between the morning and following evening observations, the probable error in this case being more than twice as great.

Thompson and Astrographic Combined.		*Theoretical Weight.*
Evening to Morning comparisons only,	$8''.807 \pm ''.0036$	58·5
Morning to Evening comparisons only,	$8·801 \pm ·0080$	35·4

The probable error for unit weight of an evening to morning comparison is $\pm''·028$, and for a morning to evening comparison is $\pm''·048$. The greater part of this difference in the respective probable errors must be attributed to the errors of star-places.

In a former paper, "On the errors of a photographed réseau" (*Monthly Notices*, vol. lxvii. p. 175), it has been shown that the division errors of the central lines of the réseau between which Eros falls affect systematically the deduced value of the solar parallax. Details were there given of the determination of these errors. The corrections there deduced (additional to those which had been provisionally employed), amounting in the mean to $+''·065$ for Astrographic photographs, were applied to the positions of Eros before deducing the preceding results. This quantity was very closely confirmed by a discussion of the residuals of the comparison stars, which indicated $+''·068$. The probable errors of the two results separately are, however, about $\pm''·006$ and $\pm''·008$ respectively. In order that the effect of the uncertainty in the determination of division errors may be estimated, the results obtained before these corrections were applied are given below.

Provisional Results uncorrected for Residual Division Error.

Astrographic photographs, Reference Stars, .	$8''.758 \pm ''.0054$
,, ,, Comparison Stars,	$8·774 \pm ·0059$
Thompson	$8·802 \pm ·0071$

It will be seen that the result from the Thompson photographs is practically unaltered by the application of the new division errors. This is due to the fact that, in the first place, the scale of the Thompson photographs is twice that of the Astrographic, and that the corrections for division error are consequently halved. Further, the accidental variations in the position of Eros relatively to the lines of the réseau were much greater than in the Astrographic photographs, and during part of the opposition the réseau was not reversed when the telescope passed from E to W of the pier. For Astrographic photographs the application of the division correction $''·065$ has increased the deduced value of the solar parallax by $''·035$.

A discussion was also made using only comparisons between plates taken with the telescope on the same side of the pier, so that the division error of the réseau was eliminated.

Telescope E only.	*Solar Parallax.*	*Theoretical Weight.*
Astrographic reference stars,	$8''.748 \pm ''.009$	12·5
,, comparison ,,	$8·783 \pm ·009$	12·5

The weight of this determination is small, a large part of the material being wasted. It, however, shows very markedly the difference between the reference and comparison stars.

From the *Declinations* of Eros the following values were obtained :—

	Solar Parallax.	*Theoretical Weight.*
Astrographic photographs, reference stars,	8″·783 ± ″·018	7·1
,, ,, comparison ,,	8·819 ± ·014	7·1
Thompson photographs ,,	8·783 ± ·016	5·5
Mean (comparison stars),	8·801 ± ·016	

No special determination of the division errors of the central lines of the réseau was needed for the declinations. The parallax being of the same sign on both sides of the meridian, comparisons were made between plates taken at a large hour-angle E or W and those near the meridian. About the same proportion of low plates were taken with the telescope E and W (réseau direct and reversed) as meridian plates; accordingly the value of the division error of the central lines of the réseau does not affect systematically the deduced parallax. The method employed for deducing the parallax was to divide the observations into groups, each extending over two or three days, and solve each group separately by least squares. The results from the individual groups for Astrographic Comparison Stars are given in Table II.

TABLE I.

Right Ascensions. Astrographic and Thompson combined.

Date.	No. of Plates compared.	Difference of Parallax.	Weight.	Correction to Solar Parallax (adopted value 8″·800).
1900. Oct. 14–15	4, 1	10″	·3	− ″·045
,, 20	3, 1	13	·4	+ ·117
,, 21	4, 2	23	2·3	+ ·007
,, 26	6, 4	25	4·8	+ ·006
,, 27	5, 4	25	4·5	+ ·038
,, 28–29	2, 5	26	3·1	− ·009
Nov. 8–9	2, 2	19	1·2	− ·012
,, 9–10	3, 3	27	3·5	+ ·012
,, 10–11	2, 3	27	2·8	+ ·012
,, 13	5, 5	28	6·3	− ·011
,, 14–15	3, 5	24	3·5	− ·067
,, 22–23	4, 5	28	5·6	+ ·009
,, 27	5, 4	15	1·6	− ·005
Dec. 6–7	9, 3	16	1·9	+ ·054
,, 9–10	3, 1	25	1·5	+ ·011

TABLE I.—*continued.*

Date.	No. of Plates compared.	Difference of Parallax.	Weight.	Correction to Solar Parallax (adopted value 8″·800).
1900.		″		″
Dec. 13	6, 3	21	2·8	− ·044
,, 15–16	6, 6	26	6·5	− ·018
,, 19	5, 5	25	5·0	+ ·003
,, 21	2, 6	27	3·5	+ ·033
,, 28–29	3, 3	12	·7	+ ·033
1901.				
Jan. 5	4, 6	12	1·1	·000
,, 8–9	7, 4	18	2·6	− ·082
,, 13–14	6, 2	22	2·4	+ ·030
,, 15	5, 5	20	3·2	+ ·034
,, 17–18	2, 3	16	1·0	(+ ·277)

Mean Correction + ″·000 ± ″·0044.

The weight is given by the formula $\frac{2mn}{m+n}\left(\frac{\delta\pi}{25''}\right)^2$ where m and n are the numbers of plates in the two groups compared, and $\delta\pi$ is the difference of parallax of Eros between the two groups. No plate is used twice over; *i.e.* if an evening plate is compared with plates taken on the preceding morning, it is not also compared with those taken on the succeeding morning.

The discordant result for Jan. 17–18 has been rejected, as there are several unsatisfactory circumstances in the Thompson photographs on which it depends. On four of the five photographs the images of Eros were noted as difficult to measure, and further, different reference stars had to be used for the two days compared, thus introducing the errors of the star-places which depend on visual observations alone.

TABLE II.

Declinations. Astrographic from Comparison Stars.

Date.	Weight.	Correction to Parallax.
1900.		″
Oct. 14–15	·2	+ ·062
,, 20–21	·4	− ·070
,, 26–27	·6	− ·023
Nov. 9–11	·9	+ ·018
,, 13–15	1·0	+ ·049
,, 22–23	·7	− ·082
Dec. 6–7	·6	+ ·047
,, 13	·5	+ ·040
,, 15–17	·5	+ ·128
,, 19–21	·7	+ ·179
,, 26–29	·3	− ·058

TABLE II.—*continued.*

Date.	Weight.	Correction to Parallax.
1901.		"
Jan. 4–5	·4	+ ·060
,, 8–9	·4	− ·040
,, 14–15	·2	− ·100

Mean Correction $+\overset{''}{\cdot}019 \pm \overset{''}{\cdot}014$.

Royal Observatory, Greenwich :
1907 *April* 9.

The Perturbations of Halley's Comet. By P. H. Cowell and A. C. D. Crommelin.

The differential equations of variation of the four elements, n, e, ϖ, ϵ, the mean motion, eccentricity, longitude of perihelion and epoch are—

$$\frac{1}{m'}\frac{dn}{n} = n_x(a^2\mathrm{X}du) + n_y(a^2\mathrm{Y}du)$$

$$\frac{1}{m'}\,de = e_x(a^2\mathrm{X}du) + e_y(a^2\mathrm{Y}du)$$

$$\frac{1}{m'}\,ed\varpi = \varpi_x(a^2\mathrm{X}du) + \varpi_y(a^2\mathrm{Y}du)$$

$$\frac{1}{m'}\left[d\epsilon - d\varpi\{1 - \sqrt{(1-e^2)}\}\right] = \epsilon_x(a^2\mathrm{X}du) + \epsilon_y(a^2\mathrm{Y}du)$$

where

$$n_x = 3\sin u \qquad n_y = -3\sqrt{(1-e^2)}\cos u$$

$$e_x = -\sqrt{(1-e^2)}\sin u\cos u \qquad e_y = \sqrt{(1-e^2)}\{1 - 2e\cos u + \cos$$

$$\varpi_x = -\sqrt{(1-e^2)}\{1 - e\cos u + \sin^2 u\} \qquad \varpi_y = \sin u\cos u - e\sin u$$

$$\epsilon_x = 2\{-\cos u + e(1+\cos^2 u) - e^2\cos u\} \qquad \epsilon_y = 2\sqrt{(1-e^2)}\{-\sin u + e\sin u$$

where $m'\mathrm{X}$, $m'\mathrm{Y}$ are the components of the disturbing force, the axes of x, y being drawn in the plane of the orbit towards perihelion and parallel to the minor axis respectively.

The quantities n_x, etc., are functions of the eccentricity e, and the eccentric anomaly u only. They are tabulated in Table I. for every odd degree of u, and for the value $\sqrt{(1-e^2)} = \cdot 254000$ or $e = \cdot 967204$. It is clearly unnecessary to tabulate beyond $u = 180°$.

All quantities in the above equations $\frac{dn}{n}$, etc., n_x, etc., $a^2\mathrm{X}du$, etc., are pure numbers of zero dimensions in space and time. All reference to such arbitrary units as the Earth's mean distance and the mean solar day is thereby avoided.

The first part of Table I. gives the comet's x and y co-ordinates divided by the semi-major axis, and also nt, the comet's mean anomaly in circular measure.

TABLE I.

First and Second Quadrants. Unit = 0·00001.

$\frac{x}{a}$	$\frac{y}{a}$	nt	$2\pi - nt$	u	$\frac{x}{a}$	$\frac{y}{a}$	nt	$2\pi - nt$
+ 32 64	+ 4 43	0 57	6282 61	91°	− 984 66	+ 253 96	621 19	5661 99
31 43	13 29	1 74	6281 44	93	1019 54	253 65	657 28	5625 91
28 99	22 14	2 97	6280 22	95	1054 36	253 03	694 54	5588 64
25 34	30 96	4 30	6278 88	97	1089 07	252 11	732 97	5550 21
20 48	39 73	5 78	6277 41	99	1123 64	250 87	772 58	5510 60
14 42	48 46	7 44	6275 75	101	1158 01	249 33	813 35	5469 84
+ 7 17	57 14	9 32	6273 87	103	1192 16	247 49	855 27	5427 91
− 1 28	65 74	11 47	6271 72	105	1226 02	245 34	898 35	5384 84
10 90	74 26	13 92	6269 26	107	1259 58	242 90	942 56	5340 62
21 68	82 69	16 72	6266 46	109	1292 77	240 16	987 90	5295 28
33 62	91 02	19 90	6263 28	111	1325 57	237 13	1034 35	5248 83
46 70	99 25	23 51	6259 68	113	1357 94	233 81	1081 91	5201 28
60 90	107 34	27 57	6255 61	115	1389 82	230 20	1130 54	5152 64
76 20	115 31	32 14	6251 05	117	1421 19	226 32	1180 25	5102 94
92 58	123 14	37 24	6245 95	119	1452 01	222 15	1231 01	5052 18
110 04	130 82	42 90	6240 28	121	1482 24	217 72	1282 79	5000 39
128 53	138 34	49 18	6234 00	123	1511 84	213 02	1335 59	4947 60
148 05	145 69	56 10	6227 08	125	1540 78	208 06	1389 38	4893 81
168 57	152 86	63 69	6219 49	127	1569 02	202 85	1444 12	4839 06
190 06	159 85	72 00	6211 19	129	1596 52	197 40	1499 82	4783 37
212 49	166 64	81 04	6202 14	131	1623 26	191 70	1556 42	4726 76
235 85	173 23	90 86	6192 32	133	1649 20	185 76	1613 92	4669 26
260 10	179 60	101 48	6181 70	135	1674 31	179 60	1672 28	4610 91
285 21	185 76	112 94	6170 25	137	1698 56	173 23	1731 47	4551 72
311 14	191 70	125 25	6157 93	139	1721 91	166 64	1791 46	4491 72
337 88	197 40	138 46	6144 73	141	1744 35	159 85	1852 23	4430 95
365 39	202 85	152 58	6130 60	143	1765 84	152 86	1913 74	4369 44
393 63	208 06	167 64	6115 54	145	1786 36	145 69	1975 96	4307 22
422 56	213 02	183 67	6099 51	147	1805 88	138 34	2038 86	4244 33
452 17	217 72	200 69	6082 50	149	1824 37	130 82	2102 39	4180 79
482 39	222 15	218 72	6064 47	151	1841 82	123 14	2166 54	4116 65
513 21	226 32	237 77	6045 41	153	1858 21	115 31	2231 25	4051 93
544 59	230 20	257 88	6025 31	155	1873 51	107 34	2296 50	3986 68
576 47	233 81	279 06	6004 13	157	1887 71	99 25	2362 25	3920 94
608 84	237 13	301 31	5981 87	159	1900 78	91 02	2428 46	3854 73
641 64	240 16	324 68	5958 51	161	1912 72	82 69	2495 09	[illegible]
674 83	242 90	349 15	5934 04	163	1923 51	74 26	2562 [illegible]	[illegible]
708 38	245 34	374 75	5908 44	165	1933 13	65 74	2629 [illegible]	[illegible]
742 25	247 49	401 49	5881 70	167	1941 57	57 14	[illegible]	[illegible]
776 40	249 33	429 38	5853 81	169	1948 83	48 46	[illegible]	[illegible]
810 77	250 87	458 42	5824 76	171	1954 89	39 [illegible]	[illegible]	[illegible]
845 34	252 11	488 63	5794 56	173	1959 75	30 [illegible]	[illegible]	[illegible]
880 05	253 03	520 01	5763 18	175	1963 40	22 [illegible]	[illegible]	[illegible]
914 87	253 65	552 56	5730 63	177	1965 83	13 [illegible]	[illegible]	[illegible]
− 949 75	+ 253 96	586 29	5696 90	179	− 1967 05	+ 4 [illegible]	[illegible]	[illegible]

TAB—continued.

			First Qua	ght.	Correction to Parallax.		
u	n_z	e_z	ϖ_z	i_z	+ ·060	− ·040	− ·100
1°	+ 52 36	− 1 13	− 8 44	− 2 15			
3	157 01	3 37	9 36	2 14			
5	261 47	5 60	11 19	2 12	14.		
7	365 61	7 80	13 93	2 03	756 3		
9	469 30	9 97	17 57	1 83	752 62		
11	572 43	12 08	22 09	1 46	748 00	1	
13	674 85	14 14	27 48	− 0 83	742 47	16	
15	776 46	16 13	33 72	+ 0 17	736 04	16 39	
17	877 12	18 04	40 78	1 64	728 70	16 42	
19	976 70	19 86	48 64	3 71	720 48	16 51	
21	1075 10	21 58	57 27	6 52	711 39	16 67	
23	1172 19	23 20	66 64	10 24	701 42	16 94	
25	1267 86	24 71	76 71	15 03	690 61	17 33	
27	1361 97	26 10	87 46	21 06	678 95	17 86	
29	1454 43	27 36	98 83	28 53	666 46	18 56	
31	1545 11	28 48	110 80	37 62	653 16	19 46	
33	1633 92	29 47	123 31	48 54	639 07	20 58	
35	1720 73	30 31	136 32	61 50	624 19	21 96	
37	1805 44	31 01	149 79	76 72	608 56	23 60	1
39	1887 96	31 55	163 67	94 40	592 18	25 56	1
41	1968 18	31 94	177 92	114 76	575 09	27 86	1
43	2046 00	32 18	192 47	138 03	557 29	30 52	1
45	2121 32	32 26	207 28	164 42	538 82	33 57	1
47	2194 06	32 18	222 31	194 15	519 68	37 05	2
49	2264 13	31 94	237 50	227 42	499 92	40 98	2
51	2331 44	31 55	252 80	264 44	479 54	45 38	2
53	2395 91	31 01	268 16	305 41	458 58	50 30	2
55	2457 46	30 31	283 53	350 51	437 06	55 74	3
57	2516 01	29 47	298 86	399 94	415 02	61 74	3
59	2571 50	28 48	314 09	453 84	392 46	68 32	3
61	2623 86	27 36	329 20	512 39	369 42	75 49	4
63	2673 02	26 10	344 12	575 72	345 94	83 29	4
65	2718 92	24 71	358 81	643 96	322 04	91 72	4
67	2761 52	23 20	373 23	717 23	297 74	100 80	5
69	2800 74	21 58	387 34	795 61	273 08	110 54	5
71	2836 56	19 86	401 10	879 18	248 08	120 96	6
73	2868 91	18 04	414 46	968 00	222 79	132 06	6
75	2897 78	16 13	427 40	1062 11	197 22	143 85	6
77	2923 11	14 14	439 88	1161 52	171 41	156 33	7
79	2944 88	12 08	451 88	1266 22	145 40	169 47	7
81	2963 06	9 97	463 35	1376 19	119 20	183 35	8
83	2977 64	7 80	474 29	1491 39	92 86	197 89	8
85	298[illegible]8 58	5 60	484 66	1611 72	66 41	213 11	8
87	2995 [illegible]89	3 37	494 45	1737 12	39 88	228 98	9
89	+ 2099 54	− 1 13	− 503 64	+ 1867 44	− 13 30	+ 245 50	− 9

TABLE I. ?ontinued.

First and Second Quadrants. Unit = 0·00001.

$\frac{x}{a}$	$\frac{y}{a}$	nt	$2\pi-nt$	n_y	e_y	ϖ_y	ϵ_y
32 64 +	4 43	0 57	6282 61 55	+ 13 30	+262 65	−984 51	−516 50
31 43	13 29	1 74	6281 42 30	39 88	280 41	1018 14	532 98
28 99	22 14	2 97 6?9-	2286 48	66 41	298 75	1050 35	548 73
25 34	30 96	4 ?17	2434 89	92 86	317 65	1080 96	563 65
20 48	39 73	540 22	2587 30	119 20	337 08	1109 80	577 66
14 42	48 46 6	545 63	2743 45	145 40	357 00	1136 74	590 70
7 17	57 1 14	550 41	2903 07	171 41	377 38	1161 60	602 68
1 28	6?6 13	554 57	3065 87	197 22	398 18	1184 25	613 52
10 90	18 04	558 12	3231 52	222 79	419 37	1204 54	623 18
21 68 6	19 86	561 06	3399 71	248 08	440 89	1222 34	631 57
33 62 74	21 58	563 42	3570 07	273 08	462 70	1237 53	638 64
46 70 52	23 20	565 21	3742 24	297 74	484 76	1249 99	644 34
60 90 92	24 71	566 46	3915 85	322 04	507 02	1259 61	648 60
76 23 02	26 10	567 18	4090 49	345 94	529 42	1266 29	651 38
92 ?3 86	27 36	567 40	4265 75	369 42	551 91	1269 96	652 65
110 ?1 50	28 48	567 15	4441 23	392 46	574 44	1270 53	652 36
128 16 01	29 47	566 46	4616 50	415 02	596 95	1267 94	650 48
148 457 46	30 31	565 35	4791 10	437 06	619 38	1262 13	646 98
16? 395 91	31 01	563 85	4964 62	458 58	641 69	1253 07	641 86
1? 2331 44	31 55	562 01	5136 60	479 54	663 80	1240 73	635 09
2 2264 13	31 94	559 85	5306 59	499 92	685 67	1225 09	626 67
2 2194 06	32 18	557 41	5474 14	519 68	707 23	1206 15	616 60
2121 32	32 26	554 72	5638 80	538 82	728 43	1183 92	604 88
2046 00	32 18	551 81	5800 13	557 29	749 20	1158 41	591 53
1968 18	31 94	548 74	5957 68	575 09	769 49	1129 68	576 56
1887 96	31 55	545 52	6111 01	592 18	789 25	1097 76	560 00
1805 44	31 01	542 20	6259 70	608 56	808 41	1062 71	541 88
1720 73	30 31	538 80	6403 33	624 19	826 92	1024 61	522 23
1633 92	29 47	535 38	6541 48	639 07	844 73	983 55	501 11
1545 11	28 48	531 96	6673 75	653 16	861 78	939 62	478 55
1454 43	27 36	528 57	6799 78	666 46	878 04	892 93	454 62
1361 97	26 10	525 24	6919 18	678 95	893 44	843 61	429 38
1267 86	24 71	522 02	7031 61	690 61	907 94	791 78	402 88
1172 19	23 20	518 92	7136 73	701 42	921 50	737 59	375 21
1075 10	21 58	515 97	7234 24	711 39	934 08	681 18	346 44
976 70	19 86	513 21	7323 85	720 48	945 65	622 72	316 64
877 12	18 04	510 65	7405 28	728 70	956 16	562 38	285 90
776 46	16 13	508 31	7478 30	736 04	965 58	500 33	254 32
674 85	14 14	506 23	7542 68	742 47	973 89	436 76	221 97
572 43	12 08	504 40	7598 24	748 00	981 06	371 85	188 96
469 30	9 97	502 86	7644 79	752 62	987 08	305 81	155 38
365 61	7 80	501 61	7682 20	756 32	991 91	238 83	121 34
261 47	5 60	500 66	7710 36	759 10	995 54	171 12	86 94
157 01	3 37	500 03	7729 18	760 96	997 97	102 88	52 27
+ 052 36	+1 13	−499 71	+7738 60	+ 761 88	+999 19	− 34 33	− 17 44

TABLE I.—*continued.*

Second Quadrant. Unit = 0·0001.

u	$\left(\frac{a}{r}\right)^3$	$\frac{3a^3x^2}{r^5}$	$\frac{3a^3xy}{r^5}$	$\frac{3a^3y^2}{r^5}$	u	$\left(\frac{a}{r}\right)^3$	$\frac{3a^3x^2}{r^5}$	$\frac{3a^3xy}{r^5}$
91°	95 10	267 50	−68 99	1779	137°	20 09	59 65	6 09
93	86 23	243 60	60 60	1507	139	19 31	57 39	5 55
95	78 45	222 54	53 42	1282	141	18 61	55 36	5 08
97	71 57	203 77	47 16	1092	143	17 96	53 48	4 63
99	65 53	187 27	41 80	933	145	17 36	51 73	4 22
101	60 16	172 48	37 14	800	147	16 83	50 19	3 85
103	55 40	159 32	33 08	687	149	16 34	48 77	3 50
105	51 16	147 58	29 53	591	151	15 90	47 48	3 17
107	47 36	136 96	26 41	509	153	15 49	46 28	2 87
109	43 98	127 54	23 69	440	155	15 13	45 24	2 59
111	40 95	119 05	21 30	381	157	14 80	44 28	2 33
113	38 22	111 36	19 17	330	159	14 51	43 43	2 08
115	35 76	104 40	17 29	286	161	14 25	42 66	1 85
117	33 56	98 20	15 64	249	163	14 02	42 00	1 62
119	31 55	92 49	14 15	216	165	13 82	41 41	1 41
121	29 74	87 34	12 83	188	167	13 64	40 88	1 20
123	28 10	82 67	11 64	164	169	13 50	40 48	1 01
125	26 61	78 40	10 59	143	171	13 37	40 09	0 81
127	25 26	74 54	9 63	125	173	13 28	39 83	0 63
129	24 02	70 97	8 77	109	175	13 21	39 63	0 45
131	22 90	67 75	8 00	95	177	13 16	39 48	0 27
133	21 88	64 83	7 30	82	179	13 14	39 42	− 0 9
135	20 95	62 15	6 67	72				

When the disturbing force is due to a planet whose mass is m' times that of the Sun, X and Y are the differentials of

$$V = \frac{1}{\rho} - \frac{xx' + yy' + zz'}{r'^3}$$

where ρ is the distance between planet and comet, x', y', z', r', the heliocentric coordinates and distance of the planet.

We may write $\quad V = V_1 + V_2$

or $\quad V = V_3 + V_4$

where $\quad V_1 = \frac{1}{\rho} \qquad V_2 = -\frac{xx' + yy' + zz'}{r'^3}$

$$V_3 = \frac{1}{r} + (xx' + yy' + zz')\left(\frac{1}{r^3} - \frac{1}{r'^3}\right)$$

$$V_4 = \frac{1}{\rho} - \frac{1}{r} - \frac{xx' + yy' + zz'}{r^3}$$

The advantage of the latter form is that V_3 can be dealt with in finite terms, whereas the disturbing forces arising from V_4 are of order $\frac{r'^2}{r^4}$, where r is large. The form $V = V_3 + V_4$ is therefore convenient for the parts of the orbit lying considerably beyond the disturbing planet.

The integrals arising from V_3 are

$$\frac{1}{m'}\,\frac{1}{n}\int dn = A_1 + \alpha_1'\left(\frac{1}{na}\frac{dx'}{dt}\right) + \alpha_1\frac{x'}{a} + \beta_1'\left(\frac{1}{na}\frac{dy'}{dt}\right) + \beta_1\frac{y'}{a}$$

$$\frac{1}{m'}\quad\int de = A_2 + \alpha_2'\left(\frac{1}{na}\frac{dx'}{dt}\right) + \alpha_2\frac{x'}{a} + \beta_2'\left(\frac{1}{na}\frac{dy'}{dt}\right) + \beta_2\frac{y'}{a}$$

$$\frac{1}{m'}\,e\int d\varpi = A_3 + \alpha_3'\left(\frac{1}{na}\frac{dx'}{dt}\right) + \alpha_3\frac{x'}{a} + \beta_3'\left(\frac{1}{na}\frac{dy'}{dt}\right) + \beta_3\frac{y'}{a}$$

$d\epsilon$ is not integrable, but

$$\frac{1}{m'}\left[\int(2\pi - nt)\frac{dn}{n} + \int d\epsilon - \{1 - \surd(1 - e^2)\}\int d\varpi - \frac{2\pi - nt}{n}\int dn\right]$$

$$= A_4 + \alpha_4'\left(\frac{1}{na}\frac{dx'}{dt}\right) + \alpha_4\frac{x'}{a} + \beta_4'\left(\frac{1}{na}\frac{dy'}{dt}\right) + \beta_4\frac{y'}{a}.$$

It is convenient for brevity to call the left-hand side of this equation

$$\frac{1}{m'}\left[\int d\zeta + \surd(1 - e^2)\int d\varpi - \frac{2\pi - nt}{n}\int dn\right]$$

so that $\int d\zeta$ is the perturbation of the mean anomaly at the next perihelion passage.

[The effect in arc of the elementary disturbance dn after an interval $\frac{2\pi}{n} - t$ is $(2\pi - nt)\frac{dn}{n}$.]

In the above formulæ

$$A_1 = -\frac{3a}{r} \qquad A_2 = h\frac{dy}{dt} \qquad A_3 = -h\frac{dx}{dt} \qquad A_4 = -u + \pi$$

$$\alpha_1' = -\frac{3}{na}\frac{dx}{dt} \qquad \alpha_1 = \frac{3}{n^2a}\frac{d^2x}{dt^2} \qquad \beta_1' = -\frac{3}{na}\frac{dy}{dt} \qquad \beta_1 = \frac{3}{n^2a}\frac{d^2y}{dt^2}$$

$$\alpha_2' = -\frac{y}{na^2}\frac{dy}{dt} \qquad \beta_2' = \frac{1}{na^2}\left(2x\frac{dy}{dt} - y\frac{dx}{dt}\right)$$

$$\alpha_2 = \frac{1}{n^2a^2}\left\{y\frac{d^2y}{dt^2} + \left(\frac{dy}{dt}\right)^2\right\} \qquad \beta_2 = \frac{1}{n^2a^2}\left\{-x\frac{d^2y}{dt^2} - \frac{dy}{dt}\cdot\frac{dx}{dt}\right\}$$

$$\alpha_3' = \frac{1}{na^2}\left\{2y\frac{dx}{dt} - x\frac{dy}{dt}\right\} \qquad \beta_3' = -\frac{x}{na^2}\frac{dx}{dt}$$

$$\alpha_3 = \beta_2 \qquad \beta_3 = \frac{1}{n^2a^2}\left\{x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2\right\}$$

$$\alpha_4' = -\frac{2x}{a} \qquad \alpha_4 = -\frac{1}{na}\frac{dx}{dt} \qquad \beta_4' = -\frac{2y}{a}, \qquad \beta_4 = -\frac{1}{na}\frac{dy}{dt}$$

Arbitrary constants may, of course, be added to the A's. If, however, any constant be added to A_1, we shall have a term proportional to the time in A_4. In A_4 the constant $+\pi$ was introduced to create symmetry about aphelion. It is clearly convenient to introduce the $V = V_3 + V_4$ method, and to drop it at points equidistant from aphelion.

h is twice the area conserved $= na^2\sqrt{(1-e^2)}$.

In this paper the perturbations by Jupiter between the returns of 1835 and 1910 are considered.

A preliminary calculation having shown that the comet will return about 1910 May, the disturbing forces due to Jupiter are calculated on the assumption that the comet moves in an undisturbed ellipse for which $\sqrt{(1-e^2)} = \cdot254000$, and with a mean motion less than Jupiter's in the ratio $1 : 2\pi \times \cdot999151$.

The disturbing forces are calculated for every odd degree of the comet's eccentric anomaly. Jupiter is supposed to move in an undisturbed ellipse, and the changes of Jupiter's mean anomaly are $360° \times \cdot999151$ times the changes of nt as given in Table I.

Jupiter's mean anomaly when $u = 0$ is assumed to have been $79°\cdot327$.

Jupiter's eccentricity is taken as 0·0482538. The ratio of mean motions implies a ratio of mean distances 0·2939437, for

$$(2\pi \times \cdot999151)^2\,(\cdot2939437)^3 = 1 + \frac{1}{1047\cdot35}.$$

The relative position of the two ellipses may be described as follows:—

The comet's ascending node on Jupiter's orbit is 46° 35′ in advance of Jupiter's perihelion. The motion of the comet is retrograde and its inclination to Jupiter's orbit is 180° − 18° 45′. The comet's perihelion is 113° 48′ in the direction of its own motion from its node on Jupiter's orbit.

If ξ', η' be Jupiter's coordinates referred to its own axes,

$$\frac{\xi'}{a'} = -\frac{3e'}{2} + \left(1 - \frac{3}{8}e'^2\right)\cos g' + \left(\frac{1}{2}e' - \frac{1}{3}e'^3\right)\cos 2g' + \frac{3}{8}e'^2\cos 3g' + \frac{1}{3}e'^3\cos 4g'$$

$$\frac{\eta'}{a'} = \left(1 - \frac{5}{8}e'^2\right)\sin g' + \left(\frac{1}{2}e' - \frac{5}{12}e'^3\right)\sin 2g' + \frac{3}{8}e'^2\sin 3g' + \frac{1}{3}e'^3\sin 4g'$$

Expressions for

$$\frac{a'^2\xi'}{r'^3},\ \frac{a'^2\eta'}{r'^3}$$

can be derived by a double differentiation.

Resolving parallel to the comet's axes

$$\frac{x'}{a} = +\cdot1034\ 614\ \frac{\xi'}{a'} - \cdot2611\ 992\ \frac{y'}{a'}$$

$$\frac{y'}{a} = -\cdot2664\ 364\ \frac{\xi'}{a'} - \cdot1181\ 556\ \frac{\eta'}{a'}$$

$$\frac{z'}{a} = -\cdot0686\ 316\ \frac{\xi'}{a'} + \cdot0649\ 395\ \frac{\eta'}{a'}$$

and

$$\frac{a^2x'}{r'^3}du = +\cdot1421\ 979\ \frac{a'^2\xi'}{r'^3} - \cdot3589\ 935\ \frac{a'^2\eta'}{r'^3}$$

$$\frac{a^2y'}{r'^3}du = -\cdot3661\ 915\ \frac{a'^2\xi'}{r'^3} - \cdot1623\ 937\ \frac{a'^2\eta'}{r'^3}$$

$$\frac{a^2z'}{r'^3}du = -\cdot0943\ 276\ \frac{a'^2\xi'}{r'^3} + \cdot0892\ 532\ \frac{a'^2\eta'}{r'^3}$$

where $du = 2°$ the interval between successive values of u.

Table II. gives these quantities with argument g' the mean anomaly of Jupiter.

The table is strictly appropriate to the revolution 1835–1910 only; but small linear corrections would make it available for any other revolution: moreover, the small corrections need not be applied except when the distance between Jupiter and the comet is small.

TABLE II.

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r^3}du$	$\frac{a^2y'}{r^3}du$	$\frac{a^2z'}{r^3}du$
0°	+ 98 47	− 253 58	− 65 32	+ 156 98	− 404 25	− 104 13
1	93 67	255 70	64 12	149 32	407 62	102 21
2	88 83	257 73	62 90	141 60	410 83	100 26
3	83 97	259 67	61 65	133 83	413 86	98 26
4	79 07	261 51	60 38	126 01	416 73	96 23
5	74 15	263 27	59 09	118 14	419 43	94 15
6	69 20	264 93	57 78	110 22	421 96	92 04
7	64 22	266 49	56 45	102 27	424 31	89 88
8	59 23	267 97	55 10	94 28	426 50	87 70
9	54 21	269 35	53 73	86 25	428 51	85 49
10	49 17	270 63	52 34	78 20	430 34	83 24
11	44 12	271 82	50 94	70 13	432 00	80 95
12	39 05	272 91	49 51	62 04	433 49	78 65
13	33 97	273 91	48 07	53 93	434 79	76 31
14	28 87	274 81	46 61	45 81	435 93	73 94
15	23 76	275 61	45 14	37 68	436 89	71 55
16	18 65	276 32	43 64	29 56	437 66	69 13
17	13 53	276 93	42 14	21 42	438 27	66 69
18	8 41	277 44	40 62	13 30	438 70	64 23
19	+ 3 28	277 86	39 08	+ 5 19	438 95	61 74
20	− 1 85	278 18	37 53	− 2 91	439 03	59 23
21	6 98	278 40	35 97	10 99	438 93	56 72
22	12 11	278 52	34 39	19 06	438 66	54 17
23	17 23	278 55	32 81	27 09	438 22	51 61
24	22 34	278 48	31 21	35 10	437 61	49 04
25	27 45	278 31	29 60	43 08	436 82	46 46
26	32 55	278 04	27 98	51 02	435 86	43 86
27	37 64	277 68	26 35	58 92	434 75	41 25
28	42 71	277 22	24 71	66 77	433 46	38 63
29	47 77	276 67	23 06	74 59	431 99	36 01
30	52 82	276 02	21 41	82 34	430 38	33 39
31	57 84	275 28	19 74	90 04	428 59	30 74
32	62 84	274 44	18 08	97 70	426 65	28 11
33	67 83	273 50	16 40	105 28	424 54	25 45
34	72 79	272 48	14 72	112 80	422 28	22 81
35	77 72	271 35	13 03	120 26	419 86	20 16
36	82 63	270 14	11 34	127 64	417 29	17 52
37	87 51	268 83	9 65	134 95	414 57	14 88
38	92 36	267 43	7 95	142 19	411 70	12 23
39	97 18	265 94	6 25	149 34	408 70	9 60
40	101 96	264 36	4 54	156 41	405 54	6 97
41	106 72	262 69	2 84	163 41	402 23	4 34
42	111 43	260 93	− 1 14	170 31	398 79	− 1 74
43	116 11	259 08	+ 0 57	177 12	395 22	+ 0 87
44	120 74	257 14	2 28	183 84	391 50	3 47
45	− 125 34	− 255 12	+ 3 98	− 190 46	− 387 67	+ 6 05

TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r'^3}du$	$\frac{a^2y'}{r'^3}du$	$\frac{a^2z'}{r'^3}du$
45°	−125 34	−255 12	+ 3 98	−190 46	−387 67	+ 6 05
46	129 89	253 01	5 68	197 00	383 70	8 63
47	134 41	250 82	7 39	203 42	379 60	11 18
48	138 87	248 54	9 08	209 75	375 38	13 72
49	143 29	246 18	10 78	215 98	371 04	16 25
50	147 66	243 74	12 47	222 09	366 58	18 76
51	151 98	241 21	14 16	228 11	362 01	21 26
52	156 26	238 61	15 85	234 02	357 32	23 73
53	160 48	235 92	17 52	239 81	352 54	26 19
54	164 64	233 16	19 20	245 50	347 64	28 62
55	168 75	230 32	20 86	251 07	342 64	31 04
56	172 81	227 40	22 52	256 52	337 54	33 44
57	176 81	224 42	24 17	261 86	332 34	35 81
58	180 75	221 35	25 82	267 09	327 05	38 15
59	184 63	218 22	27 45	272 19	321 67	40 48
60	188 46	215 01	29 08	277 17	316 20	42 77
61	192 22	211 73	30 70	282 05	310 65	45 04
62	195 92	208 38	32 30	286 79	305 01	47 30
63	199 55	204 97	33 90	291 41	299 30	49 51
64	203 12	201 49	35 48	295 91	293 51	51 70
65	206 62	197 94	37 06	300 29	287 65	53 86
66	210 06	194 34	38 62	304 54	281 71	56 00
67	213 43	190 66	40 17	308 67	275 71	58 10
68	216 73	186 93	41 71	312 67	269 66	60 18
69	219 97	183 14	43 23	316 55	263 53	62 21
70	223 13	179 29	44 74	320 32	257 35	64 23
71	226 22	175 38	46 24	323 94	251 11	66 22
72	229 24	171 42	47 72	327 45	244 82	68 16
73	232 19	167 40	49 18	330 83	238 49	70 09
74	235 06	163 33	50 63	334 08	232 10	71 97
75	237 86	159 20	52 07	337 20	225 68	73 81
76	240 59	155 03	53 49	340 21	219 20	75 64
77	243 24	150 81	54 89	343 09	212 70	77 42
78	245 81	146 54	56 27	345 84	206 16	79 18
79	248 31	142 23	57 64	348 46	199 59	80 89
80	250 73	137 87	58 99	350 97	192 98	82 57
81	253 07	133 47	60 32	353 34	186 35	84 22
82	255 33	129 03	61 63	355 60	179 69	85 83
83	257 52	124 55	62 92	357 74	173 01	87 41
84	259 62	120 03	64 19	359 75	166 31	88 95
85	261 65	115 47	65 45	361 63	159 59	90 46
86	263 60	110 88	66 68	363 40	152 85	91 93
87	265 46	106 25	67 89	365 04	146 11	93 37
88	267 24	101 60	69 08	366 57	139 36	94 76
89	268 95	96 91	70 26	367 98	132 59	96 13
90	−270 57	− 92 19	+71 41	−369 27	−125 81	+ 97 45

TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r^3}du$	$\frac{a^2y'}{r^3}du$	$\frac{a^2z'}{r^3}du$
90°	−270 57	−92 19	+71 41	−369 27	−125 81	+97 45
91	272 11	87 44	72 54	370 44	119 04	98 74
92	273 56	82 67	73 64	371 50	112 26	100 00
93	274 94	77 87	74 73	372 42	105 48	101 22
94	276 23	73 05	75 79	373 26	98 71	102 41
95	277 44	68 20	76 83	373 96	91 93	103 55
96	278 57	63 34	77 84	374 55	85 17	104 66
97	279 61	58 46	78 84	375 03	78 42	105 74
98	280 58	53 56	79 81	375 41	71 67	106 77
99	281 45	48 64	80 75	375 66	64 94	107 79
100	282 25	43 71	81 67	375 81	58 22	108 75
101	282 96	38 77	82 57	375 85	51 52	109 68
102	283 59	33 82	83 45	375 79	44 82	110 57
103	284 13	28 85	84 30	375 62	38 17	111 44
104	284 59	23 88	85 12	375 34	31 51	112 26
105	284 97	18 90	85 92	374 96	24 88	113 05
106	285 26	13 91	86 69	374 48	18 28	113 80
107	285 48	8 92	87 44	373 90	11 70	114 52
108	285 61	− 3 93	88 17	373 21	− 5 16	115 21
109	285 65	+ 1 06	88 86	372 41	+ 1 36	115 85
110	285 62	6 05	89 54	371 53	7 85	116 46
111	285 50	11 04	90 18	370 55	14 31	117 05
112	285 30	16 03	90 81	369 47	20 74	117 59
113	285 02	21 02	91 40	368 29	27 14	118 11
114	284 65	25 99	91 97	367 03	33 49	118 59
115	284 20	30 96	92 51	365 67	39 81	119 03
116	283 68	35 92	93 03	364 21	46 10	119 44
117	283 07	40 88	93 52	362 67	52 35	119 82
118	282 38	45 81	93 98	361 03	58 55	120 15
119	281 62	50 74	94 42	359 31	64 72	120 46
120	280 77	55 65	94 83	357 50	70 84	120 73
121	279 84	60 55	95 21	355 60	76 92	120 98
122	278 84	65 43	95 56	353 63	82 95	121 19
123	277 76	70 29	95 90	351 56	88 94	121 37
124	276 59	75 13	96 20	349 41	94 89	121 52
125	275 35	79 95	96 47	347 18	100 78	121 63
126	274 04	84 75	96 72	344 87	106 64	121 72
127	272 64	89 52	96 94	342 48	112 43	121 77
128	271 18	94 27	97 14	340 02	118 18	121 79
129	269 63	98 99	97 30	337 47	123 88	121 78
130	268 02	103 69	97 44	334 86	129 53	121 75
131	266 32	108 36	97 56	332 16	135 13	121 68
132	264 56	112 99	97 64	329 39	140 67	121 58
133	262 72	117 60	97 71	326 55	146 16	121 42
134	260 80	122 17	97 74	323 64	151 60	121 28
135.	−258 82	+126 71	+97 74	−320 65	+156 98	+121 09

TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r^3}du$	$\frac{a^2y'}{r^3}du$	$\frac{a^2z'}{r^3}du$
135°	−258 82	+126 71	+97 74	−320 65	+156 98	+121 09
136	256 77	131 22	97 72	317 60	162 30	120 88
137	254 64	135 68	97 68	314 48	167 56	120 62
138	252 44	140 12	97 60	311 29	172 77	120 35
139	250 18	144 51	97 50	308 03	177 92	120 04
140	247 85	148 86	97 37	304 72	183 01	119 71
141	245 45	153 18	97 22	301 33	188 04	119 35
142	242 99	157 45	97 04	297 88	193 02	118 96
143	240 45	161 68	96 83	294 37	197 94	118 54
144	237 86	165 86	96 60	290 80	202 79	118 09
145	235 20	170 00	96 34	287 17	207 58	117 62
146	232 47	174 10	96 05	283 48	212 30	117 13
147	229 68	178 14	95 74	279 74	216 97	116 61
148	226 83	182 14	95 40	275 93	221 58	116 06
149	223 92	186 09	95 04	272 06	226 12	115 48
150	220 95	189 99	94 65	268 15	230 60	114 87
151	217 92	193 84	94 24	264 19	235 01	114 25
152	214 83	197 64	93 80	260 16	239 36	113 59
153	211 68	201 38	93 33	256 09	243 64	112 91
154	208 48	205 07	92 84	251 96	247 86	112 21
155	205 22	208 71	92 33	247 78	252 00	111 47
156	201 90	212 28	91 79	243 56	256 09	110 72
157	198 53	215 81	91 22	239 27	260 11	109 95
158	195 11	219 27	90 64	234 95	264 06	109 14
159	191 64	222 68	90 02	230 58	267 94	108 32
160	188 11	226 02	89 39	226 16	271 76	107 47
161	184 54	229 31	88 73	221 71	275 51	106 60
162	180 91	232 54	88 04	217 20	279 18	105 70
163	177 24	235 70	87 33	212 64	282 80	104 79
164	173 52	238 80	86 60	208 05	286 33	103 84
165	169 75	241 84	85 85	203 41	289 80	102 88
166	165 94	244 81	85 08	198 74	293 20	101 89
167	162 09	247 72	84 28	194 02	296 53	100 88
168	158 19	250 56	83 46	189 26	299 79	99 85
169	154 25	253 34	82 61	184 47	302 98	98 80
170	150 27	256 04	81 75	179 63	306 10	97 72
171	146 25	258 68	80 86	174 77	309 14	96 63
172	142 19	261 26	79 95	169 86	312 12	95 51
173	138 09	263 76	79 02	164 92	315 01	94 38
174	133 96	266 19	78 07	159 95	317 84	93 21
175	129 79	268 56	77 10	154 93	320 59	92 04
176	125 59	270 85	76 11	149 89	323 27	90 84
177	121 35	273 07	75 10	144 81	325 88	89 61
178	117 08	275 22	74 06	139 71	328 40	88 38
179	112 78	277 29	73 01	134 57	330 87	87 12
180	−108 45	+279 29	+71 94	−129 41	+333 25	+ 85 84

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TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r^3}du$	$\frac{a^2y'}{r^3}du$	$\frac{a^2z'}{r^3}du$
180°	− 108 45	+ 279 29	+ 71 94	− 129 41	+ 333 25	+ 85 84
181	104 10	281 22	70 85	124 21	335 55	84 54
182	99 71	283 07	69 74	118 99	337 78	83 22
183	95 29	284 85	68 62	113 73	339 94	81 89
184	90 86	286 56	67 47	108 45	342 01	80 54
185	86 40	288 18	66 31	103 15	344 01	79 16
186	81 92	289 74	65 13	97 81	345 94	77 77
187	77 41	291 21	63 93	92 46	347 79	76 36
188	72 88	292 61	62 72	87 08	349 56	74 93
189	68 34	293 93	61 49	81 67	351 26	73 49
190	63 78	295 17	60 24	76 25	352 86	72 02
191	59 20	296 34	58 98	70 81	354 40	70 54
192	54 60	297 42	57 70	65 34	355 85	69 03
193	49 99	298 43	56 41	59 84	357 23	67 52
194	45 37	299 35	55 10	54 34	358 52	65 99
195	40 73	300 20	53 77	48 81	359 74	64 44
196	36 08	300 97	52 44	43 27	360 87	62 88
197	31 42	301 66	51 08	37 72	361 92	61 29
198	26 76	302 27	49 72	32 14	362 90	59 70
199	22 09	302 79	48 34	26 53	363 79	58 08
200	17 41	303 24	46 95	20 94	364 60	56 45
201	12 73	303 61	45 54	15 32	365 32	54 80
202	8 04	303 89	44 13	9 69	365 96	53 14
203	− 3 35	304 10	42 70	− 4 05	366 51	51 47
204	+ 1 33	304 22	41 26	+ 1 60	366 99	49 78
205	6 02	304 26	39 81	7 26	367 38	48 07
206	10 71	304 22	38 35	12 94	367 68	46 35
207	15 39	304 10	36 88	18 61	367 90	44 61
208	20 07	303 90	35 40	24 30	368 04	42 87
209	24 75	303 61	33 90	29 99	368 09	41 11
210	29 42	303 25	32 40	35 69	368 04	39 33
211	34 08	302 80	30 90	41 38	367 92	37 54
212	38 73	302 27	29 38	47 11	367 70	35 74
213	43 37	301 66	27 85	52 82	367 41	33 93
214	47 99	300 97	26 32	58 52	367 02	32 09
215	52 61	300 20	24 78	64 23	366 54	30 26
216	57 21	299 34	23 24	69 94	365 97	28 41
217	61 80	298 40	21 68	75 65	365 32	26 54
218	66 37	297 39	20 12	81 36	364 58	24 68
219	70 92	296 29	18 56	87 07	363 74	22 79
220	75 45	295 11	16 99	92 76	362 81	20 89
221	79 96	293 85	15 42	98 45	361 80	18 98
222	84 45	292 51	13 84	104 13	360 69	17 07
223	88 92	291 10	12 26	109 82	359 50	15 14
224	93 36	289 60	10 68	115 48	358 20	13 20
225	+ 97 78	+ 288 02	+ 9 09	+ 121 13	+ 356 82	+ 11 25

TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r^3}du$	$\frac{a^2y'}{r^3}du$	$\frac{a^2z'}{r^3}du$
225°	+ 97 78	+288 02	+ 9 09	+121 13	+356 82	+ 11 25
226	102 16	286 36	7 50	126 78	355 34	9 30
227	106 53	284 63	5 90	132 41	353 78	7 34
228	110 86	282 81	4 31	138 03	352 11	5 36
229	115 16	280 92	2 72	143 64	350 35	3 38
230	119 43	278 95	+ 1 12	149 22	348 51	+ 1 39
231	123 69	276 90	− 0 48	154 79	346 56	− 0 60
232	127 87	274 78	2 07	160 34	344 52	2 61
233	132 04	272 58	3 67	165 86	342 39	4 61
234	136 16	270 30	5 26	171 37	340 16	6 62
235	140 26	267 95	6 86	176 86	337 84	8 65
236	144 31	265 53	8 45	182 31	335 41	10 68
237	148 32	263 03	10 04	187 74	332 90	12 71
238	152 29	260 45	11 62	193 15	330 29	14 75
239	156 22	257 80	13 20	198 52	327 58	16 78
240	160 10	255 08	14 78	203 88	324 78	18 83
241	163 94	252 29	16 36	209 19	321 88	20 88
242	167 74	249 43	17 93	214 47	318 89	22 93
243	171 48	246 50	19 49	219 71	315 79	24 98
244	175 18	243 49	21 05	224 93	312 60	27 04
245	178 82	240 42	22 61	230 09	309 31	29 09
246	182 42	237 28	24 15	235 23	305 93	31 15
247	185 96	234 07	25 69	240 31	302 44	33 21
248	189 45	230 79	27 23	245 37	298 86	35 27
249	192 89	227 45	28 75	250 37	295 19	37 33
250	196 27	224 04	30 27	255 33	291 41	39 38
251	199 59	220 56	31 78	260 23	287 54	41 43
252	202 86	217 03	33 28	265 09	283 58	43 49
253	206 06	213 43	34 77	269 90	279 52	45 54
254	209 21	209 77	36 24	274 66	275 36	47 58
255	212 30	206 04	37 71	279 36	271 10	49 63
256	215 32	202 26	39 17	284 00	266 75	51 66
257	218 28	198 42	40 62	288 58	262 29	53 70
258	221 18	194 52	42 05	293 11	257 76	55 73
259	224 02	190 56	43 47	297 57	253 12	57 74
260	226 78	186 55	44 88	301 97	248 38	59 77
261	229 48	182 48	46 28	306 30	243 56	61 77
262	232 11	178 36	47 66	310 57	238 63	63 77
263	234 68	174 18	49 03	314 77	233 62	65 76
264	237 17	169 96	50 38	318 89	228 51	67 74
265	239 59	165 68	51 72	322 94	223 31	69 71
266	241 94	161 35	53 04	326 92	218 01	71 67
267	244 22	156 98	54 35	330 82	212 64	73 62
268	246 43	152 56	55 64	334 64	207 16	75 56
269	248 56	148 09	56 91	338 38	201 60	77 48
270	+250 61	+143 57	−58 17	+342 03	+195 95	− 79 39

TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r^3}du$	$\frac{a^2y'}{r^3}du$	$\frac{a^2z'}{r^3}du$
270°	+250 61	+143 57	−58 17	+342 03	+195 95	− 79 39
271	252 60	139 02	59 41	345 60	190 21	81 29
272	254 50	134 42	60 63	349 09	184 38	83 16
273	256 32	129 78	61 83	352 48	178 47	85 03
274	258 07	125 10	63 02	355 78	172 47	86 87
275	259 74	120 38	64 18	358 99	166 39	88 70
276	261 33	115 63	65 33	362 11	160 23	90 51
277	262 84	110 84	66 45	365 12	153 99	92 31
278	264 27	106 02	67 56	368 04	147 65	94 09
279	265 62	101 16	68 64	370 86	141 25	95 84
280	266 88	96 27	69 70	373 57	134 78	97 57
281	268 06	91 35	70 74	376 18	128 21	99 27
282	269 16	86 41	71 76	378 68	121 58	100 96
283	270 18	81 43	72 76	381 07	114 88	102 62
284	271 11	76 44	73 73	383 35	108 10	104 26
285	271 95	71 41	74 68	385 52	101 26	105 87
286	272 71	66 37	75 61	387 58	94 34	107 45
287	273 39	61 30	76 51	389 51	87 37	109 01
288	273 97	56 22	77 39	391 33	80 32	110 54
289	274 48	51 12	78 25	393 02	73 21	112 04
290	274 89	46 00	79 08	394 60	66 05	113 51
291	275 22	40 86	79 88	396 05	58 83	114 95
292	275 45	35 72	80 66	397 37	51 54	116 36
293	275 60	30 56	81 41	398 57	44 21	117 74
294	275 67	25 39	82 14	399 64	36 83	119 08
295	275 64	20 21	82 84	400 57	29 39	120 38
296	275 52	15 03	83 52	401 37	21 91	121 66
297	275 32	9 84	84 16	402 05	14 40	122 89
298	275 03	+ 4 65	84 78	402 57	+ 6 83	124 10
299	274 64	− 54	85 38	402 97	− 0 77	125 26
300	274 17	5 73	85 94	403 23	8 42	126 39
301	273 61	10 92	86 48	403 35	16 09	127 48
302	272 96	16 11	86 98	403 33	23 79	128 53
303	272 22	21 29	87 46	403 16	31 52	129 53
304	271 39	26 47	87 92	402 84	39 26	130 50
305	270 47	31 63	88 34	402 39	47 04	131 42
306	269 47	36 79	88 73	401 78	54 84	132 30
307	268 37	41 93	89 10	401 03	62 64	133 13
308	267 18	47 06	89 43	400 14	70 46	133 93
309	265 91	52 17	89 73	399 09	78 29	134 68
310	264 55	57 27	90 01	397 89	86 12	135 38
311	263 10	62 35	90 26	396 54	93 96	136 03
312	261 56	67 40	90 47	395 05	101 80	136 64
313	259 93	72 44	90 65	393 40	109 62	137 20
314	258 22	77 45	90 81	391 60	117 46	137 71
315	+256 42	− 82 43	−90 93	+389 64	−125 25	−138 17

TABLE II.—*continued.*

Argument—Jupiter's Mean Anomaly. Unit 0·00001.

g'	$\frac{x'}{a}$	$\frac{y'}{a}$	$\frac{z'}{a}$	$\frac{a^2x'}{r'^3}du$	$\frac{a^2y'}{r'^3}du$	$\frac{a^2z'}{r'^3}du$
315°	+256 42	− 82 43	− 90 93	+389 64	− 125 25	− 138 17
316	254 53	87 38	91 02	387 52	133 04	138 59
317	252 56	92 31	91 09	385 26	140 82	138 95
318	250 50	97 21	91 12	382 85	148 57	139 26
319	248 36	102 07	91 12	380 29	156 29	139 52
320	246 13	106 90	91 09	377 57	163 98	139 73
321	243 82	111 69	91 03	374 70	171 64	139 88
322	241 42	116 44	90 93	371 67	179 26	139 99
323	238 95	121 15	90 81	368 49	186 83	140 04
324	236 39	125 83	90 66	365 16	194 37	140 04
325	233 75	130 46	90 47	361 68	201 86	139 98
326	231 03	135 04	90 25	358 04	209 28	139 87
327	228 23	139 58	90 01	354 26	216 66	139 71
328	225 36	144 07	89 73	350 34	223 97	139 49
329	222 40	148 51	89 42	346 26	231 23	139 22
330	219 37	152 90	89 08	342 04	238 40	138 89
331	216 26	157 23	88 71	337 67	245 51	138 51
332	213 08	161 51	88 30	333 15	252 54	138 07
333	209 83	165 74	87 87	328 50	259 49	137 57
334	206 50	169 90	87 41	323 70	266 36	137 02
335	203 10	174 02	86 92	318 76	273 14	136 42
336	199 63	178 06	86 39	313 70	279 83	135 76
337	196 09	182 05	85 84	308 49	286 42	135 05
338	192 48	185 98	85 26	303 14	292 92	134 27
339	188 80	189 83	84 64	297 67	299 31	133 46
340	185 06	193 62	84 00	292 07	305 59	132 57
341	181 25	197 35	83 33	286 33	311 77	131 64
342	177 38	201 00	82 63	280 48	317 84	130 65
343	173 45	204 59	81 90	274 50	323 79	129 61
344	169 46	208 10	81 14	268 40	329 62	128 51
345	165 40	211 54	80 35	262 18	335 33	127 37
346	161 29	214 91	79 54	255 85	340 91	126 16
347	157 13	218 20	78 69	249 41	346 37	124 91
348	152 90	221 41	77 82	242 86	351 69	123 61
349	148 63	224 54	76 92	236 21	356 88	122 25
350	144 30	227 60	76 00	229 44	361 92	120 84
351	139 92	230 58	75 04	222 59	366 83	119 39
352	135 49	233 47	74 06	215 64	371 60	117 88
353	131 01	236 28	73 06	208 59	376 21	116 32
354	126 49	239 01	72 03	201 46	380 68	114 72
355	121 92	241 66	70 97	194 24	385 01	113 07
356	117 31	244 21	69 89	186 93	389 17	111 37
357	112 66	246 69	68 78	179 55	393 18	109 62
358	107 96	249 07	67 65	172 10	397 03	107 84
359	103 24	251 37	66 50	164 58	400 72	106 01
360	+ 98 47	− 253 58	− 65 32	+156 98	− 404 25	− 104 13

Table III. now gives, for argument u, the values of g' Jupiter's mean anomaly and $a^2\mathrm{X}du$, $a^2\mathrm{Y}du$, $a^2\mathrm{Z}du$ for the first and fourth quadrants; and $a^2\mathrm{X}_4du$, $a^2\mathrm{Y}_4du$, $a^2\mathrm{Z}_4du$ for the second and third quadrants.

The $\mathrm{V}=\mathrm{V}_3+\mathrm{V}_4$ method is used for the second and third quadrants. The discontinuity in the numerical values as X, Y, Z are replaced by X_4, Y_4, Z_4, illustrates the value of the method.

The values of Z are not required in this paper, but will be made use of when we compute the change of the plane of the orbit. We have

$$a^2\mathrm{X}_4=\left(\frac{a^3}{\rho^3}-\frac{a^3}{r^3}\right)(x'-x)+3\frac{a^3x^2}{r^5}\cdot\frac{x'}{a}+3\frac{a^3xy}{r^5}\cdot\frac{y'}{a}$$

$$a^2\mathrm{Y}_4=\left(\frac{a^3}{\rho^3}-\frac{a^3}{r^3}\right)(y'-y)+3\frac{a^3xy}{r^5}\quad\frac{x'}{a}+3\frac{a^3y^2}{r^5}\quad\frac{y'}{a}$$

The computation requires the values of $\frac{a^3}{r^3}$, $\frac{3a^3x^2}{r^5}$, $\frac{3a^3xy}{r^5}$ and $\frac{3a^3y^2}{r^5}$ for the second (and third) quadrants. These functions have therefore been included in Table I.

The third and fourth quadrants are written out in the reverse order, so as to preserve the order of the auxiliary quantities given in Table I.

TABLE III.

First Quadrant. Unit 0·0001.

u	g'	$a^2\mathrm{X}du$	$a^2\mathrm{Y}du$	$a^2\mathrm{Z}du$	u	g'	$a^2\mathrm{X}du$	$a^2\mathrm{Y}du$	$a^2\mathrm{Z}du$
°	°				°	°			
1	79·532	+ 5 56	+ 457	− 2 10	47	119·951	39 42	115 57	66 86
3	79·953	6 54	401	2 28	49	124·379	68 44	119 74	80 19
5	80·395	7 34	333	2 41	51	129·130	99 10	105 91	80 82
7	80·874	8 02	2 54	2 50	53	134·209	115 15	78 26	65 00
9	81·406	8 59	1 60	2 54	55	139·626	110 61	51 70	41 81
11	82·003	9 05	+ 53	2 52	57	145·392	94 21	35 36	21 78
13	82·679	9 43	− 71	2 47	59	151·514	75 93	28 34	8 43
15	83·453	9 71	2 14	2 35	61	157·999	60 18	26 78	+ 66
17	84·334	9 91	3 79	2 18	63	164·852	47 43	27 74	− 3 53
19	85·341	10 01	5 68	1 93	65	172·085	37 07	29 68	5 55
21	86·485	10 03	7 86	1 57	67	179·703	28 35	31 78	6 26
23	87·783	9 93	10 39	1 11	69	187·707	20 65	33 69	6 14
25	89·244	9 74	13 31	− 48	71	196·113	13 52	35 17	5 47
27	90·888	9 40	16 71	+ 36	73	204·914	6 73	36 06	4 39
29	92·722	8 93	20 71	1 48	75	214·122	+ 11	36 26	2 98
31	94·758	8 32	25 45	2 99	77	223·741	− 6 41	35 60	− 1 30
33	97·017	7 59	31 11	5 03	79	233·773	12 82	33 97	+ 60
35	99·506	6 80	37 97	7 85	81	244·218	19 03	31 22	2 67
37	102·236	6 13	46 33	11 76	83	255·084	24 87	27 21	4 87
39	105·225	5 94	56 66	17 27	85	266·372	30 11	21 83	7 11
41	108·477	7 05	69 32	25 03	87	278·080	34 39	15 02	9 28
43	112·009	11 02	80 31	35 85	89	290·212	− 37 27	− 6 83	+ 11 24
45	115·829	20 62	92 17	50 22					

TABLE III.—*continued.*

Second Quadrant. Unit 0·0001.				*Third Quadrant.* Unit 0·0001.				
g'	a^2X_4du	a^2Y_4du	a^2Z_4du	u	g'	a^2X_4du	a^2Y_4du	a^2Z_4du
302°77	+ 47	− 21	+ 15	269°	315°91	+ 25	+ 30	+ 14
315·75	43	− 8	14	267	302·94	36	22	13
329·15	36	+ 4	12	265	289·53	41	12	11
342·97	23	12	9	263	275·71	39	+ 2	8
357·22	+ 8	16	6	261	261·46	32	− 7	5
11·88	− 5	15	3	259	246·80	21	13	+ 2
26·96	15	+ 9	1	257	231·72	+ 9	16	0
42·46	17	0	0	255	216·22	− 3	15	− 1
58·36	− 12	− 10	1	253	200·32	13	10	− 1
74·67	+ 1	18	5	251	184·01	16	− 3	0
91·38	16	19	9	249	167·30	14	+ 6	+ 2
108·48	26	− 13	11	247	150·20	− 5	13	6
125·98	26	0	11	245	132·70	+ 8	15	8
143·86	17	+ 9	9	243	114·83	20	11	9
162·11	+ 5	11	5	241	96·57	23	+ 2	8
180·74	− 5	8	2	239	77·94	16	− 7	4
199·73	9	+ 3	0	237	58·95	+ 6	9	1
219·08	7	− 2	0	235	39·60	− 3	7	0
238·77	− 3	5	0	233	19·91	6	− 1	0
258·80	+ 2	6	1	231	359·88	− 5	+ 3	1
279·16	6	5	2	229	339·52	+ 1	5	2
299·84	8	− 2	2	227	318·83	5	4	2
320·84	7	+ 1	2	225	297·85	8	+ 1	2
342·13	+ 3	3	2	223	276·56	7	− 1	2
3·71	− 2	3	1	221	254·97	+ 5	3	1
25·56	5	+ 1	0	219	233·11	0	4	0
47·69	− 3	− 3	0	217	210·99	− 3	− 3	0
70·07	+ 2	5	1	215	188·61	5	0	0
92·69	7	− 4	3	213	165·99	− 3	+ 3	1
115·54	9	0	3	211	143·13	+ 2	5	3
138·62	+ 6	+ 3	3	209	120·06	7	+ 3	3
161·90	0	4	2	207	96·78	8	− 1	3
185·37	− 2	+ 2	0	205	73·31	+ 5	3	1
209·02	4	− 1	0	203	49·67	− 1	3	0
232·83	− 1	3	0	201	25·85	3	− 1	0
256·80	+ 1	3	1	199	1·89	− 2	+ 2	0
280·90	4	− 2	1	197	337·78	+ 1	2	1
305·13	4	0	1	195	313·55	4	+ 1	1
329·47	+ 2	+ 2	1	193	289·21	5	− 1	1
353·90	− 1	2	1	191	264·78	+ 3	2	1
18·42	3	+ 1	0	189	240·27	0	3	0
43·00	− 2	− 2	0	187	215·68	− 2	− 2	0
67·63	+ 2	3	1	185	191·05	3	+ 1	0
92·30	6	− 2	2	183	166·38	− 1	3	1
116·99	+ 6	+ 1	+ 3	181	141·69	+ 3	+ 3	+ 2

TABLE III.—*continued.*

Fourth Quadrant. Unit 0·0001.

u	g'	a^2Xdu	a^2Ydu	a^2Zdu	u	g'	a^2Xdu	a^2Ydu
359°	179°·146	− 1 31	− 4 52	− 1 30	313°	138°·731	33 81	10 33
357	178·726	− 19	5 50	1 80	311	134·300	37 04	14 23
355	178·287	+ 1 19	6 31	2 21	309	129·552	40 83	18 57
353	177·805	1 97	6 96	2 59	307	124·469	45 36	23 22
351	177·276	3 04	7 48	2 92	305	119·052	50 76	27 80
349	176·679	4 12	7 88	3 21	303	113·286	56 97	31 61
347	176·003	5 22	8 18	3 48	301	107·168	63 38	33 76
345	175·229	6 33	8 37	3 73	299	100·683	68 57	33 57
343	174·344	7 48	8 47	3 93	297	93·827	70 59	31 38
341	173·337	8 66	8 46	4 16	295	86·597	68 39	28 82
339	172·193	9 89	8 36	4 35	293	78·979	62 60	27 71
337	170·899	11 18	8 14	4 52	291	70·972	54 83	28 75
335	169·435	12 51	7 81	4 69	289	62·569	46 38	31 51
333	167·779	13 91	7 35	4 84	287	53·768	37 75	35 12
331	165·960	15 38	6 75	4 97	285	44·560	28 94	38 75
329	163·921	16 93	5 97	5 08	283	34·941	19 86	41 72
327	161·662	18 56	4 98	5 16	281	24·909	10 42	43 45
325	159·173	20 30	3 83	5 21	279	14·460	+ 74	43 50
323	156·442	22 13	2 33	5 18	277	3·598	− 8 88	41 52
321	153·457	24 09	− 58	5 11	275	352·310	18 03	37 36
319	150·202	26 20	+ 1 52	4 94	273	340·602	26 00	31 08
317	146·670	28 48	4 01	4 68	271	328·470	− 32 36	+ 22 97
315	142·850	31 00	6 93	4 28				

Table IV. gives for argument u

$$\frac{1}{m'}\frac{dn}{n}\quad \frac{1}{m'}\,de,\ \frac{1}{m'}\,ed\varpi,\ \frac{1}{m'}\left[d\epsilon - d\varpi\{1-\sqrt{(1-e^2)}\}\right]\text{ and }\frac{1}{m'}\,\frac{dn}{n}\times(2\pi$$

The mechanical quadratures for the second and third quadran require to be supplemented by the definite integrals arising from V

TABLE IV.

First Quadrant.

u	$\frac{1}{m'}\frac{dn}{n}$	$\frac{1}{m'}de$	$\frac{1}{m'}ed\varpi$	$\frac{de}{m'}-\frac{d\varpi}{m'}\{1-\sqrt{(1-e^2)}\}$	$\frac{1}{m'}\frac{dn}{n}\times(2\pi-nt)$
1°	− ·0319	+ ·0007	− ·0005	− ·0001	− 0·200
3	·0202	·0005	·0005	·0001	0·127
5	− ·0061	+ ·0002	·0007	·0003	− 0·038
7	+ ·0101	− ·0002	·0010	·0003	+ 0·063
9	·0283	·0006	·0014	·0003	0·178
11	·0478	·0010	·0020	·0001	0·300
13	·0689	·0014	·0026	− ·0001	0·432
15	·0912	·0020	·0033	+ ·0002	0·572
17	·1145	·0024	·0039	·0006	0·718
19	·1387	·0029	·0045	·0012	0·869
21	·1637	·0035	·0048	·0021	1·025
23	·1893	·0041	·0047	·0033	1·185
25	·2154	·0047	·0041	·0050	1·347
27	·2415	·0055	− ·0024	·0073	1·510
29	·2679	·0062	+ ·0005	·0104	1·673
31	·2948	·0074	·0052	·0145	1·840
33	·3228	·0086	·0124	·0200	2·012
35	·3540	·0104	·0229	·0272	2·204
37	·3926	·0128	·0378	·0369	2·442
39	·4476	·0164	·0581	·0506	2·780
41	·5375	·0216	·0841	·0705	3·334
43	·6731	·0280	·1080	·0966	4·168
45	·9340	·0376	·1268	·1385	5·774
47	1·4655	·0555	·1535	·2226	9·043
49	2·1482	·0710	·1187	·3234	13·228
51	2·8184	·0794	+ ·0276	·4257	17·318
53	3·1178	·0751	− ·0804	·4844	19·114
55	2·9442	·0623	·1469	·4835	18·005
57	2·5171	·0496	·1563	·4481	15·353
59	2·0637	·0410	·1287	·4065	12·552
61	1·6779	·0367	·0852	·3716	10·176
63	1·3638	·0355	− ·0364	·3435	8·245
65	1·1035	·0364	+ ·0135	·3195	6·649
67	·8775	·0386	·0628	·2957	5·269
69	·6704	·0417	·1115	·2687	4·010
71	·4707	·0452	·1592	·2346	2·805
73	·2734	·0488	·2048	·1907	1·622
75	+ ·0747	·0522	·2476	·1346	+ 0·441
77	− ·1264	·0548	·2857	+ ·0634	− 0·743
79	·3281	·0561	·3168	− ·0242	1·921
81	·5267	·0553	·3382	·1289	3·068
83	·7152	·0520	·3463	·2499	4·144
85	·8854	·0448	·3373	·3842	5·103
87	1·0243	·0332	·3072	·5251	5·870
89	− 1·1170	− ·0164	+ ·2526	− ·6619	− 6·363
Sums,	+ 24·3392	− 1·2575	+ 3·0688	+ 3·5259	+ 150·679

TABLE IV.—*continued.*

Second Quadrant. V_4 only.

u	$\frac{1}{m'}\frac{dn}{n}$	$\frac{1}{m'}de$	$\frac{1}{m'}ed\varpi$	$\frac{de}{m'}-\frac{d\varpi}{m'}\{1-\sqrt{(1-e^2)}\}$	$\frac{1}{m'}\frac{dn}{n}(2\pi-nt)$
91	+ ·0141	− ·0006	− ·0003	+ ·0105	+ 0·080
93	·0129	− ·0002	·0014	·0096	0·073
95	·0108	+ ·0001	·0023	·0080	0·060
97	·0069	·0004	·0025	·0049	0·038
99	+ ·0026	·0005	·0022	+ ·0012	+ 0·014
101	− ·0013	·0005	·0014	− ·0023	− 0·007
103	·0042	+ ·0003	− ·0002	·0049	0·023
105	·0049	·0000	+ ·0009	·0052	0·026
107	·0036	− ·0004	·0019	− ·0033	0·019
109	− ·0001	·0008	·0021	+ ·0014	− 0·001
111	+ ·0040	·0009	+ ·0015	·0069	+ 0·021
113	·0069	− ·0004	− ·0001	·0104	0·036
115	·0071	+ ·0001	·0015	·0102	0·037
117	·0048	·0005	·0021	·0063	0·024
119	+ ·0017	·0006	·0017	+ ·0014	+ 0·009
121	− ·0010	·0005	− ·0007	− ·0027	− 0·005
123	·0022	+ ·0002	+ ·0001	·0044	0·011
125	·0018	− ·0001	·0007	·0033	0·009
127	− ·0009	·0003	·0008	− ·0012	− 0·004
129	+ ·0002	·0004	·0006	+ ·0014	+ 0·001
131	·0012	·0003	+ ·0003	·0035	0·006
133	·0017	− ·0001	− ·0002	·0045	0·008
135	·0016	+ ·0001	·0005	·0038	0·007
137	+ ·0008	·0002	·0005	+ ·0015	+ 0·004
139	− ·0002	·0002	− ·0002	− ·0014	− 0·001
141	·0008	+ ·0001	+ ·0002	·0032	0·004
143	− ·0007	− ·0002	·0005	− ·0017	− 0·003
145	·0000	·0004	+ ·0004	+ ·0016	0·000
147	+ ·0008	− ·0003	·0000	·0048	+ 0·003
149	·0014	·0000	− ·0005	·0060	0·006
151	·0011	+ ·0003	·0036	+ ·0040	0·005
153	+ ·0003	·0004	·0003	− ·0002	+ 0·001
155	− ·0002	+ ·0002	− ·0001	·0015	− 0·001
157	·0006	− ·0001	+ ·0003	·0029	0·002
159	·0003	·0003	·0003	− ·0006	− 0·001
161	− ·0001	·0003	+ ·0001	+ ·0008	0·000
163	+ ·0003	− ·0002	− ·0001	·0031	+ 0·001
165	·0003	·0000	·0002	·0030	0·001
167	+ ·0002	+ ·0002	− ·0002	+ ·0015	+ 0·001
169	·0000	·0002	·0000	− ·0008	0·000
171	·0000	+ ·0001	+ ·0002	·0023	0·000
173	− ·0003	− ·0002	+ ·0001	− ·0015	− 0·001
175	·0001	·0003	·0000	+ ·0016	0·000
177	− ·0001	− ·0002	− ·0003	·0046	0·000
179	+ ·0001	+ ·0001	− ·0003	+ ·0046	0·000
Sums,	+ ·0584	− ·0012	− ·0094	+ ·0777	+ 0·318

TABLE IV.—*continued.*

Third Quadrant. V_4 only.

u	$\frac{dn}{m'\ n}$	$\frac{1}{m'}de$	$\frac{1}{m'}ed\varpi$	$\frac{de}{m'}-\frac{d\varpi}{m'}\{1-\sqrt{(1-e^2)}\}$	$\frac{1}{m'}\frac{dn}{n} \times (2\pi-nt)$
269°	− ·0075	+ ·0008	+ ·0017	+ ·0065	− 0·005
267	·0107	·0006	+ ·0003	·0089	0·007
265	·0122	·0004	− ·0009	·0101	0·008
263	·0116	+ ·0001	·0019	·0096	0·009
261	·0096	− ·0002	·0025	·0079	0·007
259	·0064	·0005	·0026	·0050	0·005
257	− ·0029	·0006	·0024	+ ·0016	− 0·002
255	+ ·0006	·0006	·0016	− ·0018	+ 0·001
253	·0035	·0004	− ·0005	·0048	0·003
251	·0044	− ·0001	+ ·0005	·0056	0·004
249	·0041	+ ·0003	·0015	·0046	0·004
247	+ ·0018	·0006	·0019	− ·0011	+ 0·002
245	− ·0017	·0008	·0014	+ ·0041	− 0·002
243	·0049	+ ·0005	+ ·0003	·0089	0·006
241	·0059	·0000	− ·0010	·0099	0·007
239	·0044	− ·0004	·0018	·0066	0·006
237	− ·0019	·0005	·0014	+ ·0022	− 0·003
235	+ ·0004	·0004	− ·0007	− ·0019	+ 0·001
233	·0014	− ·0001	+ ·0002	·0031	0·002
231	+ ·0013	+ ·0002	·0007	− ·0024	+ 0·002
229	·0000	·0003	·0005	+ ·0008	0·000
227	− ·0009	·0003	+ ·0002	·0029	− 0·001
225	·0016	+ ·0001	− ·0003	·0046	0·003
223	·0015	− ·0001	·0005	·0040	0·003
221	·0012	·0002	·0006	+ ·0028	− 0·002
219	− ·0002	·0003	·0004	− ·0002	0·000
217	+ ·0003	− ·0002	− ·0001	·0021	+ 0·001
215	·0009	·0000	+ ·0003	·0032	0·002
213	+ ·0007	+ ·0003	·0005	− ·0018	+ 0·001
211	·0000	·0004	+ ·0004	+ ·0015	0·000
209	− ·0008	+ ·0003	− ·0001	·0049	− 0·002
207	·0012	− ·0001	·0005	·0055	0·003
205	·0008	·0003	·0005	+ ·0034	− 0·002
203	− ·0001	·0003	− ·0001	− ·0008	0·000
201	+ ·0002	− ·0001	+ ·0001	·0022	0·000
199	+ ·0003	+ ·0002	+ ·0002	− ·0014	+ 0·001
197	·0000	·0002	·0000	+ ·0008	0·000
195	− ·0002	+ ·0001	− ·0001	·0030	− 0·001
193	·0004	− ·0001	·0003	·0038	0·001
191	·0003	·0002	·0003	+ ·0023	0·001
189	·0002	·0003	− ·0001	·0000	− 0·001
187	− ·0001	− ·0002	+ ·0001	− ·0015	0·000
185	+ ·0002	+ ·0001	·0002	·0023	+ 0·001
183	·0002	·0003	+ ·0001	− ·0008	0·001
181	+ ·0002	+ ·0003	− ·0001	+ ·0023	+ 0·001
Sums,	− ·0687	+ ·0010	− ·0102	+ ·0823	− 0·060

Table IV.—*continued.*

Fourth Quadrant.

u	$\frac{1}{m'}\frac{dn}{n}$	$\frac{1}{m'}de$	$\frac{1}{m'}ed\varpi$	$\frac{de}{m'}-\frac{d\varpi}{m'}\{1-\sqrt{(1-e^2)}\}$	$\frac{1}{m'}\frac{dn}{n} \times (2\pi - nt)$
°					
359	+ ·0351	− ·0008	+ ·0001	0000	0·000
357	·0422	·0009	·0001	− ·0001	0·000
355	·0448	·0009	+ ·0001	·0001	0·000
353	·0454	·0010	− ·0001	·0002	0·000
351	·0420	·0009	·0003	·0004	0·000
349	·0353	·0008	·0007	·0005	0·000
347	·0255	·0006	·0013	·0005	0·000
345	+ ·0125	·0004	·0021	·0007	0·000
343	− ·0039	− ·0001	·0034	·0008	0·000
341	·0236	+ ·0003	·0048	·0009	0·000
339	·0468	·0007	·0067	·0009	− 0·001
337	·0740	·0012	·0090	·0007	0·002
335	·1047	·0017	·0116	− ·0002	0·003
333	·1396	·0023	·0147	+ ·0006	0·004
331	·1787	·0029	·0182	·0018	0·007
329	·2226	·0036	·0222	·0037	0·010
327	·2715	·0045	·0264	·0064	0·013
325	·3254	·0054	·0310	·0102	0·018
323	·3853	·0064	·0355	·0154	0·025
321	·4514	·0075	·0401	·0222	0·033
319	·5244	·0088	·0445	·0287	0·043
317	·6050	·0104	·0483	·0434	0·055
315	·6949	·0123	·0516	·0589	0·071
313	·7955	·0147	·0537	·0787	0·090
311	·9097	·0176	·0546	·1041	0·114
309	1·0410	·0213	·0544	·1367	0·144
307	1·1933	·0258	·0538	·1779	0·182
305	1·3689	·0309	·0543	·2294	0·229
303	1·5646	·0363	·0583	·2915	0·289
301	1·7623	·0412	·0683	·3614	0·354
299	1·9232	·0441	·0841	·4305	0·421
297	1·9955	·0445	·0994	·4861	0·474
295	1·9523	·0433	·1032	·5189	0·503
293	1·8112	·0424	·0866	·5296	0·505
291	1·6141	·0436	− ·0490	·5253	0·486
289	1·3938	·0473	+ ·0052	·5115	0·453
287	1·1612	·0532	·0701	·4878	0·405
285	·9150	·0604	·1414	·4499	0·343
283	·6520	·0680	·2143	·3925	0·262
281	·3701	·0749	·2840	·3086	0·159
279	− ·0738	·0799	·3449	·1954	− 0·034
277	+ ·2258	·0815	·3905	+ ·0523	+ 0·110
275	·5140	·0786	·4149	− ·1175	0·267
273	·7665	·0703	·4125	·3020	0·424
271	+ ·9676	+ ·0560	+ ·3811	− ·4896	+ 0·567
Sums,	− 23·7926	+ 1·1374	+ 1·4670	+ 5·5443	− 4·364

In order to calculate the definite integrals arising from V_3 we equire values of Jupiter's coordinates and velocities for $u=90^\circ$ nd $u=270^\circ$. These are also set down for $u=180^\circ$ so that the wo quadrants may be exhibited separately.

	$u=90^\circ$	$u=180^\circ$	$u=270^\circ$
A_1	−3·000	−1·525	−3·000
a_1'	+3·000	0·000	−3·000
a_1	+2·902	+0·775	+2·902
β_1'	0·000	+0·387	0·000
β_1	−0·762	0·000	+0·762
A_2	0·000	−0·0328	0·000
a_2'	0·000	0·000	0·000
a_2	−0·0645	+0·0167	−0·0645
β_2'	+0·2540	+0·5080	+0·2540
$a_2-\beta_2$	−0·2457	0·000	+0·2457
A_3	+0·2540	0·000	−0·2540
a_3'	−0·5080	−0·2540	−0·5080
β_3'	−0·9672	0·000	+0·9672
β_3	+0·0646	−0·5083	+0·0646
A_4	+1·571	0·000	−1·571
a_4'	+1·934	+3·934	+1·934
a_4	+1·000	0·000	−1·000
β_4'	−0·508	0·000	+0·508
β_4	0·000	+0·129	0·000
$\frac{1}{na}\frac{dx'}{dt}$	−0·0712	+0·5777	−0·8825
$\frac{x'}{a}$	+0·2755	−0·2691	+0·2409
$\frac{1}{na}\frac{dy'}{dt}$	−1·8676	+1·6911	−1·7005
$\frac{y'}{a}$	+0·0127	+0·1007	−0·1177

Hence for the integrals arising from V_3

	$u=90^\circ$	$u=180^\circ$	$u=270^\circ$
$\frac{1}{m'}\cdot\frac{1}{n}\int dn$	−2·424	−1·080	+0·257
$\frac{1}{m'}\int de$	−0·4953	+0·8218	−0·4763
$\frac{1}{m'}e\int d\varpi$	+2·0296	−0·1979	−1·3988
$\frac{1}{m'}\left[\int d\zeta+\sqrt{(1-e^2)}\int d\varpi-\frac{2\pi-nt}{n}\int dn\right]$	+2·658	+2·286	−4·383

Hence for $\frac{1}{m'} \cdot \frac{1}{n}\int dn$,

First quadrant	. . . .	$+24{\cdot}3392$
Second quadrant (V_4)	. . . .	$+\ {\cdot}0584$
" (V_3)	. . . .	$+\ 1{\cdot}344$
Third quadrant (V_4)	. . . .	$-\ 0{\cdot}0687$
" (V_3)	. . . .	$+\ 1{\cdot}337$
Fourth quadrant	. . . .	$-23{\cdot}7926$
	Sum	$+\ 3{\cdot}217$

or substituting $m' = \dfrac{1}{1047{\cdot}35}$

$$\frac{1}{n}\int dn = +{\cdot}003072 \left[\text{de Pontécoulant} = +{\cdot}003441\right].$$

Similarly

$$\int de = m'\Big[-1{\cdot}2575 - {\cdot}0012 + 1{\cdot}3171 + {\cdot}0010 - 1{\cdot}2981 + 1{\cdot}1374\Big]$$

$$= m' \times -0{\cdot}1013 = -{\cdot}000097$$

$$\int d\varpi = \frac{m'}{e}\Big[+3{\cdot}0688 - {\cdot}0094 - 2{\cdot}2275 - {\cdot}0102 - 1{\cdot}2009 + 1{\cdot}4670\Big]$$

$$= \frac{m'}{e} \times +1{\cdot}0878 = +{\cdot}001074 \text{ radians} \Big[\text{de Pontécoulant} = +{\cdot}00$$

Lastly, in order to calculate $\int d\zeta$; in extracting the results the mechanical quadratures we use the formula

$$\int d\zeta = \int (2\pi - nt)\frac{dn}{n} + \left[\int d\epsilon - \left\{1 - \sqrt{(1-e^2)}\right\}\int d\varpi\right] - \frac{\sqrt{(1-e^2)}}{e}$$

in extracting the results of the integrations arising from V_3 we use the formula

$$\int d\zeta = \left[\int d\zeta + \sqrt{(1-e^2)}\int d\varpi - \frac{2\pi - nt}{n}\int dn\right] - \frac{\sqrt{(1-e^2)}}{e}e\int d\varpi + (2\pi -$$

From the mechanical quadratures

$$\int\left(2\pi - nt\right)\frac{dn}{n} = m'\Big[+150{\cdot}679 + 0{\cdot}318 - 0{\cdot}060 - 4{\cdot}364\Big]$$

$$= m' \times +146{\cdot}573 = +0{\cdot}139947 \text{ radians}$$

Also

$$\int de - 0{\cdot}746\int d\varpi = m'\Big[+3{\cdot}5259 + 0{\cdot}0777 + 0{\cdot}0823 + 5{\cdot}5443\Big]$$

$$= m' \times +9{\cdot}2302 = +0{\cdot}008813 \text{ radians.}$$

Hence $\int d\zeta$ (part from mechanical quadratures only)

$$= +0{\cdot}139947 \qquad +0{\cdot}008813 \qquad -0{\cdot}000273$$
$$= +0{\cdot}148487 \text{ radians.}$$

From the definite integrals we have for $\frac{1}{m'}\int d\zeta$

Integral between limits of $\int d\zeta + \sqrt{(1-e^2)}\int d\varpi - \frac{2\pi - nt}{n}\int dn =$	$-\ 7{\cdot}041$
$-\frac{\sqrt{(1-e^2)}}{e} \times e\int d\varpi$ taken between limits . . . =	$+\ 0{\cdot}900$
$+\left(2\pi - nt\right)\frac{1}{n}\int dn$ at the upper limit . . . . =	$+\ 0{\cdot}155$
minus ditto at the lower limit . . . . . =	$+13{\cdot}767$
Sum	$+\ 7{\cdot}781$

Hence $\int d\zeta$ (part from integral only) $= +0{\cdot}007429$ radians.

Hence the whole perturbation $\int d\zeta = +{\cdot}15592$ radians.

It should be stated that several approximations have been introduced; in particular the square of the mass of Jupiter has been neglected, or in other words the disturbing forces have been computed as if the comet was in a certain mean undisturbed path.

Our result for $\int de$ is one fifty-fifth part of that given by de Pontécoulant, *C.R.*, lviii. p. 827. This renders it extremely probable that de Pontécoulant's error consisted in the omission of the factor, circular measure of one degree.

De Pontécoulant's value of $\int d\zeta$ is $+{\cdot}15302$ radians; this implies that, considering Jupiter's action only, his value of the next perihelion passage is thirteen days later than ours.

Magnitude of o Ceti (Mira), 1906 *December* 14–1907 *February* 16, *as observed at the Radcliffe Observatory, Oxford.*

(*Communicated by the Radcliffe Observer.*)

Information having been received early in December that the increase in the light of Mira Ceti suggested a maximum of abnormal brightness, observations of this variable were undertaken by Mr Wickham and Mr Robinson.

These observations are made by Argelander's method, and are all naked-eye estimations.

When the observations commenced the variable had already considerably exceeded its brightness on the occasion of the last maximum, but a curve drawn to represent the observed magnitudes does not clearly indicate a maximum, the epoch of which may have occurred previous to December 14. Unfortunately, observations were rendered impracticable on December 11–13 by bad weather which prevailed. The date of maximum derived from Chandler's "Revision of Elements of Third Catalogue of Variable Stars" (*A. J.* 553) is 1906 December 19.*

The variable was so bright on the earlier nights that it was necessary to take comparison-stars situated at considerable distances from it, and at various altitudes, but these are distributed in such a way that the resulting mean will, it is hoped, be fairly free from the effects of atmospheric absorption. The proximity of α Ceti, the colour of which was similar to that of the variable, was found advantageous, but the observers met with the usual difficulty in comparing the dull yellow light of the variable with that of lustrous white stars.

Some notes on colour are included amongst the "Observers' Remarks."

It is worth mentioning that in the *Harvard Annals, Photometry*, vol. xiv. p. 121, α Ceti is described as "O," or pronounced orange, whilst in the recently published vol. xvii. of the Potsdam Astrophysical Observatory this star is characterised as "G—," or between light yellow and yellow.

* It may be well to point out here that the second term of the periodic inequality as given in the footnote to the catalogue is incorrectly printed. The coefficient should be 12·3 (not $12^{\mathrm{m}}\cdot 3$), the unit being a day.

TABLE I.

Stars used for Comparison.

Name of Star.	R.A. 1900.	N.P.D. 1900.	Adopted Mag.	Ref. No.	Name of Star.	R.A. 1900.	N.P.D. 1900.	Adopted Mag.
	h m	° ′				h m	° ′	
α Androm.	0 3·2	61 28	2·15	20	α Persei	3 17·2	40 30	1·90
γ Pegasi	0 8·1	75 22	2·87	21	ε Eridani	3 28·2	99 48	3·82
β Ceti	0 38·6	108 32	2·26	22	δ Eridani	3 38·5	100 6	3·72
γ Cassiop.	0 50·7	29 49	2·25	23	γ Eridani	3 53·4	103 48	3·27
η Ceti	1 3·6	100 43	3·61	24	Aldebaran	4 30·2	73 41	1·06
β Androm.	1 4·1	54 55	2·37	25	ι Aurigæ	4 50·5	57 0	2·86
θ Ceti	1 19·0	98 42	3·83	26	β Eridani	5 2·9	95 13	2·87
η Piscium	1 26·1	75 10	3·71	27	Rigel	5 9·7	98 19	0·34
β Arietis	1 49·1	69 41	2·73	28	γ Orionis	5 19·8	83 44	1·70
α Piscium	1 56·9	87 43	3·94	29	β Tauri	5 20·0	61 29	1·78
γ Androm.	1 57·8	48 9	2·20	30	δ Orionis	5 26·9	90 22	2·48
α Arietis	2 1·5	67 1	2·23	31	ε Orionis	5 31·1	91 16	1·68
ξ² Ceti	2 22·8	81 59	4·34	32	ζ Orionis	5 35·7	92 0	1·89
δ Ceti	2 34·4	90 6	4·00	33	κ Orionis	5 43·0	99 42	2·18
γ Ceti	2 38·1	87 11	3·58	34	α Orionis	5 49·8	82 37	0·93
η Eridani	2 51·5	99 18	4·06	35	β Aurigæ	5 52·2	45 4	2·07
α Ceti	2 57·1	86 18	2·82	36	γ Cygni	20 18·6	50 4	2·32
Algol (β Pers.)	3 1·7	49 26	2·31	37	β Pegasi	22 58·9	62 28	2·61
ζ Eridani	3 11·0	99 11	4·88	38	α Pegasi	22 59·8	75 20	2·57

The adopted magnitudes are the means of all values found in the following *Harvard Annals*:—

1. "Harvard Photometry" (vol. xiv.).
2. "Photometric Revision of the D.M." (vol. xxiv.).
3. "Miscellaneous Photometric Measurements" (vol. xxiv.).
4. "Photometric Revision of the Harvard Photometry" (vol. xliv.).
5. "A Photometric D.M." (vol. xlv.).

TABLE II.

Observations of the Magnitude of o Ceti (Mira).

Day and Hour.		Obsr.	Estimations.	Resulting Magnitude.
1906.	h			
Dec. 14	7	R	·2 f 29; ·1 b 12; ·4 b 17; =3	2·19
14	9¾	W	much f 24, 30, 31, 32, 35; much b 10	...
21	8½	W	=18; =11; ·5 f 12; ·33 b 17; ·67 f 35; ·5 f 30, 31, 32; 1·5 b 10; much f 24	2·50
26	6¾	R	·2 f 29; ·2 f 12; ·2 f 3; ·7 b 17; ·7 b 9	2·18

TABLE II.—*continued.*

Observations of the Magnitude of o Ceti (Mira).

Day and Hour.		Obsr.	Estimations.	Resulting Magnitude.
1906.	h			
Dec. 26	8½	W	·5 f 12 ; ·25 b 30 ; ·25 f 31 ; ·25 f 32 ; = 11 ; much f 24, 27, 34	2·25
27	8½	W	·2 f 12 ; ·5 b 17 ; = 31, 32 ; ·3 b 30 ; ·2 b 11	2·06
27	8½	R	= 12 ; ·2 f 1 ; ·3 b 38 ; ·7 b 37 ; ·7 b 17 ; = 38 (2nd obs.)	2·24
1907.				
Jan. 2	6½	R	·2 f 1 ; ·2 f 12 ; ·6 b 9 ; ·4 b 17 ; ·4 f 29	2·30
3	6	R	·3 b 17 ; ·4 b 38 ; ·1 f 37 ; ·6 b 9 ; ·3 f 1 ; ·3 f 12 ; ·6 b 2	2·40
3	6¼	W	·3 f 1 ; = 6, 11, 12, 36 ; ·4 b 17 ; ·3 f 20 ; ·2 f 35	2·31
6	8¼	R	·3 b 17 ; ·3 f 12 ; ·6 b 9 ; ·3 f 6	2·46
6	8⅙	W	·5 f 28 ; ·4 f 12 ; ·3 b 9 ; ·3 f 20 ; ·25 b 17 ; ·3 f 36, 35 ; ·2 f 29, 30 ; ·1 b 4 ; = 33 ; ½ (1 + 2)	2·39
11	6	R	·5 b 9 ; ·5 f 12 ; ·1 b 17	2·56
11	7	W	·4 f 12 ; ·2 b 9 ; ·3 f 6 ; ·2 b 17 ; ·2 f 32 ; ·2 b 25	2·53
17	8	W	= 17 ; ½ (9 + 12)	2·59
17	8½	R	·25 b 9 ; ·3 f 17 ; ·5 b 2 ; ·4 f 33 ; ·4 b 26 ; ·5 b 23	2·63
18	7	R	·15 f 17 ; ·2 b 9 ; ·3 b 2 ; ·4 b 26 ; ·2 f 30	2·64
22	7	R	·4 f 17 ; ·3 b 23 ; ·2 b 26 ; ·4 f 33 ; ·8 b 5 ; ·9 b 7 ; 1·0 b 15 ; ·4 f 30 ; ·15 b 2	2·82
23	6½	R	·2 b 26 ; ·4 f 33 ; ·4 f 17	2·82
29	7	R	= 26 ; ·5 f 17 ; ·45 b 15 ; ·2 f 9 ; ·1 f 23 ; ·7 f 33 ; ·1 f 2 ; ·8 b 10	3·08
31	6¾	R	·2 f 23 ; ·2 b 15 ; ·6 f 17 ; ·3 b 10 ; = 26 ; ·45 f 2 ; ·55 f 9 ; ·3 b 22 ; ·4 b 21 ; ·4 b 14	3·38
Feb. 2	6¾	R	·7 f 17 ; ·25 b 15 ; ·5 b 14, 10 ; ·4 b 16, 21 ; ·4 f 23, 2 ; ·15 b 22 ; ·5 f 9 ; ·2 f 26 ; ·3 b 8	3·42
2	7	W	1·0 f 17 ; ½ (14 + 17)	3·55
10	8	R	= 16, 21 ; ·2 b 14 ; ·1 b 10	3·88
13	7	R	·1 f 14 ; ·2 f 10 ; ·2 b 13 ; ·6 b 19 ; ·2 f 16, 21	4·16
16	7	R	much f 14 ; estimated mag.	4·50

Observers' Remarks.

1906. Dec. 14. Half weight to comparison with No. 3. Colour of Mira a mean tint between the pale yellow of Capella and the deep orange of Aldebaran. (R.)

(With small telescope, aperture 1¾-inch, the image of the variable showed distinct carmine fringes, which were not visible in the comparison stars, with two exceptions, α Ceti and Aldebaran, where the colour and fringes were much fainter. [W.])

Dec. 21, 27. The dull yellow colour of the variable and some

of the comparison stars is obvious to the naked eye, and comparison with brilliant white stars, such as those of the belt of Orion, is difficult. (W.)

Dec. 26. Observations rather difficult owing to the proximity of the Moon. Slight haze at times. Half weight to comparison with No. 3. (R.)

Dec. 27. Mira similar in colour to α Arietis, yellow. (R.) Half weight to comparison with No. 17. (W.)

1907. Jan. 2. Colour of o Ceti similar to that of α Arietis. (R.)

Jan. 3. α Ceti is reddish; o Ceti is yellow, closely resembling α Arietis in colour. (R.)

Jan. 6. Colour of o Ceti same as α Arietis, but α Ceti and β Androm. are orange-red. (R.) The yellow colour of Mira is still manifest to the naked eye. (W.)

Jan. 11. α Ceti reddish; α Arietis and o Ceti yellow. (R.)

(W. and R. examined Mira Ceti with the Barclay Equatorial, aperture 10-inch, power 88. Both observers noted that the image of Mira showed markedly red spiculæ around the margin, but this margin was not so broad nor so deep a red as the same observers saw around Nova Persei in 1901. Two other coloured stars were examined for comparison. α Ceti exhibited but little red in its fringe, whilst a light orange or canary colour prevailed over the main image. Aldebaran showed reddish orange with a narrow margin of red, but this of a light tint only. The image of Mira Ceti was therefore quite distinct in appearance from these stars when viewed with the same optical means.

Observer R. notes, in addition, that "with the naked eye Mira is not so deep an orange as α Ceti, but with the Barclay telescope it is decidedly deeper than α and not quite so bright.")

Jan. 17. Sky itself transparently clear, but a ground fog prevailed, diminishing from the horizon towards Cetus, but even there it obscured all stars of the fifth magnitude. (W.)

The fog seemed to render stars of an orange colour more conspicuous to-night; α Ceti was, probably from this cause, relatively brighter than usual. The observations were made from the summit of the Observatory tower, 100 feet above the ground. (R.)

Jan. 18. Foggy; estimations difficult; but probably the mean is reliable. The observations were made from the summit of the Observatory tower. (R.)

Jan. 22. Bright Moon effectually screened during the observations. (R.)

Jan. 23. A brief comparison only. (R.)

Jan. 29. Observed in intervals of cloud and in bright moonlight. Estimations rather difficult, but mean is reliable. (R.)

Jan. 31. Sky hazy; observation difficult. (R.)

Feb. 16. Seen with difficulty in moonlight and through low haze. (R.)

Radcliffe Observatory, Oxford:
1907 *April* 9.

Early and Late Perseids. By W. F. Denning.

The following is a list of the apparent paths of probable and possible Perseids observed at Bristol during the 16 nights July 7 to 22 and 9 nights August 17 to 25 inclusive in the thirty years from 1876 to the present time. A proportion of these meteors were undoubtedly not true Perseids, but belonged to the other showers existing in Cassiopeia, Andromeda, Pisces, Aries, Camelopardus, Taurus, Auriga, etc., at the same epoch. There are a great number of such radiants visible—certainly more than a hundred—and it is very difficult to attribute correct positions for some of the individual meteors which have appeared at this time of the year.

The materials here furnished may afford some help in determining the dates of beginning and ending of the Perseid shower. These are very doubtful at present, but I believe that the weight of evidence favours the conclusion that true Perseids may be occasionally recognised after the first week in July. The display is very feeble at that time, and may not supply one meteor during a watch of several hours by a single observer. But on July 19 the stream becomes well pronounced, and its radiant capable of being definitely ascertained, though not in every year. The date just mentioned is in fact the earliest one on which I can confidently say that Perseid meteors are sometimes visible in sufficient numbers to enable a good radiant to be obtained by an individual observer.

In the last column of the list I have ascribed the radiants as they appear in my MS. book of observations. These positions are not correct in all cases. But I have reprojected the apparent paths, and have affixed the letter P to distinguish those which may be regarded as Perseids.

If other observers will supply similar data it will be possible to deduce the place of the radiant on every night from July 7 to 22. But it cannot be safely said that the shower begins so early as July 7; it will be very difficult to learn when the earth really encounters the first shots from this widely-distended stream.

Further materials for the above period, and for the concluding stages from August 17 to 25, will throw an interesting light on the actual duration of the shower, and on the positions of the radiant near the limiting dates of its visibility.

We might soon determine these features but for the fact that there are many other secondary displays yielding similar meteors and radiating from the same region of the sky. It is often impossible to say whether meteors are Perseids or not, though their flight-directions may conform with the position of the great August shower. At the earlier stages of the display there are well-defined minor radiants at about

$23^\circ+43^\circ$	$33^\circ+52^\circ$	$50^\circ+31^\circ$
$23+57$	$43+22$	$54+71$
$29+36$	$44+57$	$60+58$
$33+18$	$47+43$	$61+48$

—and many others, varying in strength from year to year.

With regard to the concluding phases of this system, I have recorded meteors from the right position up to August 25, but there are doubts as to whether the shower is really prolonged to that date. It is certainly continued to August 20, but on subsequent nights the evidence gleaned is not sufficiently ample to enable one to absolutely affirm its visible continuance. From a discussion of my combined observations in various years, I have deduced the following radiants of Perseids :—

Aug. 16	53°+58°	21 ↓ s	Aug. 20	57°+59°	10 ↓ s
,, 17	54+60	5 ,,	,, 21	64+59	12 ,,
,, 18	55+59	11 ,,	,, 22	70+61	7 ,,
,, 19	57+59	5 ,,			

Companion showers (in addition to those above given for the earlier stages) are well pronounced at about—

39°+28°	70°+66°	75°+15°
58+ 9	71+52	77+32
61+36	74+42	87+43
63+22	77+58	106+52

The earliest radiant I have obtained, presumably for the Perseids (though a shower of Cassiopeids may also be involved) is for the period July 7–9, 10° + 45½°, ten meteors. Quite possibly the shower continues during fifty nights July 7–August 25.

The late Mr J. Kleiber gave the theoretical place of the Perseid radiant as, on July 8, 9° + 46°, and August 16, 54° + 59°—(*Monthly Notices*, lii. p. 351).

Observed Paths of Early Perseids.

	Date.		G.M.T. h m	mag.	From α δ	To α δ	Path.	Notes.	
7	July	7	13 21	2	349°+25°	345°+20°	6½°	R K	P
2		,,	11 44	3	318+56	283+49	22	R K	P
5		8	12 1	♃	240+61	218+37	27	v R B K	30+8
8		,,	11 4	4	47+71	87+73	12	R	345±0
		,,	12 7	4	61+59½	81+70	7	R K	
		,,	12 36	3	355+22	349⅔+10½	11½	R K	P
		,,	13 22	2	332+72½	320+75	4½	R B K	P
2		9	13 11	3	310+20	303½+14½	8	R K	P
7		11	11 18	2	339+26	316+3	31	v RK	47+43
5		,,	11 62	2	278+45	260+32	18	v R	P
8		,,	11 36	3	344+58	334+59	5½	S B K	P
		,,	11 49	4	344+59½	333+61½	6	S K	P

Observed Paths of Early Perseids—continued.

Date.		G.M.T. h m	mag.	From α δ	To α δ	Path.	Notes.	
1904 July	11	14 14	2	18+36	20½+32½	4	R K	P
1877	12	11 43	3	335+48	313+40	17	R	47+43
	,,	11 57	3	336+30	313+11	28	v R	P
	,,	12 17	3	340+64	308+66	13	v R K	23+43
1885	,,	11 18	3	281−15	275−21½		S K	P
1888	,,	12 5	3	44+58½	54+60	6	R K	P
1885	13	12 9	2	293½+35½	276½+20	21	v R K	P
	,,	12 49	3	356+41	350+37½	6	R K	P
	14	12 35	3	324+64	279+62	20	v R B K P	
1901	15	13 5	>1	270+53	248½+32	24	R K	23+43, 2
1876	16	11 17	1	260+80	225+67	15	R K	23+43,
	,,	11 33	2	310+51	286+35	22	v R K	P 47+
	,,	12 19	2	321+42	304+30	17	R B K	P
1898	,,	11 4	3	24+53	26+57	4	S K	P
1902	,,	13 47	5	15½+55	14½+58	3	R K	20+43
1898	,,	12 12	3	348+53	339+53½	5½	R K	23+43
	,,	12 42	4	8+66	0+71	6	S B K	23+43
1898	17	12 42	4	25½+54½	27+59	5	R K	23+43
1876	18	10 35	4	16+47	15+40	7	R	27+71
	,,	11 20	>1	211+49	206+18	31	R K	P
1881	,,	11 9	1	110+70	144+58½	19	R K	P
1887	,,	11 1	>1	3+34½	0+31	4	S B K	P
1900	,,	10 38	5	314+46½	306+42	8	R K	P
1876	*19	10 58	♃	346+25	337+12	20	B K	P
1887	,,	11 43	1	358½+38	351+30	10	S B K	P
	,,	12 25	1	350+62	330½+64	10	S B K	P
	,,	13 8	4	356+47	347+44	7	R K	P
1900	†,,	11 43	♃	344+33	329+18	20	R K	P
1901	,,	11 41	1	335+68	289+67¼	12½	v R K	23+43, 2
	,,	12 58	4	346+35	339½+30	7	R K	P
1876	20	11 29	1	341+6	335−8	15	K	P
1901	,,	11 24	4	338+60	302+58½	8	v R K	23+43, 2
	,,	11 59	2	328+59	292+54¼	20	v R K	23+43, 2
1876	21	11 21	2	344+60	315+57	15	R K	P
1887	,,	12 40	3	5+44	359½+39	6½	v R	P

* Observed also by J. Lucas, Oxford. Radiant, 22°+54°.

† Observed also by Prof. A. S. Herschel. Radiant, 17+50.

Observed Paths of Early Perseids—continued.

Date.		G.M.T. h m	mag.	From α δ	To α δ	Path.	Notes.	
1901 July	21	11 57	5	53+62½	70+65½	8	R K	23+43
	,,	12 2	5	25+36	25+32	4	R K	23+43
	,,	13 40	3	311+52	297½+45	11	R K	P
	,,	13 43	1	349½+15	341+1½	16	R K	P
	,,	14 0	3	26+53½	31+53¾	3	S K	P
1887	22	11 2	4	6+43	1+40	5	R K	P
	,,	12 21	♃	356+45	332+29	24½	R B K	P
	,,	13 2	5	25+47	27+43	4	S	P
	,,	13 55	3	331+77	281+75	12	R K	P
1900	,,	11 15	2	13+55	356+59¾	10	S K	P

Late Perseids.

Date.		G.M.T. h m	mag.	From α δ	To α δ	Path.	Notes.	
1901 August	17	15 21	2	90+43	98+35	10	R	P
1885	,,	10 59	4	346+59	318+46	21	R K	P
	,,	12 44	4	304½+46	298½+39	8	v R K	P
1898	,,	9 36	3	55½+69	57½+74	5	S K	P
1899	,,	15 8	2	96+43	103+36	8½	R K	P
1885	18	13 37	3	335+37	328+27½	11	R K	P
1893	,,	12 25	♀	330+31	315½+9	26	R K	P
	,,	12 37	1	38+10	37½+1	11½	R B K	P
1901	,,	10 28	3	327+40	320+21	23	R K	P
	,,	10 54	3	329+50	313+34½	17	R K	P
	,,	11 44	3	241+84	236+72	12	R K	P
	,,	12 16	5	4½+10	0+2	9	R	P
	,,	12 48	5	69+63½	83+65½	6	R K	P
	,,	12 53	2	37+64½	25+66¾	5½	S B K	P
	,,	13 4	2	245+86	236+71	15	R K	P
1884	19	11 10	3	88+59	92+59½	2½	v R	77+58
	,,	11 27	3	39+68	23+70	6	K 4°	P
	,,	12 21	4	341+78½	308+73¾	9	B K	P
1901	,,	11 48	4	353½+16	348+9	9	R K	P
	,,	12 29	3	22½+58½	14+57	5	S K	P
1885	20	11 41	3	28+46	24+42	5	R K	P
	,,	13 55	3	352½+15	347½+6	10	R K	P

Late Perseids—continued.

Date.			G.M.T. h	m	mag.	From α δ	To α δ	Path.	Notes.
1887	August	20	10	1	5	356+67	297+14	64	S K 77+58
1901		,,	10	16	>1	349+43	327½+19½	29	R B K P
		,,	10	35	3	79+56½	92+53	8	S P
		,,	11	2	4	28½+22½	25½+15½	8	R P
		,,	11	41	3	72+67½	82+71	5	S BK P
		,,	11	45	5	357½+32½	350+22	11	R K P
		,,	13	58	4	308½+50	299½+40	11½	R P
1884		21	11	25	3	16+56	3+54	8	R K 71+52
		,,	11	40	2	42+81	309+84	11	R BK 71+52
1887		,,	10	37	5	11+50½	0+44	10	R K 77+58
		,,	12	58	4	27+49	18+45½	7	R K 71+52
1901		,,	10	0	4	312½+51½	303+41	12	R K P
		,,	12	20	2	14+45	359+33½	15½	R K P
		,,	12	48	2	22½+17	16+3½	15	R K P
		,,	13	8	4	17+15	11½+3	13	R K P
		,,	15	11	5	66+36	66+31	5	R K P
1903		,,	13	4	4	30+37½	27+33	5½	R K P
1884		22	12	21	4	72+48¾	69+43½	5½	not R K 77+58
1900		,,	10	20½	3	287+78½	264+67	13	v R K P
		,,	11	39	4	17½+28	11+17	13	R K P
		,,	11	48	3	11+44¼	3+37½	8	R K P
1901		,,	10	52	4	334+62	319+52½	12	R K P
		,,	12	5	3	11½+10	7+2	9	R K P
1906		,,	11	8	>♃	18+38	7½+28	13	S K P
		,,	11	9	5	345+33	336+22	15	v R K P
1884		23	11	59	1	345+19	335½+2	19½	v R K 40+59
1887		,,	12	27	3	93+62½	109+60¼	8	R K 72+62
1900		,,	11	3	4	16+22	12+16	7	R 77+58
1903		,,	11	0	4	25+27	22+21	7	not R K 70+65
1887		24	11	10	3	66+70	6+81½	17½	R K 77+58
1900		,,	9	26	2	25+51	10+42	13	v R K 77+58
1879		25	11	0	3	243+30	243¾+16	14	R K 62+35
1903*		,,	9	35	3	27+29	20+19	12	R K 79+59

Abbreviations—R, rapid ; S, slow ; B, bright ; K, streak ; v, very ; P, Perseid.

* Also observed by Mr C. L. Brook at Meltham, Huddersfield.

The Electric Arrangements of an Observatory.
By W. Ernest Cooke, M.A.

A system of electric connections for the numerous instruments has been evolved at the Perth observatory of so simple and convenient a character that a description may be interesting and perhaps useful, especially to those who contemplate establishing a new observatory.

There is only one sidereal clock and one battery used for all purposes, such as chronographs, clock dials, clock control, equatorial control, chronograph control, transit keys, and small electric lights.

The battery consists of two large accumulator cells having an E.M.F. of 4 volts. There are in reality two of these used alternately, so that one may be always available. It is found advisable to change the battery at 9 a.m. daily, and to recharge the one not in use. The charging is performed very simply by means of a motor transformer, worked from the ordinary electric light supply current.

By means of a three-way switch either battery can be turned on or off, or connected with the charging transformer, instantly.

From the battery a pair of heavy mains are run round the buildings and connected to a pair of terminals in each room, wherein any instruments which require an electric current are located. These terminals are labelled A and B.

From the contacts of the sidereal clock a current passes through a relay, and the contacts made by the armature of this relay practically indicate the adopted seconds of the clock. It has been considered advisable, in the case of the clock contacts only, to use a separate battery, and two cells of the gravity type are found to give sufficient current. But even in this case a switch has been provided for substituting the current from A B if the gravity battery is out of order.

One of the terminals of the secondary contact of the relay is connected to A, and the other to a third main wire which runs round all the buildings and is labelled C. The function of the clock relay is thus to make a momentary connection between A and C at every beat of the sidereal clock.

We have thus, in every instrument room, the three terminals A, B, and C, and from these all necessary connections can be made. Thus, for a chronograph or clock dial, wires must be run direct from C B, and this gives an electric impulse every second, since C and A are joined once a second by the sidereal clock relay. For an instrument which works by means of a key, or any form of hand control, wires must be run from A B. If a chronograph be of a

single pen type, *i.e.* if both the clock beats and the key taps are to be registered by means of the same electro-magnet, it will be necessary to introduce a relay; for otherwise the key will connect A and C, and will send an extra impulse along the clock circuit at every tap. In this case the primary terminals of the relay are connected with C B direct, and the secondary with A B through the chronograph. The secondary of the relay is thus placed in parallel with the key.

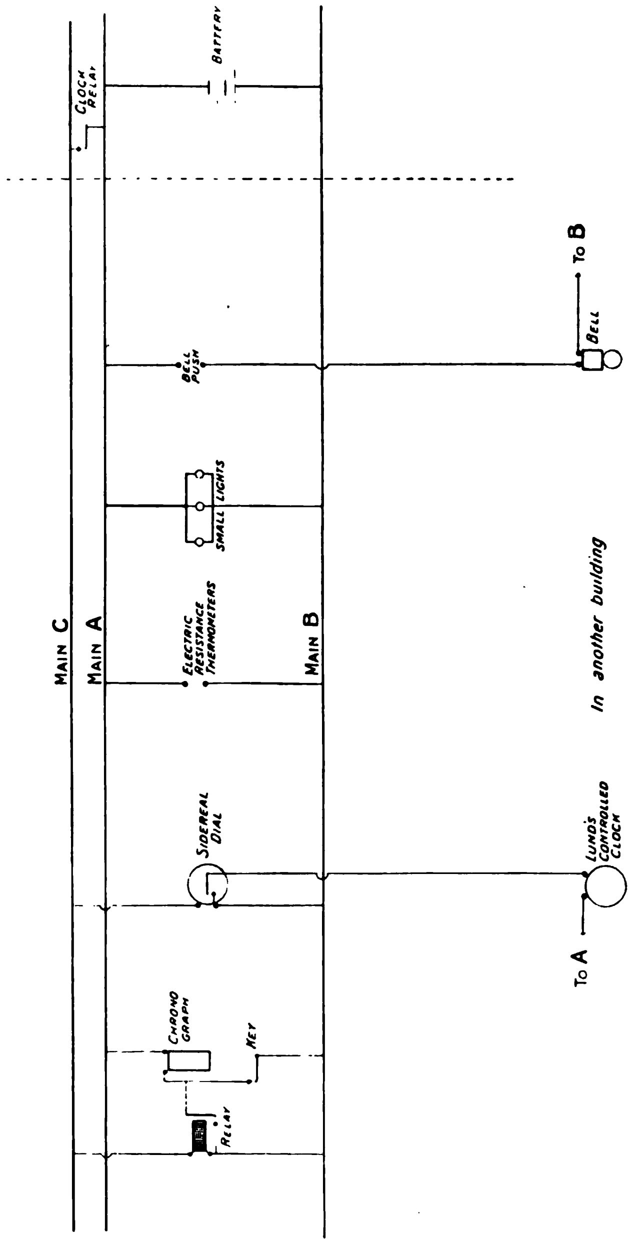
It may be mentioned that one of these relays is used as a key for transmitting clock beats in determining differences of longitude.

The system is especially convenient when it is desired to control an instrument in one building by means of a key or instrument in another building. For instance, there is a clock dial in the transit room, worked from the sidereal clock by simply connecting its terminals with B C. This dial contains a pair of contacts which are joined for a couple of seconds at each sidereal hour. These are required to control, by means of Lund's clips, a clock keeping sidereal time in another building. The connections are very simple. One of the primary contacts of the dial is already connected with B. It is now also joined to one of the secondary contacts. The other secondary is connected to a single line wire which leads to one of the terminals of Lund's control, and the other terminal is joined to the nearest A. The arrangement is shown in the diagram. Even the line wire might be dispensed with and an earth used if this instrument only were concerned, but it will be found advisable to keep away from earth connections entirely.

The accompanying diagram gives a rough sketch of the manner in which the system is used, but conveys no idea of the number of instruments which may be connected. At the Perth Observatory the following are all worked by the one battery :—

Two sidereal dials.
Three sidereal chronographs.
One mean time chronograph.
Three transit keys.
One complete longitude system for short distances.
One Lund's controlled clock.
One mean time multiple relay.
One electric recorder (for platinum thermometers).
Two complete Grubb's controls (Astrograph and Chronograph), using the sidereal clock instead of an auxiliary pendulum.
One control for mean time clock.
Several small lamps on instruments.
Two sounders for beating seconds in dark rooms.
Four call bells.
Two alarm bells on Astrograph.

All electric connections are, in fact, worked from the A B C terminals except the hourly current from the mean time clock.


Main C
Main A
Main B
Clock Relay
Battery
To B
Bell
Bell Push
Small Lights
Electric Resistance Thermometers
In another building
Sidereal Dial
Lund's Controlled Clock
To A
Chronograph
Key
Relay

Even that is connected through A B to the multiple relay, but thence the signals have to be distributed outside, and the observatory is only supposed to make a contact once per hour,—each outside user supplying his own battery. We ourselves take one of these circuits and run it through various meteorological instruments, fire a gun, and control a public clock, but for this purpose Leclanché batteries are used.

Perth Observatory, Western Australia.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXVII. MAY 10, 1907. No. 7

H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Harold A. H. Christie, B.A., Royal Observatory, Greenwich (proposed by Sir W. H. M. Christie);
Bertram Francis Eardley Keeling, The Observatory, Helwan, near Cairo, Egypt (proposed by H. H. Turner); and
Frederick Alexander Lindemann, Sidholme, Sidmouth, Devon (proposed by A. F. Lindemann).

Sixty-nine presents were announced as having been received since the last meeting, including, amongst others:—

Maurice Farman, Mesures d'étoiles doubles, presented by the author; Harvard Observatory Annals, vol. 52, part i, Eclipses of Jupiter's satellites; vol. 52, part 2, Second Catalogue of variable stars, presented by the Observatory; Heidelberg Astrophysical Observatory, Publications, vol. 2, Nos. 2-10, presented by the Observatory; Meudon Observatory, Annals, vol. 3, part i, presented by the Observatory; 20 charts of the Astrographic Chart of the heavens, presented by the Royal Observatory, Greenwich.

Note on Le Verrier's Tables of Saturn. By A. M. W. Downing, D.Sc., F.R.S.

My attention has been called to the rapidly-increasing discordance between the places of Saturn derived from Le Verrier's tables and those derived from the tables of Hill, as shown by the latest available data, *i.e.* the Ephemerides for 1909.

The following statement gives the corrections to Le Verrier's geocentric places, near the time of opposition in 1909, as given in the *Connaissance des Temps* for that year, deduced from their comparison with the corresponding places given in the *Nautical Almanac* :—

Day 1909.	R.A.	Dec.
	s	"
Oct. 2	−1·09	−6·8
10	−1·10	−6·9
18	−1·08	−6·8

The corresponding corrections to the heliocentric longitudes are :—

Day 1909.	Longitude
Oct. 2	−16″·0
10	−15·9
18	−15·9

It will be remembered that M. Gaillot has published "Tables rectifiées du mouvement de Saturne" in tome xxiv. of the *Annales de l'Observatoire de Paris*, and it is of interest to exhibit the resulting corrections to Le Verrier's heliocentric longitudes according to Hill, and according to Gaillot, at convenient epochs, as follows :—

Epoch.	Hill.	Gaillot.
1894 March 6	− 3″·8	″ ...
1901 Jan. 1	− 3·5	− 4·9
1909 Oct. 10	−15·9	−16·2

The observed mean correction to Le Verrier's longitudes of Saturn, derived from the Greenwich Observations of 1900, is − 4″, whilst the observed mean correction to Hill's longitudes, derived from the Greenwich Observations of the following year (the first year in which Hill's tables were used in the *Nautical Almanac*), is about half a second in the same sense; thus demonstrating the superior accuracy of Hill and Gaillot at the commencement of the century. But it is important to note the rapid increase in the discordance between Le Verrier and the other two authorities since that time.

Computation of Secular Perturbations. By R. T. A. Innes.

I.

It is well known that the Lagrangian equations for the variation of a planet's elements (the planet being considered of infinitesimal dimensions) are exact. It is in the integration of these equations that difficulties arise. To avoid these difficulties several assumptions are made, such as that the planets can be considered in pairs (*i.e.* the problem of three bodies) and do not react on each other; that the elements on the right-hand side of the equations are constant, and that the perturbative function can be developed in a converging series. When we are concerned with the mutual perturbations of the eight major planets, the above assumptions are sufficiently good; the errors introduced by the first two assumptions are eliminated later by taking into account the powers of the masses higher than the first, the third assumption is nearly justified by the smallness of the eccentricities and mutual inclinations of the eight major planets. In cases of difficulty, the Lagrangian equations are abandoned and the perturbations found by other processes; thus Hill found that the Lagrangian method as used by Le Verrier for the theory of the motion of Jupiter and Saturn was insufficient, so that in his theory he adopted the processes developed by Hansen.

Gauss showed that the secular part of the perturbations (to the first power of the masses) could be found without using a development of the perturbative function. The rationale of Gauss's method is as follows:—Let us imagine that the perturbative function is developed in a series as follows,

$$P_0 = P_1 + P_s \sin M + P_c \cos M + P_{2s} \sin 2M + \text{etc.}$$

where the coefficients P contain, besides certain quantities depending on the elements of the disturbed planet, the elements and position of the disturbing body. Multiplying each side by dM and integrating around the circumference gives

$$\frac{1}{2\pi}\int_0^{2\pi} P_0 dM = P_1$$

Then writing $P_1 = p_1 + p_s \sin M_1 + p_c \cos M_1 + \text{etc.}$, where the coefficients p depend only on the elements of both planets and do not contain the time explicitly, a second integration gives

$$\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} P_0 dM dM_1 = p_1$$

It is on the quantity p_1 that the secular perturbations depend. In the general case it is impossible to find this quantity by algebraical expansions. Gauss proved that one of these integrations could be

reduced to elliptic integrals; the second has to be a mechanical integration. Gauss, as is well-known, gave a geometrical title to his paper and did not prepare his formulæ for numerical application. Bour and others have given further geometrical interpretations of Gauss's formulæ, which, interesting as they are, call for no attention here.

Since Dr. G. W. Hill published his first paper on Gauss's Method of Computing the Secular Perturbations of the Planets (Astron. Papers prepared for the use of the American Ephemeris, 1882; Collected Works, T. ii., 1906) many numerical applications of this method have been made. Hill did not leave the method in final shape, for no sooner had he finished his paper than he proceeded to give a substantial modification in an addendum by making use of Gauss's arithmetico-geometrical mean. Both of Hill's solutions require extensive tables of the elliptic integrals for their easy application. Hill provided these tables for his first solution, and Callandreau and the writer did the same for Hill's second method. Previously to Hill's work there had been but little use made of Gauss's method. We learn from Bode's *Astronomisches Jahrbuch*, 1819, p. 229, that before Gauss had actually published his memoir, Nicolai had computed the sec. var. of the elements of the Earth's orbit by this method, but no details of the work were given. The late Professor J. C. Adams applied the method to the perturbation of the orbit of the November Meteors (see *M. N.*, 1867, Apl.; Collected Works, T. ii. pp. 194–200). It is well known that the method as expounded by Gauss requires the solution of a cubic equation, but Adams avoids the cubic by neglecting a portion of the terms factored by the square of the eccentricity of the disturbing body, and so reduces the equation to a quadratic. This is no doubt the modification which Adams stated greatly facilitates the application of Gauss's formulæ. As a practical step this modification was quite justified, but it takes away from the interest of the problem, and, as will appear immediately, is of no real advantage. Halphen, in 1886 (C.R., T. ciii.) announced that the problem could be completely solved without the labour of obtaining the roots of the cubic; he gave an exposition of his method in his *Fonctions Elliptiques*, 1888, T. ii. I was unable to follow Halphen's exposition, to which Tisserand's explanation added nothing (*Mec. Cel.*, T. i.), and it was only on obtaining by chance a copy of Dr Louis Arndt's paper on Halphen's solution (*Recherches . . . . Pert. Seculaires*, Neuchâtel, 1896) that the solution was grasped. The formulæ for numerical application given by Arndt assume that g_3 is always positive, so that his solution will only apply to the easier cases, although Halphen drew special attention to the case of g_3 being negative. Arndt has provided tables for the cases in which Legendre's modular angle θ will not exceed 45°. Hill has again revised his solution, and shows that by using Jacobi's nome q tables are practically unnecessary, but the roots of the cubic are still required (*Ast. Jour.*, No. 511, 1901).

As already stated, Gauss's method is rigorous to the first power

of the masses. It has been applied to most of the mutual actions of the major planets, but unnecessarily so in many cases, as the tables given by Newcomb and Le Verrier are ample and much simpler to use unless the ratio of the mean distances exceeds one-half; in other words, the small inclinations and eccentricities of the orbits of the major planets permit algebraical expansion, which only becomes long when this ratio exceeds one-half.

This method has its rôle in computing the first order secular perturbations of the periodical comets and minor planets. An interesting application to Eros has recently been made by Herr W. Dziewulski (*Säkulare Marsstörungen . . . des Eros*, Cracovie, 1906). He shows incidentally that this method gives better results in the case of Eros than the Lagrangian method, which takes into account all powers of the masses, but not of the inclinations and eccentricities. In fact, the Lagrangian approximation will in some cases not even give the correct signs, far less the correct figures. Tolerable approximations are only given in the cases of the planets with preponderating masses.

Mr C. J. Merfield, of Sydney, has recently announced the completion of a computation of the secular variation of Eros.

In this paper Hill's first exposition has been adopted as a working basis, and in the introduction of the elliptic functions Schwarz's notation, as developed in Padé's translation (*Formules et Propositions*, Paris, 1894), has been followed, except that the quantity h is replaced by Jacobi's letter q. No proofs of formulæ found in previous papers are given.

The aim throughout has been simplicity in numerical application. The final formulæ have been framed for that end, and have been submitted to numerical tests. The pure mathematician dismisses one with a series guaranteed to be absolutely convergent for certain values of the argument, but a trial often proves that the series, though it may be absolutely convergent, is arithmetically absolutely useless. It will appear that if an accuracy of six places of figures suffices, the use of series can be entirely evaded, a result by no means to be expected *a priori*.

The two main points in this paper are (1) the simplification of the formulæ by the introduction of the Weierstrassian cubic; (2) the new series for the computation of the periods of the elliptic functions,—series which are remarkably inert, nearly all the variation being thrown on to a simple trigonometrical factor of the series.

Thus the computer has the choice of two main methods in applying Gauss's principle,—(1) solving the cubic and using Jacobi's nome q (Hill, *Ast. Jour.*, No. 511); or (2) using the invariants of the cubic and two hypergeometrical series. Either of these methods is much shorter than the earlier solutions; which of the two is the better in actual practice cannot be answered offhand. It may be remarked that the former uses almost entirely trigonometrical logarithms, which the latter does not, and this alone is a considerable advantage.

2.

All the equations and formulæ wanted in practice are collected later on. Here it is unnecessary to write down the six Lagrangian equations; as a type we confine our attention to that for the eccentricity

$$\frac{de}{dt} = \cos\phi\frac{d\phi}{dt} = \frac{a^2 n\cos\phi}{1+m}\left[\sin v\mathrm{R} + (\cos v + \cos \mathrm{E})\mathrm{S}\right]$$

We have to find that part of the values of R, S, and W which is independent of the time (or of the positions of the planets in their orbits).

As already stated, two integrations are required, one of which will be done mechanically. The above equation is therefore written

$$\left[\frac{d\phi}{dt}\right]_{00} = \frac{m_1 n}{1+m}\mathrm{M_E}\left[\cos\phi\sin\mathrm{E}\frac{a}{r}\mathrm{R}_0 - \left(\frac{3}{2}e - 2\cos\mathrm{E} + \frac{e}{2}\cos 2\mathrm{E}\right)\frac{a}{r}\mathrm{S}_0\right]$$

The mechanical integration need only be made on the quantities

$\frac{a}{r}\mathrm{R}_0$, $\frac{a}{r}\mathrm{S}_0$, and $\frac{a}{r}\mathrm{W}_0$, where

$$\frac{a}{r}\mathrm{R}_0 = \mathrm{A}_0^{(c)} + \frac{1}{2}\mathrm{A}_1^{(c)}\cos\mathrm{E} + \frac{1}{2}\mathrm{A}_1^{(s)}\sin\mathrm{E} + \frac{1}{2}\mathrm{A}_2^{(c)}\cos 2\mathrm{E} + \text{etc.}$$

with similar equations for $\frac{a}{r}\mathrm{S}_0$ and $\frac{a}{r}\mathrm{W}_0$.

Thus the final equations take the form

$$\left[\frac{d\phi}{dt}\right]_{00} = \frac{m_1 n}{1+m}\left[\frac{1}{2}\mathrm{A}_1^{(s)}\cos\phi - \frac{3}{2}e\mathrm{B}_0^{(c)} + \mathrm{B}_1^{(c)} - \frac{e}{4}\mathrm{B}_2^{(c)}\right]$$

The addendum to Hill's first paper may be referred to for further matter concerning these equations.

We have

$$\frac{a}{r}\mathrm{R}_0 = \frac{1}{2\pi}\int_0^{2\pi}\frac{a^2}{r}\ \frac{xx_1 + yy_1 - r^2}{\Delta^3}(1 - e_1\cos\mathrm{E}_1)d\mathrm{E}_1$$

with corresponding expressions of the same form for $\frac{a}{r}\mathrm{S}_0$ and $\frac{a}{r}\mathrm{W}_0$.

Putting

$$\frac{a}{a_1} = \alpha \qquad r_0 = \alpha(1 - e\cos E) = \frac{r}{a_1}$$

$$\begin{aligned} k\cos(v+K) &= A_c & \qquad k_1\cos\phi_1\sin(v+K_1) &= A_s \\ -k\sin(v+K) &= B_c & k_1\cos\phi_1\cos(v+K_1) &= B_s \\ \sin\Pi_1 &= C_c & \cos\phi_1\cos\Pi_1 &= C_s \end{aligned}$$

$$\begin{aligned} A_0 &= 1 + r_0^2 + 2e_1 r_0 A_c \\ B_0\cos\epsilon &= r_0 A_c + e_1 \\ B_0\sin\epsilon &= r_0 A_s \\ C_0 &= e_1^2 \end{aligned}$$

Note, Hill's $A = a_1^2 A_0$, etc.

$$\Delta_0^2 = A_0 - 2B_0\cos(E_1 - \epsilon) + C_0\cos^2 E_1$$

Hill's $\Delta = a_1\Delta_0$

then

$$\frac{a}{r}R_0 = \frac{1}{2\pi}\int_0^{2\pi} \alpha^2\frac{A_c(\cos E_1 - e_1) + A_s\sin E_1 - r_0}{\Delta_0^3}(1 - e_1\cos E_1)dE_1$$

$$\frac{a}{r}S_0 = \frac{1}{2\pi}\int_0^{2\pi} \alpha^2\frac{B_c(\cos E_1 - e_1) + B_s\sin E_1}{\Delta_0^3}(1 - e_1\cos E_1)dE_1$$

$$\frac{a}{r}W_0 = \frac{1}{2\pi}\int_0^{2\pi} \alpha\sin J r_0\frac{C_c(\cos E_1 - e_1) + C_s\sin E_1}{\Delta_0^3}(1 - e_1\cos E_1)dE_1$$

If we remove the constant factors α^2, α^2, and $\alpha\sin J$ respectively, the numerators of the fractions on the right may be written as

$$f - ge_1 + [g(1 + e_1^2) - fe_1]\cos E_1 + h\sin E_1 - he_1\sin E_1\cos E_1 - ge_1\cos^2 E_1,$$

where f, g, and h have the respective values

	$\frac{1}{\alpha^2}\frac{a}{r_0}R_0$	$\frac{1}{\alpha^2}\frac{a}{r_0}S_0$	$\frac{1}{\alpha\sin J}\frac{a}{r_0}W_0 \times \frac{1}{r_0}$
f	$-r_0$	0	0
g	A_c	B_c	C_c
h	A_s	B_s	C_s

3.

Altering the variable from E_1 to T changes the equations for $\frac{1}{a^2}\frac{a}{r_0}R_0$, etc., to the form

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{\Gamma_1+\Gamma_2\sin^2 T+\Gamma_3\cos^2 T}{\Delta_0^3}\,dT \quad . \quad . \quad (a)$$

wherein

$$\Delta_0^2 \text{ is now } G_1 - G_2\sin^2 T + G_3\cos^2 T$$

but we may now omit the subscript $_0$ as unnecessary.

The elliptic integrals, using the limits 0 to $\frac{\pi}{2}$, are

$$\int_0^{\frac{\pi}{2}}\frac{dT}{\Delta^3}, \quad \int_0^{\frac{\pi}{2}}\frac{\sin^2 T\,dT}{\Delta^3} \quad \text{and} \quad \int_0^{\frac{\pi}{2}}\frac{\cos^2 T\,dT}{\Delta^3}$$

The G's are the roots of the equation

$$x^3-(A-C)x^2+(B^2-AC)x+CB^2\sin^2\epsilon=0$$

or $\quad x^3-k_1x^2+k_2x-k_3=0$

Putting

$$e_1=G_1-\frac{k_1}{3}, \qquad e_2=G_2-\frac{k_1}{3}, \qquad e_3=-G_3-\frac{k_1}{3}$$

transforms

$$G_1-G_2\sin^2 T+G_3\cos^2 T \quad \text{into} \quad e_1-e_2\sin^2 T-e_3\cos^2 T$$

Introducing the Weierstrassian functions by means of

$$\sin^2 T=\frac{e_1-e_3}{e_2-e_3}\cdot\frac{s-e_2}{s-e_1} \qquad \left(\text{when } T=\begin{cases}0, & s=e_2\\ \frac{\pi}{2} & s=e_3\end{cases}\right.$$

gives

$$\cos^2 T=\frac{e_2-e_1}{e_2-e_3}\cdot\frac{s-e_3}{s-e_1}$$

and

$$\frac{dT}{ds}=\frac{1}{2}\sqrt{\frac{(e_1-e_3)(e_2-e_1)}{(s-e_2)(s-e_3)}}\cdot\frac{1}{s-e_1}$$

$$\Delta^2=(e_1-e_2)(e_1-e_3)/(s-e_1)$$

$$\frac{dT}{\Delta^3}=\frac{1}{2}\,\frac{s-e_1}{(e_1-e_2)(e_1-e_3)}\cdot\frac{ds}{\sqrt{(s-e_1)(s-e_2)(s-e_3)}}$$

Putting

$$s = \wp u \quad \text{and therefore} \quad ds = \wp' u du$$

and noting that

$$\wp' u = 2\sqrt{(\wp u - e_1)(\wp u - e_2)(\wp u - e_3)}$$

we have

$$\frac{dT}{\Delta^3} = \frac{\wp u - e_1}{(e_1 - e_2)(e_1 - e_3)} du$$

Having $\wp u = e_3$ when $u = \omega'$ and to e_2 when $u = \omega''$ or $\omega + \omega'$ and writing $\sqrt{G} = (e_1 - e_2)(e_1 - e_3)(e_2 - e_3)$ and noting that

$$-\wp u du = d\frac{\mathfrak{S}_1}{\mathfrak{S}} u$$

or

$$\int \wp u du = -\frac{\mathfrak{S}_1}{\mathfrak{S}} u$$

so that

$$\int_{\omega''}^{\omega'} \wp u du = \eta$$

we obtain

$$\int_0^{\frac{\pi}{2}} \frac{dT}{\Delta^3} = \frac{e_2 - e_3}{\sqrt{G}}(e_1\omega + \eta)$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 T dT}{\Delta^3} = \frac{e_1 - e_3}{\sqrt{G}}(e_2\omega + \eta) \quad . \quad . \quad . \quad (b)$$

$$\int_0^{\frac{\pi}{2}} \frac{\cos^2 T dT}{\Delta^3} = \frac{e_2 - e_1}{\sqrt{G}}(e_3\omega + \eta)$$

The proofs of the above formulæ will be found in Padé's or Schwarz's paper already referred to. The italic G is not to be confounded with the three G roots.

The last set of formulæ correspond to Dr. Hill's $\frac{M}{m^3}$, $\frac{M}{m^3}\cos^2\kappa$ and $\frac{M}{m^3}\sin^2\kappa$ in the *Amer. Journ. of Math.*, t. xxiii. pp. 321–2.

Should the cubic equation be solved, the numerical value of these formulæ may be found as follows:—

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\frac{d\mathrm{T}}{\Delta^3}=\sqrt{\frac{e_2-e_3}{G}}\;\frac{1}{4\sqrt{q}}\;\frac{1+3^2q^2+5^2q^6+7^2q^{12}+\text{etc.}}{(1+q^2+q^6+q^{12}+\text{etc.})^3}$$

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\frac{\sin^2\mathrm{T}d\mathrm{T}}{\Delta^3}=\sqrt{\frac{e_1-e_3}{G}}\;8q\;\frac{1+2^2q^3+3^2q^8+4^2q^{15}+\text{etc.}}{(1+2q+2q^4+2q^9+\text{etc.})^3}$$

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\frac{\cos^2\mathrm{T}d\mathrm{T}}{\Delta^3}=\sqrt{\frac{e_1-e_2}{G}}\;8q\;\frac{1-2^2q^3+3^2q^8-4^2q^{15}+\text{etc.}}{(1-2q+2q^4-2q^9+\text{etc.})^3}$$

where q is to be derived as follows:—Let

$$\frac{e_1-e_2}{e_1-e_3}=\cos^2\theta \text{ and } \tfrac{1}{2}l=\left[\frac{\sin\frac{\theta}{2}}{1+\sqrt{\cos\theta}}\right]^2$$

then

$$q=\tfrac{1}{2}l+2(\tfrac{1}{2}l)^5+15(\tfrac{1}{2}l)^9+150(\tfrac{1}{2}l)^{13}+\text{ etc.}$$

or in practice

$$\log q=\log\tfrac{1}{2}l+[9\cdot9388](\tfrac{1}{2}l)^4$$

(see *M. N.*, R.A.S., vol. lxii. p. 494).

All these formulæ are extraordinarily convergent. They can also be derived from the formulæ given on p. 333 of Hill's paper (*A.J.*, No. 511) in virtue of the relations

$$k=4\sqrt{q}\left[\frac{1+q^2+q^6+q^{12}+\text{ etc.}}{1+2q+2q^4+2q^9+\text{ etc.}}\right]^2\qquad k_1=\left[\frac{1-2q+2q^4-2q^9+\text{ etc.}}{1+2q+2q^4+2q^9+\text{ etc.}}\right]^2$$

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\frac{d\mathrm{T}}{\sqrt{1-k^2\sin^2\mathrm{T}}}=(1+2q+2q^4+2q^9+\text{ etc.})^2$$

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\sqrt{1-k^2\sin^2\mathrm{T}}\,d\mathrm{T}=\left[\frac{1+3^2q^2+5^2q^6+7^2q^{12}+\text{ etc.}}{1-3q^2+5q^6-7q^{12}+\text{ etc.}}\right]\times\sqrt{k_1}$$

the last formula being one of some half dozen representations of Legendre's E by means of Jacobi's q function.

4.

Introducing the equations (b) into the equation (a) and putting

$$\mathrm{M} = (e_2 - e_3)\Gamma_1 + (e_1 - e_3)\Gamma_2 + (e_2 - e_1)\Gamma_3$$
$$\mathrm{N} = e_1(e_2 - e_3)\Gamma_1 + e_2(e_1 - e_3)\Gamma_2 + e_3(e_2 - e_1)\Gamma_3$$

enables us to write

$$\frac{1}{a^2}\frac{a}{r}\mathrm{R}_0 = \frac{2}{\pi}\frac{1}{\sqrt{G}}\left[\mathrm{M}\eta + \mathrm{N}\omega\right] \quad . \quad . \quad . \quad (c)$$

with similar simple expressions for the two other components. If M and N are written in determinant form they are

$$\mathrm{M} = \begin{vmatrix} \Gamma_1 & -\Gamma_2 & -\Gamma_3 \\ e_1 & e_2 & e_3 \\ 1 & 1 & 1 \end{vmatrix} . \qquad \mathrm{N} = \begin{vmatrix} e_1\Gamma_1 & -e_2\Gamma_2 & -e_3\Gamma_3 \\ e_1 & e_2 & e_3 \\ 1 & 1 & 1 \end{vmatrix}$$

5.

We now proceed to the elimination of the roots of the cubic (e_1, e_2 and e_3) by means of the invariants g_2 and g_3. The following formulæ are collected here for easy reference:—

$$\mathrm{G}_1 + \mathrm{G}_2 - \mathrm{G}_3 = k_1 . \quad \mathrm{G}_1\mathrm{G}_2 - \mathrm{G}_1\mathrm{G}_3 - \mathrm{G}_2\mathrm{G}_3 = k_2 . \quad -\mathrm{G}_1\mathrm{G}_2\mathrm{G}_3 = k_3$$
$$\tfrac{3}{4}g_2 = \lambda = k_1^2 - 3k_2 = -3(e_1e_2 + e_1e_3 + e_2e_3)$$
$$\tfrac{1}{4}g_3 = \tfrac{1}{27}(2k_1^3 - 9k_1k_2 + 27k_3) = e_1e_2e_3$$
$$16G = g_2^3 - 27g_3^2 = 16(e_1 - e_2)(e_1 - e_3)(e_2 - e_3)$$

or for the last

$$\sqrt{G} = \tfrac{1}{4}g_3 \times \begin{vmatrix} 1, & 1, & 1 \\ \frac{1}{e_1} & \frac{1}{e_2} & \frac{1}{e_3} \\ e_1 & e_2 & e_3 \end{vmatrix} \qquad = \tfrac{1}{4}g_3 \times \begin{vmatrix} 1, & 1, & 1 \\ \frac{1}{e_1} & \frac{1}{e_2} & \frac{1}{e_3} \\ \frac{e_2e_3}{e_1} & \frac{e_1e_3}{e_2} & \frac{e_1e_2}{e_3} \end{vmatrix}$$

We have now four determinants; let us multiply the first by the third, and then the second by the fourth. The first result is written at length,

$$\mathrm{M}\sqrt{G} = \begin{vmatrix} \Gamma_1 - \Gamma_2 - \Gamma_3, & e_2e_3\Gamma_1 - e_1e_3\Gamma_2 - e_1e_2\Gamma_3, & e_1\Gamma_1 - e_2\Gamma_2 - e_3\Gamma_3 \\ 0 & 3e_1e_2e_3 & -2(e_1e_2 + e_1e_3 + e_2e_3) \\ 3 & e_1e_2 + e_1e_3 + e_2e_3 & 0 \end{vmatrix}$$

To abbreviate put

$$\begin{aligned} \Gamma_1 - \Gamma_2 - \Gamma_3 &= \psi \\ e_2e_3\Gamma_1 - e_1e_3\Gamma_2 - e_1e_2\Gamma_3 &= \chi \\ e_1\Gamma_1 - e_2\Gamma_2 - e_3\Gamma_3 &= \phi \end{aligned} \quad . \quad . \quad . \quad (d)$$

and replace therein the constituents in e_1, e_2, and e_3 by their appropriate equivalents in g_2 and g_3; then expanding we obtain finally

$$\mathrm{M}\sqrt{G} = \psi\tfrac{1}{8}g_2^2 + \tfrac{3}{2}g_2\chi - \tfrac{9}{4}g_3\phi$$

A similar process gives

$$\mathrm{N}\sqrt{G} = -\psi\tfrac{3}{16}g_2g_3 - \tfrac{9}{4}g_3\chi + \tfrac{1}{8}g_2^2\phi$$

so that (*c*) becomes

$$\frac{1}{a^2}\,\frac{a}{r}R_0 = \frac{2}{\pi}\,\frac{1}{\sqrt{G}}\left[\tfrac{1}{2}(g_2\eta - \tfrac{3}{2}g_3\omega)(\tfrac{1}{4}g_2\psi + 3\chi) + \tfrac{1}{4}(\tfrac{1}{2}g_2^2\omega - 9g_3\eta)\phi\right]$$

with similar equations for S_0 and W_0.

6.

Bruns first pointed out that the periods of the elliptic functions could be found without a knowledge of the three roots (H. Bruns, *Perioden der Ellip. Integrale*, Dorpat, 1875).

This remarkable investigation does not deserve the neglect it has received at the hands of English mathematicians. Bruns does not adapt his formulæ for numerical use, and I have found it better to adopt another absolute invariant. If we write

$$\cos\iota = 3\sqrt{3}\,\frac{g_3}{g_2^{\frac{3}{2}}} \qquad (0^\circ < \iota < 180^\circ)$$

we have

$$\frac{2}{\pi}\,\frac{1}{\sqrt{G}}\,\frac{1}{2}\left(g_2\eta - \frac{3}{2}g_3\omega\right) = \frac{5}{8}\,\frac{1}{\lambda^{\frac{1}{4}}}\,\frac{1}{\cos^2\frac{\iota}{2}}\,F\left(\frac{1}{6},\ \frac{5}{6},\ 2.\ \sin^2\frac{\iota}{2}\right)$$

$$\frac{2}{\pi}\,\frac{1}{\sqrt{G}}\,\frac{1}{4}\left(\frac{1}{2}g_2^2\omega - 9g_3\eta\right) = \frac{7}{8}\,\frac{1}{\lambda^{\frac{1}{4}}}\,\frac{\lambda^{\frac{1}{4}}}{\cos^2\frac{\iota}{2}}\,F\left(-\frac{1}{6},\ \frac{7}{6},\ 2.\ \sin^2\frac{\iota}{2}\right)$$

Hence

$$\frac{1}{a^2}\,\frac{a}{r}R_0 = \frac{1}{8}\,\frac{1}{\cos^2\frac{\iota}{2}}\,\frac{1}{\lambda^{\frac{1}{4}}}\left[\left(\frac{1}{4}g_2\psi + 3\chi\right)5F\left(\frac{1}{6}\right) + \lambda^{\frac{1}{4}}\phi\,7F\left(-\frac{1}{6}\right)\right]$$

with similar equations for S_0 and W_0. The abbreviations $F\left(\frac{1}{6}\right)$ and $F\left(-\frac{1}{6}\right)$ require no explanation.

7.

It remains to find the values of ψ, χ, and ϕ. Replacing the *e*'s by the G's in (*d*) we have

$$\psi = \Gamma_1 - \Gamma_2 - \Gamma_3.$$

$$\phi = \left(G_1 - \frac{k_1}{3}\right)\Gamma_1 - \left(G_2 - \frac{k_1}{3}\right)\Gamma_2 + \left(G_3 + \frac{k_1}{3}\right)\Gamma_3 = G_1\Gamma_1 - G_2\Gamma_2 + G_3\Gamma_3 - \frac{k_1}{3}\psi$$

$$\chi = -G_2G_3\Gamma_1 + G_1G_3\Gamma_2 - G_1G_2\Gamma_3 + \frac{k_1}{3}\phi - \left(\frac{k_1}{3}\right)^2\psi$$

Hill and Arndt give the formulæ for these quantities in G which are as follows:—

	$\frac{1}{a^2}\frac{a}{r}R_0$	$\frac{1}{a^2}\frac{a}{r}S_0$	$\frac{1}{a \sin J}\frac{a}{r}W_0$
$\Gamma_1 - \Gamma_2 - \Gamma_3 = \psi$	$-r_0$	o	o
$G_1\Gamma_1 - G_2\Gamma_2 + G_3\Gamma_3 = \phi + \frac{k_1}{3}\psi$	$-r_0(A_0 - e_1B_0\cos\epsilon) + A_cD + A_sB_0\sin\epsilon$	$o + B_cD + B_sB_0\sin\epsilon$	$o + C_cD + C_sB_0\sin\epsilon$
$-G_2G_3\Gamma_1 + G_1G_3\Gamma_2 - G_1G_2\Gamma_3 = \chi - \frac{k_1}{3}\phi + \frac{k_1^2}{9}\psi$	$-e_1r_0B_0\sin\epsilon(A_sA_c - A_cA_s) = o$	$-e_1r_0B_0\sin\epsilon(B_sA_c - B_cA_s) = -e_1r_0B_0\sin\epsilon\cos\phi_1\cos J$	$-e_1r_0B_0\sin\epsilon(C_sA_c - C_cA_s) = -e_1r_0B_0\sin\epsilon\cos\phi_1\cos(\nu + \Pi)$

[where $D = (1 + e_1^2)B_0\cos\epsilon - e_1(A_0 + C_0)$].

These formulæ simplify in use. Working formulæ and two numerical applications follow.

Working Formulæ.

9.

Let

a = semi-axis major corrected for the constant part of its perturbations
n = mean motion
$e = \sin\phi$ = eccentricity
π = long. of perihelion from equinox
i = inclination of orbit to ecliptic
Ω = long. of ascending node
χ = long. of perihelion from a fixed point in orbit
$\omega = \pi - \Omega$
r = radius-vector
E = excentric anomaly
ν = true anomaly
m = mass (Sun's mass unity)

of the disturbed body; the addition of a suffix ($_1$) indicates similar quantities appertaining to the disturbing body.

10.

$$\begin{aligned}
-\sin\tfrac{1}{2}J\sin\tfrac{1}{2}(\Phi+\Psi) &= \sin\tfrac{1}{2}(\Omega_1-\Omega)\sin\tfrac{1}{2}(i_1+i)\\
-\sin\tfrac{1}{2}J\cos\tfrac{1}{2}(\Phi+\Psi) &= \cos\tfrac{1}{2}(\Omega_1-\Omega)\sin\tfrac{1}{2}(i_1-i)\\
\cos\tfrac{1}{2}J\sin\tfrac{1}{2}(\Phi-\Psi) &= \sin\tfrac{1}{2}(\Omega_1-\Omega)\cos\tfrac{1}{2}(i_1+i)\\
\cos\tfrac{1}{2}J\cos\tfrac{1}{2}(\Phi-\Psi) &= \cos\tfrac{1}{2}(\Omega_1-\Omega)\cos\tfrac{1}{2}(i_1-i)
\end{aligned}$$

check formulæ

$$\begin{aligned}
\cos p\sin q &= \sin i_1\cos(\Omega_1-\Omega)\\
\cos p\cos q &= \cos i_1\\
\cos p\sin r &= -\cos i_1\sin(\Omega_1-\Omega)\\
\cos p\cos r &= \cos(\Omega_1-\Omega)\\
\sin p &= -\sin i_1\sin(\Omega_1-\Omega)
\end{aligned}$$

then

$$\begin{aligned}
\sin J\sin\Phi &= \sin p\\
\sin J\cos\Phi &= \cos p\sin(i-q)\\
\sin J\sin(\Psi-r) &= \sin p\cos(i-q)\\
\sin J\cos(\Psi-r) &= \sin(i-q)\\
\cos J &= \cos p\cos(i-q)
\end{aligned}$$

$$\Pi = \omega - \Phi. \qquad \Pi_1 = \omega_1 - \Psi$$

$$k\cos(K-\Pi) = \cos\Pi_1. \qquad k_1\cos(K_1-\Pi) = \cos\Pi_1\cos J$$
$$k\sin(K-\Pi) = -\sin\Pi_1\cos J. \qquad k_1\sin(K_1-\Pi) = -\sin\Pi_1$$
$$\text{check: } k^2+k_1^2 = 1+\cos^2 J$$

When the Earth is the disturbing planet, these equations simplify to

$$\begin{aligned}
J &= i\\
\Phi &= 0\\
\Pi &= \pi - \Omega\\
\Pi_1 &= \pi_1 - \Omega
\end{aligned}$$

and when the Earth is the disturbed planet, to

$$\begin{aligned} J &= i_1 \\ \Psi &= 180^\circ \\ \Pi &= 180^\circ + \pi - \Omega_1 \\ \Pi_1 &= 180^\circ + \pi_1 - \Omega_1 \end{aligned}$$

II.

The various functions of E which follow should now be computed for $4j$ equal parts of the circumference. The larger the number of parts is, the more exact will be the results of the mechanical integration, but j should not be taken larger than necessary. A measure of the accuracy attained is given by adding the odd values and the even values of each of the quantities: if these differ, more values of E are required. Reference to Hill's original example or to those of later computers will assist the judgment.

Take $\alpha = a/a_1$, then omitting the suffix $_0$ used in the demonstration, when no confusion will be created thereby, we have

$$r_0 \cos \nu = \alpha(\cos E - e). \qquad r_0 \sin \nu = \alpha \cos \phi \sin E$$

$$r_0 = \alpha(1 - e \cos E). \text{ (check)}$$

$$k \cos (\nu + K) = A_c \qquad k_1 \cos \phi_1 \sin (\nu + K_1) = A_s$$

$$-k \sin (\nu + K) = B_c \qquad k_1 \cos \phi_1 \cos (\nu + K_1) = B_s$$

$$A = 1 + r_0^2 + 2e_1 r_0 A_c.$$

$$B \cos \epsilon = r_0 A_c + e_1. \qquad B \sin \epsilon = r_0 A_s$$

$$k_1 = A - e_1^2. \qquad k_2 = B_2 - Ae_1^2. \qquad -k_3 = e_1^2 B^2 \sin^2 \epsilon$$

$$\tfrac{3}{4} g_2 = \lambda = k_1^2 - 3k_2 \qquad g_3 = \tfrac{4}{27}(2k_1^3 - 9k_1k_2 + 27k_3)$$

$$\cos \iota = \frac{\sqrt{27}\, g_3}{g_2^{\frac{3}{2}}}. \qquad (0^\circ < \iota < 180^\circ)$$

$$D = (1 + e_1^2) B \cos \epsilon - e_1(A + e_1^2)$$

$$\phi_R = -r_0\left(A - e_1 B \cos \epsilon - \frac{k_1}{3}\right) + A_c D + A_s B \sin \epsilon$$

$$\phi_S = \qquad + B_c D + B_s B \sin \epsilon$$

$$\phi_W = \qquad + \sin \Pi_1 D + \cos \phi_1 \cos \Pi_1 B \sin \epsilon$$

$$\chi_R - \frac{k_1}{3}\phi_R - \frac{k_1^2}{9} r_0 = 0$$

$$\chi_S - \frac{k_1}{3}\phi_S \qquad = -e_1 r_0 B \sin \epsilon \cos \phi_1 \cos J$$

$$\chi_W - \frac{k_1}{3}\phi_W \qquad = -e_1 r_0 B \sin \epsilon \cos \phi_1 \cos (\nu + \Pi)$$

With the aid of the tables or formulæ compute

$$\frac{5}{8}\frac{1}{\cos^2\frac{i}{2}}F\left(1/6, 5/6, 2, \sin^2\frac{i}{2}\right)=F_A$$

and

$$\frac{7}{8}\frac{1}{\cos^2\frac{i}{2}}F\left(-1/6, 7/6, 2, \sin^2\frac{i}{2}\right)=F_B$$

Then

$$\frac{1}{a_2}\frac{a}{r}R_0=\left(3\chi-\frac{1}{4}g_2r_0\right)\frac{F_B}{\lambda^{\frac{1}{2}}}+\frac{F_A}{\lambda^{\frac{3}{2}}}$$

$$\frac{1}{a_2}\frac{a}{r}S_0=(\quad 3\chi\quad)\;,,\;\;,,\;\;,,$$

$$\frac{1}{a}\frac{1}{\sin J}\frac{a}{r}W_0=[(\quad 3\chi\quad)\;,,\;\;,,\;\;,,]r_0$$

12.

With the $4j$ values of these three functions, which may be designated

$$\begin{array}{ll} R^{(0)}, R^{(1)}, R^{(2)}, \;\ldots\ldots & R^{(4j-1)} \\ S^{(0)}, S^{(1)}, \;\ldots\ldots\ldots & S^{(4j-1)} \\ W^{(0)}, W^{(1)}, \;\ldots\ldots\ldots & W^{(4j-1)} \end{array}$$

compute

$$A_0^{(c)}=\frac{1}{4j}[R^{(0)}+R^{(1)}+R^{(2)}\;\ldots\ldots\;R^{(4j-1)}]$$

$$\frac{1}{2}A_1^{(c)}=\frac{1}{4j}\left[R^{(0)}+R^{(1)}\cos\frac{1}{j}\frac{\pi}{2}+R^{(2)}\cos\frac{2}{j}\frac{\pi}{2}\;\ldots\ldots+R^{(4j-1)}\cos\frac{4j-1}{j}\frac{\pi}{2}\right]$$

$$\frac{1}{2}A^{(s)}=\frac{1}{4j}\left[\qquad R^{(1)}\sin\frac{1}{j}\frac{\pi}{2}+R^{(2)}\sin\frac{2}{j}\frac{\pi}{2}\;\ldots\ldots+R^{(4j-1)}\sin\frac{4j-1}{j}\frac{\pi}{2}\right]$$

$$\frac{1}{2}A_2^{(c)}=\frac{1}{4j}\left[R^{(0)}+R^{(1)}\cos\frac{2}{j}\frac{\pi}{2}+R^{(2)}\cos\frac{4}{j}\frac{\pi}{2}\;\ldots\ldots+R^{(4j-1)}\cos\frac{2(4j-1)\pi}{j\quad 2}\right]$$

$$\frac{1}{2}A_2^{(s)}=\frac{1}{4j}\left[\qquad R^{(1)}\sin\frac{2}{j}\frac{\pi}{2}+R^{(2)}\sin\frac{4}{j}\frac{\pi}{2}\;\ldots\ldots+R^{(4j-1)}\sin\frac{2(4j-1)\pi}{j\quad 2}\right]$$

with similar equations for $B_0^{(c)}$ $B_1^{(c)}$ .... and $C_0^{(c)}$ $C_1^{(c}$ and $C_1^{(s)}$.

(Note :— $A_2^{(s)}$ is not required.)

For any special case of j these equations become greatly simplified: thus for $j=2$ we have

$$A_0^{(c)}=\tfrac{1}{8}[R^{(0)}+R^{(1)}+R^{(2)}\;\ldots\ldots\;+R^{(7)}]$$

$$\tfrac{1}{2}A_1^{(c)}=\tfrac{1}{8}[R^{(0)}-R^{(4)}+(R^{(1)}-R^{(3)}-R^{(5)}+R^{(7)})\cos 45^\circ]$$

$$\tfrac{1}{2}A_1^{(s)}=\tfrac{1}{8}[R^{(2)}-R^{(6)}+(R^{(1)}+R^{(3)}-R^{(5)}-R^{(7)})\sin 45^\circ]$$

$$\tfrac{1}{2}A_2^{(c)}=\tfrac{1}{8}[R^{(0)}-R^{(2)}+R^{(4)}-R^{(6)}]$$

$$\tfrac{1}{2}A_2^{(s)}=\tfrac{1}{8}[R^{(1)}-R^{(3)}+R^{(4)}-R^{(6)}]$$

and so on. For other values of j the equations are easily formed, or may be found in Hansen's *Auseinandersetzung* or elsewhere.

The equation

$$\sin\phi\tfrac{1}{2}A_1^{(s)}+\cos\phi B_0^{(c)}=0$$

affords a useful check.

13.

The final equations are

$$e\left[\frac{d\phi}{dt}\right]_{00}=\frac{m_1n}{1+m}\,a^2\left[-\left(1+\frac{e^2}{2}\right)B_0^{(c)}-2e\frac{1}{2}B_1^{(c)}+\frac{e^2}{2}\,\frac{1}{2}\,B_2^{(c)}\right]$$

$$e\left[\frac{d\chi}{dt}\right]_{00}=\frac{m_1n}{1+m}\,a_2\left[eA_0^{(c)}\cos\phi-\frac{1}{2}A_1^{(c)}\cos\phi+(2-e^2)\frac{1}{2}B_1^{(s)}-\frac{e}{2}\,\frac{1}{2}\,B_2^{(s)}\right]$$

$$\left[\frac{di}{dt}\right]_{00}=\frac{m_1n}{1+m}\,a\sin J\cos\omega\left[\left(\frac{1}{2}C_1^{(c)}-eC_0^{(c)}\right)\sec\phi-\frac{1}{2}C_1^{(s)}\tan\omega\right]$$

$$i\left[\frac{d\Omega}{dt}\right]_{00}=\frac{m_1n}{1+m}\,a\sin J\cos\omega\left[\left(\frac{1}{2}C_1^{(c)}-eC_0^{(c)}\right)\sec\phi\tan\omega+\frac{1}{2}C_1^{(s)}\right]$$

$$-2\frac{r}{a}R_0\Big]_{00}=a^2\left[-(2+e^2)A_0^{(c)}+4e\frac{1}{2}A_1^{(c)}-e^2\,\frac{1}{2}A_2^{(c)}\right]$$

Calculation of $F\left(\frac{1}{6},\frac{5}{6},2,\sin^2\frac{\iota}{2}\right)$ and $F\left(-\frac{1}{6},\frac{7}{6},2,\sin^2\frac{\iota}{2}\right)$

14.

When ι is less than 90° the calculation of these series is not long, but if an accuracy of two units in the 7th place of the decimals of the logarithms is sufficient, the use of series can be avoided. This accuracy is indeed greater than either our knowledge of the planetary masses or the precision of observations requires. Besides these, the change in the secular perturbations due to higher powers of the masses than the first is of a more important character.

The approximate formulæ are

$$\mathrm{og}\,F\left(\frac{1}{6},\frac{5}{6},2.\ \sin^2\frac{\iota}{2}\right)=\log\left\{1+\frac{\frac{5}{72}\sin^2\frac{\iota}{2}}{\left(\sqrt{1-\frac{2}{3}\sin^2\frac{\iota}{2}}\right)^{1+\frac{5}{72}}}\right\}+[6\cdot0049]\left(\frac{\iota^\circ}{100}\right)^{\frac{100}{12}}$$

$$F\left(-\frac{1}{6},\frac{7}{6},2.\ \sin^2\frac{\iota}{2}\right)=\log\left\{1-\frac{\frac{7}{72}\sin^2\frac{\iota}{2}}{\left(\sqrt{1-\frac{2}{3}\sin^2\frac{\iota}{2}}\right)^{1-\frac{7}{72}}}\right\}-[6\cdot1011]\left(\frac{\iota^\circ}{100}\right)^{\frac{100}{12}}$$

The two series are inert :—

ι	$F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota}{2}\right)$	$F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota}{2}\right)$
0°	1·0000000	1·0000000
90°	0·9415120	1·0432298
180°	$0·8185111 = \frac{2}{\pi}\frac{9}{7}$	$1·1459156 = \frac{2}{\pi}\frac{9}{5}$

If ι exceeds 90° the best procedure seems to be as follows :—

Take $\iota_1 = 180° - \iota$,

then proceeding at once to the actual quantities wanted and writing

$$= \frac{2}{\pi}\frac{108}{\sin^2\frac{\iota_1}{2}} \frac{1}{\left[7F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota_1}{2}\right) + 5\cos\iota_1 F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota_1}{2}\right)\right]}$$

we have

$$\frac{5}{\cos^2\frac{\iota}{2}} F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota}{2}\right) = H - \frac{1}{\pi}\log_e\frac{1}{q}\frac{5}{\cos^2\frac{\iota_1}{2}} F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota_1}{2}\right)$$

$$\frac{7}{\cos^2\frac{\iota}{2}} F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota}{2}\right) = H\cos\iota_1 + \frac{1}{\pi}\log_e\frac{1}{q}\frac{7}{\cos^2\frac{\iota_1}{2}} F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota_1}{2}\right)$$

To compute q put

$$\frac{1}{\sqrt{3}}\tan\frac{\iota_1}{3} = \cos 2\gamma$$

and $\sqrt{\tan\gamma} = \cos 2\lambda$

then $l = \tan^2\lambda$

and $q = \frac{1}{2}l + 2(\frac{1}{2}l)^5 + \text{etc.}$

In practice it may be assumed that $\log q = \log \frac{1}{2}l$. This assumption is in defect by the following quantities :—

ι_1	Correction to $\log \frac{1}{2}l$ in units of the 7th place of decimals,
0° – 40°	+ 0
40°	1
50°	2
60°	4
70°	9
80°	17
90°	+ 30

As $\log_e \frac{1}{q}$ is the natural log of $\frac{1}{q}$, we have in ordinary logarithms—

$$\log\left(\frac{1}{\pi}\log_e\frac{1}{q}\right) = \log \text{ of } \log\frac{1}{q} + 9{\cdot}8650658$$

15.

Numerical Examples.

In the *Astron. Nachr.*, No. 4068, Dr Arthur B. Turner has computed the secular perturbations of Mars by Jupiter using Arndt's formulæ.

The preceding formulæ are applied to this case for the case of $E = 0^\circ$. We have $J = 1^\circ 26'$, $\Pi = 149^\circ{\cdot}8$, $\Pi_1 = 188^\circ{\cdot}4$, K and $K_1 = 321^\circ{\cdot}4$, k and $k_1 = 1$, $\log r_0 = 9{\cdot}4241$, $\nu = 0^\circ$, $\log A_c = 9{\cdot}893$, $\log B_c = 9{\cdot}795$, $\log A_s = 9{\cdot}794n$, $\log B_s = 9{\cdot}892$, $A_0 = 1{\cdot}09$, $\log B_0 \sin \epsilon = 9{\cdot}219$, $k_1 = 1{\cdot}088$, $k_2 = 0{\cdot}090$, $k_3 = -0{\cdot}000$, $g_2 = 1{\cdot}218$, $g_3 = 0{\cdot}251$, $\iota = 14^\circ 20'$, $F(\frac{1}{6}) = 0{\cdot}998$, $F(-\frac{1}{6}) = 1{\cdot}001$, $D = 0{\cdot}204$, and

	R_0	S_0	W_0
ϕ	0·072	0·002	0·134
χ	0·061	0·001	0·050

With these figures Turner's results are reproduced.

The next example is taken from Herr W. Dziewulski's paper on the secular perturbations of Eros by Mars. In the present position of the orbits for $E = 120^\circ$, the modular angle $\theta = 58^\circ{\cdot}9$. In the notation of the present paper we have $J = 11^\circ{\cdot}5$, $\Pi = 6^\circ{\cdot}7$, $\Pi_1 = 219^\circ{\cdot}6$, $K = 147^\circ{\cdot}6$, $K_1 = 146^\circ{\cdot}5$, $\log k = 9{\cdot}996$, $\log k_1 = 9{\cdot}995$, $\log r_0 = 0{\cdot}0268$, $\nu = 130^\circ{\cdot}6$, $\log A_c = 9{\cdot}152$, $\log B_c = 9{\cdot}992$, $\log A_s = 9{\cdot}990n$, $\log B_s = 9{\cdot}084$, $A_0 = 2{\cdot}16$, $\log B_0 \sin \epsilon = 0{\cdot}016n$, $k_1 = 2{\cdot}151$, $k_2 = 1{\cdot}120$, $k_3 = -0{\cdot}009$, $g_2 = 1{\cdot}690$, $g_3 = -0{\cdot}300$, $\iota = 135^\circ{\cdot}1$, $\iota_1 = 44^\circ{\cdot}9$, $\gamma = 40^\circ{\cdot}6$, $\lambda = 11^\circ{\cdot}1$, $\log \frac{1}{2}l = 8{\cdot}288$, $\log \frac{1}{\pi}\log_e \frac{1}{q} = 0{\cdot}099$, $F_1(\frac{1}{6}) = 0{\cdot}985$, $F_1(-\frac{1}{6}) = 1{\cdot}011$, $F(\frac{1}{6}) = 0{\cdot}881$, $F(-\frac{1}{6}) = 1{\cdot}102$, $D = 0{\cdot}044$, and

	R_0	S_0	W_0
ϕ	$-0{\cdot}490$	$-0{\cdot}083$	0·763
χ	0·195	0·041	0·472

leading to Dziewulski's results.

[For Tables, see paper by Mr Robbins.]

Johannesburg.

Tables for the Application of Mr Innes's Method.

By Frank Robbins.

As an appendix to the preceding paper on "The Computation of Secular Perturbations," the author asked me to compute for each degree of the quadrant the logarithmic values (base 10) of the two functions of *iota* required for the convenient application of his method.

In the hypergeometric series F $(\alpha\,\beta\,\gamma\,x)$ in the first case

$$\alpha \text{ has the value } -\frac{1}{6} \quad \beta=\frac{7}{6} \quad \gamma=2 \quad x=\sin^2\frac{\iota}{2}$$

and in the second case

$$\alpha=\frac{1}{6} \quad \beta=\frac{5}{6} \quad \gamma=2 \quad x=\sin^2\frac{\iota}{2}$$

For convenience of designation the tables are headed *Minus* F and *Plus* F according to the sign of α.

Vega's (1794) ten-figure logarithms, corrected by collation with the copy in use at H.M. *Nautical Almanac* Office, were used, and the natural values of the individual terms were taken out to ten places of decimals. These were obtained in duplicate for each end of the quadrant, and the whole were examined by differencing to the sixth order. Lastly, the seven-figure logarithms of the functions were taken from the eight-figure table of the Service Géographique de l'Armée (Paris, 1891), reference being made to Vega where the eighth figure was approximately five.

The log *Minus* F has been increased by 10 as customary, to avoid the inconvenience of printing negative characteristics.

The whole has been examined by Mr J. Abner Sprigge, of H.M. *Nautical Almanac* Office, so as to make it possible to use the tables with confidence in their accuracy to the seventh place.

(Iota).	Log plus F.	Δ_1	Δ_2	Log minus F.	Δ_1	Δ_2
$\overset{\circ}{1}$	0·0000023			9·9999968		
		+ 69			− 97	
2	0092		+46	9871		−63
		115			160	
3	0207		45	9711		65
		160			225	
4	0367		47	9486		65
		207			290	
5	0574		46	9196		64
		253			354	
6	0827		46	8842		64
		299			418	
7	1126		45	8424		64
		344			482	
8	0·0001470		+46	9·9997942		−65
		+390			−547	

(Iota).	Log plus F.	Δ_1	Δ_2	Log minus F.	Δ_1	Δ_2
9	0·0001860		+47	9·9997395		−64
		+437			−611	
10	2297		45	6784		64
		482			675	
11	2779		46	6109		65
		528			740	
12	3307		46	5369		64
		574			804	
13	3881		46	4565		65
		620			869	
14	4501		46	3696		64
		666			933	
15	5167		45	2763		64
		711			997	
16	5878		47	1766		65
		758			1062	
17	6636		45	9·9990704		64
		803			1126	
18	7439		46	9·9989578		65
		849			1191	
19	8288		47	88387		64
		896			1255	
20	0·0009184		45	87132		65
		941			1320	
21	0·0010125		46	85812		64
		987			1384	
22	11112		45	84428		65
		1032			1449	
23	12144		47	82979		64
		1079			1513	
24	13223		45	81466		65
		1124			1578	
25	14347		46	79888		64
		1170			1642	
26	15517		45	78246		65
		1215			1707	
27	16732		46	76539		64
		1261			1771	
28	17993		46	74768		66
		1307			1837	
29	19300		46	72931		64
		1353			1901	
30	20653		46	71030		64
		1399			1965	
31	22052		45	69065		66
		1444			2031	
32	23496		46	67034		64
		1490			2095	
33	24986		45	64939		65
		1535			2160	
34	26521		46	62779		64
		1581			2224	
35	28102		46	60555		66
		1627			2290	
36	29729		45	58265		64
		1672			2354	
37	31401		46	55911		65
		1718			2419	
38	0·0033119		+45	9·9953492		−65
		+1763			−2484	

(Iota).	Log plus F.	Δ_1	Δ_2	Log minus F.	Δ_1	Δ_2
39	0·0034882		+45	9·9951008		−65
		+1808			−2549	
40	36690		46	48459		64
		1854			2613	
41	38544		46	45846		66
		1900			2679	
42	40444		45	43167		65
		1945			2744	
43	42389		45	40423		64
		1990			2808	
44	44379		45	37615		66
		2035			2874	
45	46414		46	34741		64
		2081			2938	
46	48495		44	31803		66
		2125			3004	
47	50620		46	28799		64
		2171			3068	
48	52791		46	25731		66
		2217			3134	
49	55008		44	22597		65
		2261			3199	
50	57269		45	19398		65
		2306			3264	
51	59575		45	16134		64
		2351			3328	
52	61926		45	12806		66
		2396			3394	
53	64322		45	09412		65
		2441			3459	
54	66763		45	05953		66
		2486			3525	
55	69249		45	9·9902428		64
		2531			3589	
56	71780		44	9·9898839		66
		2575			3655	
57	74355		45	95184		64
		2620			3719	
58	76975		45	91465		66
		2665			3785	
59	79640		44	87680		65
		2709			3850	
60	82349		44	83830		65
		2753			3915	
61	85102		45	79915		65
		2798			3980	
62	87900		44	75935		65
		2842			4045	
63	90742		44	71890		65
		2886			4110	
64	93628		44	67780		65
		2930			4175	
65	96558		44	63605		65
		2974			4240	
66	0·0099532		45	59365		65
		3019			4305	
67	0·0102551		43	55060		65
		3062			4370	
68	0·0105613		+43	9·9850690		−65
					−4435	

(Iota).	Log plus F.	Δ_1	Δ_2	Log minus F.	Δ_1	Δ_2
69	0·0108718		+45	9·9846255		−65
		+3150			−4500	
70	111868		42	41755		65
		3192			4565	
71	115060		44	37190		64
		3236			4629	
72	118296		43	32561		65
		3279			4694	
73	121575		44	27867		65
		3323			4759	
74	124898		43	23108		64
		3366			4823	
75	128264		42	18285		66
		3408			4889	
76	131672		43	13396		63
		3451			4952	
77	135123		43	08444		64
		3494			5016	
78	138617		42	9·99803428		65
		3536			5081	
79	142153		42	9·99798347		64
		3578			5145	
80	145731		43	93202		64
		3621			5209	
81	149352		41	87993		64
		3662			5273	
82	153014		42	82720		63
		3704			5336	
83	156718		42	77384		64
		3746			5400	
84	160464		41	71984		64
		3787			5464	
85	164251		42	66520		62
		3829			5526	
86	168080		40	60994		64
		3869			5590	
87	171949		40	55404		62
		3909			5652	
88	175858		42	49752		64
		3951			5716	
89	179809		+40	44036		−62
		+3991			−5778	
90	0·0183800			9·9738258		

Description of the 30-inch Photographic Reflector of the Helwân Observatory. By J. H. Reynolds.

There has recently been added to the equipment of the Khedivial Observatory at Helwân, near Cairo, a photographic equatorial reflector of 30-inch aperture: before giving a description of this instrument, a little explanation as to its origin would not be out of place. About five years ago one of the 30-inch Mirrors of Standard Astrographic focal length, which were ground and figured by the late Dr Common, came into my hands. I originally purposed to employ the mirror for nebular photography in this country, and commenced designing a suitable mounting. A visit to Egypt in

1902, however, impressed on me the great advantages of a good climate for such work, and in January 1904 I went out again with the object of seeing if any arrangements could be made for setting the instrument up in Egypt and for working it.

I found an equatorial house and dome of 35 feet diameter had been erected on the hills above Helwân, about 12 miles south of Cairo: this dome was originally intended for the old 8-inch equatorial refractor, but it was much larger than was necessary for this instrument, and has proved of suitable size for the 30-inch Reflector.

The Director-General of the Survey Department, Captain H. G. Lyons, R.E., D.Sc., F.R.S., agreed, on behalf of the Egyptian Government, to accept the instrument for the Khedivial Observatory under certain conditions as to its employment, etc., and I have to express my thanks to him for the great assistance he has rendered in bringing the project to a successful issue.

The design of the mounting was commenced in June 1904 and the heavy parts were shipped and erected in the autumn of 1905. It was not until the beginning of this year, however, that photographic work was commenced, and the capabilities of the instrument tested.

The angle block carrying the polar axis is supported on a cast-iron base 7 feet high, which is in two sections, and rests on a concrete pier having a foundation of limestone. To this pier the base is securely attached by means of holding-down bolts.

Round the top of the base runs a steel gallery giving access to the driving and setting mechanism. For the convenience of the operator all the motions and clamps are on the west side of the mounting, so that the setting of the telescope can be done completely from one position.

The angle block is provided with adjusting screws in azimuth and latitude, and contains the driving clock.

The polar axis, which is 6 feet long, is of mild steel and tapers from 6 inches diameter at the top to 5 inches at the lower end. It is held in position by two solid bronze bearings mounted in a cast-iron sleeve: two rings of 1-inch ball-bearings at the upper end of the sleeve and four rings at the lower end are so adjusted as to take the transverse and longitudinal thrusts on the polar axis. This manner of reducing the friction is very efficient, and the motion of the axis, which carries almost two tons, leaves nothing to be desired.

Attached by means of a flange to the upper end of the polar axis is the hollow cast-iron fork which carries the telescope, which has been made of considerable strength to prevent flexure. Some difficulty was experienced in designing this part of the mounting, as it was found that if the declination axis was placed in the same plane as the polar axis, the telescope could not be utilised far enough to the south, owing to the low latitude. It was ultimately decided to raise the declination axis vertically so as to allow of the telescope reaching to 40° south declination, the overbalancing east and west of the meridian which would result being counterbalanced

THE 30-INCH REFLECTOR OF THE HELWÂN OBSERVATORY.

by weights at the back of the fork. The further south one gets, the more pronounced this difficulty becomes, and I do not think it would be possible to employ the open-fork type of mounting at any lower latitude in consequence.

The telescope itself consists of an octagonal cast-iron base to which the mirror cell is bolted, and a light framework tube of seamless steel tubes and aluminium rings, similar to that of the 24-inch Reflector of the Yerkes Observatory: the cell and mirror supports were constructed by the late Dr Common, and have been added to the telescope without alteration.

The flat mounting is of aluminium, to lighten the upper end of the telescope as far as possible, and this, with the photographic and optical apparatus, is contained in a separate section.

The following is done by means of a positive eyepiece with electrically-illuminated double cross wires, placed as near the photographic dark slide as possible: the double stage, with screw motions in two directions at right angles to each other, is identical in arrangement with that originated and described by Dr Common. The driving clock is provided with maintaining gear and has frictional governors of the cross-armed American type: the winding gear and starting screw are both accessible from the west side of the gallery. The driving arc runs for four hours continuously and can quickly be brought back to the starting position: differential gear for lunar rate is also provided.

In actual working this method of mounting a reflector is found to be convenient, and the clock drives at a uniform rate. There are of course one or two small defects still to be remedied, as we have only been able to work the instrument for a week or so, but we have sufficiently tested it to know that when it is in thorough working order there is no reason why satisfactory results should not be achieved.

The erection of the instrument at the Helwân Observatory was done under the superintendence of Mr B. F. E. Keeling of the Survey Department, and has been accomplished, in spite of many difficulties, in a most admirable manner. Mr Keeling is also responsible for adapting the equatorial house to the requirements of a reflector, and a false floor has been erected round the instrument near the level of the sill plate of the dome. An adjustable observing platform has also been constructed, which has proved to be very suitable for reaching the eyepiece in all positions.

The work primarily to be undertaken with the instrument is nebular photography, especially of the region lying between the equator and 40° south declination. This region includes many interesting objects which, I think, have not yet been photographed with the light grasp of such an instrument. It is intended to add a finder of good aperture, so that it may be employed on cometary photography: one piece of work which would be very suited to its capabilities is a search for Halley's Comet, as was recently suggested by Mr Crommelin.

The climate of Egypt is very suitable for astronomical photography during the greater part of the year: at times the atmosphere is so clear that stars are seen to set suddenly behind the desert horizon, and from results already obtained we know the air is extraordinarily transparent to actinic rays. Optically the mirrors are of very fair figure, especially considering that the large parabolic mirror lay in a packing-case for several years; and I am glad to have been able to mount one of Dr Common's mirrors in such a favourable situation. As far as I am aware, the Crossley Reflector of the Lick Observatory is the only reflector in similar latitudes, and almost all of this instrument was Dr Common's work.

In conclusion, I should like to thank Professor Turner for the kind encouragement and advice he has given in what was to me a rather unaccustomed task, and also Mr E. I. Jenkins for his assistance with the design.

REFERENCES TO FIGURE. (Plate 2.)

A. Concrete pier built up on limestone foundation with well for clock-weight 5 feet deep.

B^1 B^2. Cast-iron base 7 feet high attached by means of holding-down bolts to concrete pier. The openings are filled in with plate glass, and a steel door on the south side of B^1 gives access to the interior of the base. B^3 is bolted down to B^2, and has adjusting screws for latitude and azimuth.

CC. Steel gallery supported on cast iron brackets attached to B^2.

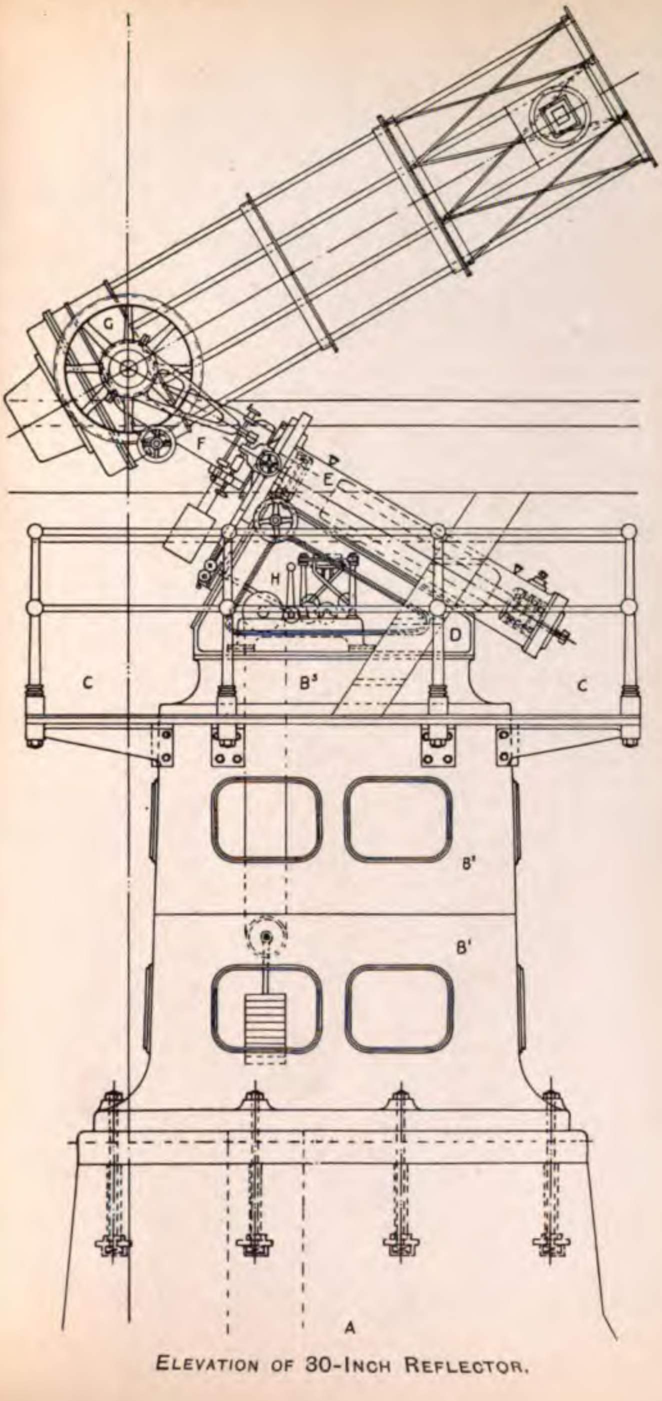
D. Angle block made in four sections bolted together, having large openings on all sides except the south, for easy access to the driving clock. The clock-winding gear is attached to the west side of D, which also carries the hand wheel and gear for the R.A. quick motion.

E. Heavy cast-iron sleeve bolted to D with solid bronze bearings and six ball-bearing rings, by means of which the weight of the polar axis, etc., is taken off the solid bearings. To the top of E is attached a flang plate for the R.A. adjustable vernier. The R.A. circle ring is bolted to a cast-iron plate keyed on to the polar axis: a rack is cut on the interior of this ring for the R.A. quick-motion pinion. The driving sector runs loose on the polar axis, and is cut with teeth 10 to the inch: on the top of the sector is a V-shaped wheel, to which the R.A. slow-motion arm is clamped.

F. Hollow cast-iron fork in which the telescope is suspended on trunnions: a plate on the west side holds the bearings for the slow-motion screws which pass through nuts let into the slow-motion arms: a double Hook's joint carries the motion of the R.A. clamp and treminates in a wheel.

G. Octagonal cast iron base of telescope tube: the trunnions are of steel bolted on to G. The west trunnion carries the declination circle: a pinion and hand wheel fixed on to the back of F give the declination quick motion. The mirror cell is bolted on to G, and is provided with adjusting screws: at the back of the cell is a counterpoise box filled with lead.

H. Driving clock: the frictional governors revolve on ball bearings, and the friction comes into play horizontally on a ring of polished tool steel.



ELEVATION OF 30-INCH REFLECTOR.

An Apparent Influence of the Earth on the Numbers and Areas of Sun-spots in the Cycle 1889–1901. By A. S. D. Maunder.

(*Communicated by Sir W. H. M. Christie, K.C.B., Astronomer Royal.*)

1. *Introductory.*

The material used in the following paper is derived from the Photo-heliographic Results published by the Astronomer Royal in the Greenwich Observations from the years 1889–1901 inclusive. During these years the results of the measures made at Greenwich of photographs of the Sun, taken daily at Greenwich, in India, and in Mauritius, are exhibited in two chief forms. In the first, the measures of positions and areas of sun-spots are given day by day. In the second, they are arranged in the form of "Ledgers" of the groups of spots, and it is these ledgers which I have utilised for the present inquiry. These ledgers of the spot groups were first given for the year 1886, but as a complete cycle of spot activity ran its course during the years 1889–1901, I have confined my present investigation to those thirteen years. The Astronomer Royal, in his last Report to the Board of Visitors, has announced that he is having similar ledgers prepared for the years 1874–1885, and when these appear I hope to treat the cycle 1878–1889 in the same way as I have here treated the cycle which followed it.

The object of my inquiry is to ascertain whether these measures of the solar photographs afford any support to the idea that the Earth exercises, or seems to exercise, any perceptible influence, either on the numbers or on the areas of the spot groups.

The following is a sample of one of the sun-spot ledgers:—

TABLE I.

Extract from Ledger of Sun-spots.

Date. Greenwich Civil Time.	Projected Area of Umbra.	Projected Area of Whole Spot.	Area for Group. Umbra.	Area for Group. Whole Spot.	Mean Longitude of Group.	Mean Latitude of Group.	Longitude from Central Meridian.
			Group 5104.				

A very fine stream following Group 5103. The stream is so compact that it is frequently measured as consisting chiefly of one very large composite spot. By November 13, two very large spots, *a* and *b*, have formed at the front and rear of the group respectively.

Nov. 4·515	17	164	59	561	168·9	−23·8	−79·0
5·452	64	458	105	752	166·8	−24·1	−68·7
6·531	92	764	96	809	164·8	−24·3	−56·6
7·466	109	1150	88	925	164·3	−24·2	−44·8
8·225	124	1274	86	879	162·8	−24·5	−36·2
9·526	210	1471	125	882	163·3	−24·4	−18·6
10·265	213	1349	122	775	163·0	−24·5	− 9·1
11·194	193	1356	109	771	163·8	−24·9	+ 4·0
12·146	233	1389	139	828	163·1	−25·3	+15·7
13·195	192	1163	126	767	163·4	−25·1	+29·9
14·561	78	682	70	588	163·9	−25·0	+48·4
15·154	96	463	102	483	164·2	−25·0	+56·5
16·228	47	266	88	462	163·7	−24·8	+70·2
17·298	3	37	12	132	159·7	−25·0	+80·2
Means ...	...	...	95	687	163·98	−24·64	...

The several columns of the ledger are described in the Introduction to the Greenwich Observations.

Spots, brought into view by the rotation of the Sun on its axis, are first seen at the east limb, and pass out of view at the west limb; the longitudes from the central meridian given in the eighth column being reckoned as negative when the spots are east of the central meridian and positive when they are west.

In the following discussion, the data of which I make use are those given in the fifth and eighth columns respectively, that is to say, the areas (expressed in millionths of the Sun's visible hemisphere, and corrected for foreshortening) of the whole spots, and the longitudes of the groups as reckoned from the central meridian of the Sun.

The period of rotation assumed for the Sun in the Greenwich Photo-heliographic Results is 25·38 days. This is, of course, the siderial period of rotation; the corresponding mean synodic rotation is 27·275 days, and the mean apparent daily motion of a spot with respect to the central meridian will be 13°·2. A long-lived spot will appear to cross the visible disc of the Sun in thirteen or fourteen days. I have therefore, for the purpose of this investigation, divided the Sun's visible hemisphere into fourteen lunes, of which seven are east of the central meridian and seven are west, each lune being 13°·2 of longitude in breadth. In the lune corresponding to the "Longitude from the Central Meridian" for each day of apparition of a spot group, I have entered the area of the spot group, corrected for the effect of foreshortening, for that day. For example, the spot group already cited would be entered as on Table II.

2. *Preponderance of Areas of Sun-spots in the Eastern Hemisphere.**

In all, 2870 spot groups have been thus treated, the total number of entries being 15,721, for few of the groups were observed for fourteen days, the average duration being about six days. Table III. gives the result of summing up the total areas for each lune, first, for the northern hemisphere, and then for the southern, and thirdly for the entire disc of the Sun.

It will be seen that in both hemispheres there is a regular progression in the spotted area from the first lune to the seventh, and a corresponding decrease from the eighth lune to the fourteenth.

The areas in the first and in the fourteenth lunes are much smaller than those in the remaining twelve. The reason for this is obvious. A spot is practically invisible when it is distant 86° from the central meridian. The effective breadth, therefore, of the first and last lunes is only half that of any of the remaining twelve. It must also be borne in mind that spot groups in general extend over several degrees of longitude, so that the centre of a group may

* Strictly speaking, this is only a quarter of the sphere, but it seems unnecessarily pedantic to coin a new term.

Table II.

Extract from the Form for entering the Areas in the Lunes.

Date when first seen.	No. of Group.	1st Lune. −79°·3 and over.	2nd Lune. −66°·1 to −79°·2	3rd Lune. −52°·9 to −66°·0	4th Lune. −39°·7 to −52°·8	5th Lune. −26°·5 to −39°·6	6th Lune. −13°·3 to −26°·4	7th Lune. −0°·1 to −13°·2	8th Lune. +0°·1 to +13°·2	9th Lune. +13°·3 to +26°·4	10th Lune. +26°·5 to +39°·6	11th Lune. +39°·7 to +52°·8	12th Lune. +52°·9 to +66°·0	13th Lune. +66°·1 to +79°·2	14th Lune. +79°·3 and over.	Total Area of Group.	Mean Hel. Long. of Group.	Mean Hel. Lat. of Group.
1903. 11.4	5104	561	752	809	925	879	882	775	771	828	767	588	483	462	132	9614	164°	−25°

Table III.

Lunes.

Eastern Hemisphere. / Western Hemisphere.	1st −79°·3 and over. / 14th +79°·3 and over.	2nd −66°·1 to −79°·2 / 13th +66°·1 to +79°·2	3rd −52°·9 to −66°·0 / 12th +52°·9 to +66°·0	4th −39°·7 to −52°·8 / 11th +39°·7 to +52°·8	5th −26°·5 to −39°·6 / 10th +26°·5 to +39°·6	6th −13°·3 to −26°·4 / 9th +13°·3 to +26°·4	7th −0°·1 to −13°·2 / 8th +0°·1 to +13°·2	Eastern Hemisphere. / Western Hemisphere.
Northern Spots.								
East.	24358	74826	89979	99224	102708	105138	106216	602449
West.	21302	71476	87947	93799	100949	103883	104679	584035
Difference E. − W.	+3056	+3350	+2032	+5425	+1759	+1255	+1537	+18414
Southern Spots.								
East.	35622	95195	111741	119763	125225	128292	128995	744833
West.	27370	90480	108816	117282	124169	128585	129869	726571
Difference E. − W.	+8252	+4715	+2925	+2481	+1056	−293	−874	+18262
All Spots.								
East.	59980	170021	201720	218987	227933	233430	235211	1347282
West.	48672	161956	196763	211081	225118	232468	234548	1310606
Difference E. − W.	+11308	+8065	+4957	+7906	+2815	+962	+663	+36676

fall within the first or last lune, though much of its area may be actually beyond the visible hemisphere. The areas in these two lunes ought therefore to be multiplied by some factor greater than 2, to make them really comparable with the other twelve.

But omitting these two lunes, we yet see a very marked progression in the other twelve; the areas steadily increase from the east to the central meridian, and diminish from the central meridian to the west.

What can be the cause of this? Two suggestions immediately present themselves. There may be some systematic error in the measurement, or there may be some systematic error in the reduction.

There are several causes which can be conceived as possibly giving rise to an error of the second kind. First of all the intense brightness of the Sun may, by irradiation, make the photographic radius systematically appear greater than it really is, with the result that the distance of the spot from the centre of the disc, expressed in terms of the radius, will be too small. An effect in the same direction would also arise from the measurer judging the centre of the spot as if he were dealing with a marking on a plane surface. This again would tend to put the position of the centre of the spot slightly too near the centre of the disc, since the effect of foreshortening will always be greatest on the side of the spot furthest from the centre. Yet again, if there is any sensible displacement of the position of a spot due to the effect of refraction in the solar atmosphere, this would also make the spot appear nearer the centre of the disc than it really is. The effect of each of these hypothetical sources of error in the measurement would be the same; the distance of a spot from the centre would be under-estimated, and too small a correction would be applied for foreshortening, and as the factor increases with the sine of the angular distance from the centre, the areas of the lunes near the limb would require to be increased in a higher proportion than those near the centre.

If the steady progression from the east limb to the central meridian, and from the central meridian to the west limb, were the only feature shown by Table III., it would be necessary to examine very carefully in detail each of the above hypothetical sources of error, trivial though they certainly must be. But there is another feature, just as clearly shown, into which any error in the correction for foreshortening cannot enter. If we compare the area of the eastern hemisphere with the area of the western, we find that the eastern is always the greater, whether we consider the north or the south. And further, if we compare each of the several lunes in the eastern hemisphere with the corresponding lune in the western, we find that, almost without exception, the eastern is the greater, usually much the greater.

This difference between the two hemispheres cannot be due to any error of foreshortening, seeing that the hemispheres and their constituent lunes are symmetrical. Neither can it be due to any personal equation in the measurement of the solar photographs;

the form of the micrometer and the method of measuring preclude that. (See Introduction to Greenwich Observations.)

The amount of the predominance of the areas in the eastern hemisphere over those in the western is too marked, both for the hemispheres as a whole, and for their constituent lunes, for it to be accidental, whatever explanation may be assigned for it.

3. *Preponderance of Numbers of Spot Groups in the Eastern Hemisphere.*

There is a simple way of getting rid of any systematic error in the measurement or reduction of the areas of the spots; *i.e.*, we may discard the areas and deal only with the number of groups. This is what has been done in Tables V. and VI. following. It will be observed that not only are the total numbers of groups counted in each lune given, but the groups are arranged in classes. All spots do not live for the same length of time. They may come into existence on the invisible hemisphere of the Sun, or on the visible hemisphere; and similarly, they may die out either on the invisible or on the visible hemisphere. They may be seen for one, two, or any number of consecutive days up to fourteen. If very long-lived, they may be so seen in a second, third, fourth, or even in a fifth rotation, but in this discussion I have treated each apparition of a spot group as a separate and independent display, except where the contrary is expressly stated. Regarding, therefore, fourteen days as the longest period for which a spot group can be followed consecutively, I have thought it well to divide the whole cycle of the sun-spots of 1889–1901 not only into those in the northern and southern hemispheres of the Sun respectively, but also into classes distinguished by the number of days during which the group was visible; that is to say, into classes of spots observed for fourteen consecutive days, for thirteen, for twelve, and so on, down to ephemeral groups which were observed only upon a single day. On the average, I found that the mean area of a group bore a direct proportion to the length of time for which it was visible. The following Table IV. gives the number of groups which existed for each of the given periods, the total area of the groups, and the mean area per group. Here it may be noted that both the number of groups and the mean area per group are slightly greater in the southern than in the northern hemisphere.

In Table V. the third section of Table IV. is dealt with, but the totals for each lune in the northern and southern hemispheres separately are added at the foot. The spots are not only divided into classes according to the length of time that they were under observation, but each lune is treated separately, and the number of groups observed in that particular lune is recorded. The total numbers observed in the eastern and western hemispheres are likewise exhibited.

From the nature of the case, the number of spots of the first class, namely, those observed for fourteen days, is the same for all

Table IV.

Total Areas and Mean Areas of Groups of Different Durations.

No. of Days.	Northern Spots.			Southern Spots.			All Spots.		
	No. of Groups.	Total Area.	Mean Area. Per Day.	No. of Groups.	Total Area.	Mean Area. Per Day.	No. of Groups.	Total Area.	Mean Area. Per Day.
14	10	55893	399	21	184142	626	31	240035	553
13	100	455948	351	199	477671	371	199	933619	361
12	93	291688	261	108	343318	265	201	635006	264
11	57	116564	186	66	129800	179	123	246364	182
10	54	65709	122	69	93245	135	123	158954	130
9	62	55491	99	61	53414	97	123	108905	98
8	61	45504	93	84	56715	84	145	102219	88
7	55	30602	80	73	41870	82	128	72472	81
6	65	17564	45	80	28300	59	145	45864	53
5	73	20919	57	85	17319	41	158	38238	48
4	88	12358	35	133	23228	44	221	35586	40
3	98	7724	26	113	11560	34	211	19284	30
2	135	6336	23	168	6169	18	303	12505	21
1	349	4184	12	410	4653	11	759	8837	12
Total,	1300	1186484	165	1570	1471404	172	2870	2657888	169

Table V.

Number of Groups in each Lune for each Class of Duration of Spot Groups.

No. of Days.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	E.	W.
14	31	31	31	31	31	31	31	31	31	31	31	31	31	31	217	217
13	109	199	199	199	199	199	199	199	199	199	199	199	199	90	1303	1284
12	33	186	201	201	201	201	201	201	201	201	201	201	168	15	1224	1188
11	18	82	116	123	123	123	123	123	123	123	123	105	41	7	708	645
10	16	63	82	112	123	123	123	123	123	123	107	60	41	11	642	588
9	12	52	70	93	114	123	123	123	123	111	71	53	30	9	587	520
8	15	67	86	96	110	137	145	145	130	78	59	49	35	8	656	504
7	8	40	61	64	77	94	124	120	88	67	64	51	34	4	468	428
6	3	39	64	81	88	96	110	102	81	64	57	49	32	4	481	389
5	9	42	59	68	84	84	66	67	82	74	65	50	32	8	412	378
4	9	47	77	104	107	83	76	69	71	83	68	48	34	8	503	381
3	2	33	58	71	53	40	52	58	63	55	59	50	31	8	309	324
2	0	30	59	51	43	56	66	54	49	52	50	55	36	5	305	301
1	1	35	64	66	65	73	94	94	83	64	50	37	28	5	398	361
Total,	266	946	1227	1360	1418	1463	1533	1509	1447	1325	1204	1038	772	213	8213	7508
North,	124	428	563	625	636	661	697	682	655	617	560	474	349	103	3734	3440
South,	142	518	664	735	782	802	836	827	792	708	644	564	423	110	4479	4068

lunes. For the second class, those observed for thirteen days, the number is the same for each lune except the first and last. Similarly for the successive classes, until we come to spots observed for less than half a complete transit, that is to say, for seven days and under. For the spots of long duration, therefore, the number of spots observed in each lune near the centre of the disc must necessarily be constant. But when we exclude these, it will be noted that not only do the numbers in the whole of the eastern quadrant exceed those in the whole of the western in nearly every case, but in most instances the number in any particular eastern lune exceeds that in the corresponding western lune.

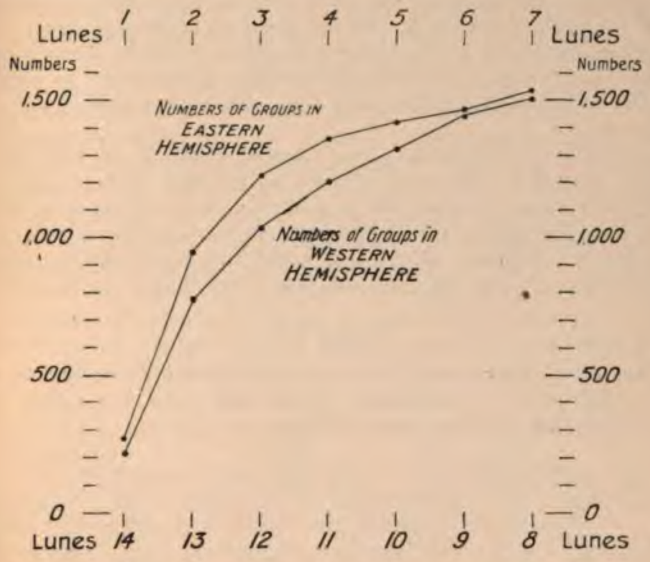


FIG. 1.—Numbers of Groups in each Lune, comparing East and West.

Table VI. (represented graphically in fig. 1) gives the differences in the number of groups between the corresponding eastern and western limbs, regardless of the class of the spot, that is to say, of the length of time that it was under observation. It will be seen that both for the northern spots and for the southern, and consequently for the two combined, the eastern lune always shows an excess over the corresponding western lune as to the number of spots observed. In no single case does the number of groups in a western lune equal those in the corresponding eastern one. The preponderance of east over west attains its maximum in the third lune from the limb, where the proportion is almost exactly 7 to 6.

TABLE VI.

Differences in Numbers of Groups found in Corresponding [illegible]
E.–W., for Northern, Southern, and All Spots.

	1–14	2–13	3–12	4–11	5–10	6–9	7–8	E.-W.
Northern,	+21	+ 79	+ 89	+ 65	+19	+ 6	+15	[illegible]
Southern,	+32	+ 95	+100	+ 91	+74	+10	+ 9	[illegible]
All Spots,	+53	+174	+189	+156	+93	+16	+24	[illegible]

4. *Preponderance of Numbers of Spot Groups rising in the* [illegible]
Hemisphere.

It has already been shown that the eastern hemisphere has [illegible] systematic preponderance over the western hemisphere in [illegible] both to total spotted area and the numbers of spot groups. [illegible] question naturally arises, Can we ascertain whether there is [illegible] difference between the visible and invisible hemispheres in [illegible] to the numbers of groups, and, if so, which hemisphere has [illegible] advantage?

Something can be learnt from the comparison of the numbers [illegible] groups seen in the two lunes nearest to the east limb on the [illegible] side, with those given by the two lunes nearest to the west [illegible] For, from the nature of the case, these numbers most nearly [illegible] the number of groups which come round into view from the [illegible] hemisphere on the one side, and pass away into it on the [illegible]

TABLE VII.

Numbers of Groups seen in the First or Second Lune, and in [illegible]
Thirteenth or Fourteenth Lune, with their differences, [illegible]
Northern, Southern, and all Spots.

	North.			South.			Whole Sun's Disc.		
No. of Days.	1st or 2nd Lune.	13th or 14th Lune.	Diff. E.-W.	1st or 2nd Lune.	13th or 14th Lune.	Diff. E.-W.	1st or 2nd Lune.	13th or 14th Lune.	Diff. E.-W.
14	10	10	0	21	21	0	31	31	[illegible]
13	100	100	0	99	99	0	199	199	[illegible]
12	85	79	+ 6	101	89	+12	186	168	+ [illegible]
12	34	23	+11	48	18	+30	82	41	+ [illegible]
10	33	12	+21	30	29	+ 1	63	41	+ [illegible]
9	27	13	+14	25	17	+ 8	52	30	+ [illegible]
8	26	14	+12	41	21	+20	67	35	+ [illegible]
7	20	13	+ 7	20	21	− 1	40	34	+ [illegible]
6	16	15	+ 1	23	17	+ 6	39	32	+ [illegible]
5	21	18	+ 3	21	14	+ 7	42	32	+ [illegible]
4	22	14	+ 8	25	20	+ 5	47	34	+ [illegible]
3	16	12	+ 4	17	19	− 2	33	31	+ [illegible]
2	9	16	− 7	21	20	+ 1	30	36	− [illegible]
1	10	13	− 3	26	20	+ 6	36	33	+ [illegible]
Total,	429	352	+77	518	425	+93	947	777	+17[illegible]

This comparison is brought out in Table VII. where, as in [illegible] IV and V., the numbers are given for northern and southern [illegible]

separately and combined, and for all classes of spots. With very few exceptions, the number of groups that come into view out of the hemisphere that is remote from the Earth exceeds that of the groups that pass out of sight from our Earth into the unseen hemisphere. It is further significant that the exceptions are almost entirely confined to spots of very short duration, not exceeding three days.

Now this persistent preponderance cannot be a mere chance effect; the signs in the third columns are too constantly positive for anything that is purely accidental. Yet the eastern and western lunes have no significance at all, when considered from a standpoint on the *Sun*; their distinctiveness lies in their different relations to the *Earth*. Putting the matter shortly, we see that, in all, 947 spot groups came round the eastern limb of the Sun into view of the Earth, and only 777 passed round from the view of the Earth at the western limb into the invisible hemisphere. And whether we consider the Sun as a whole, or its northern and southern hemispheres separately; or whether we consider the spot groups with or without regard to their length of life, we get a result to the same effect. We are driven to conclude that during the cycle, 1889–1901, the Earth was apparently responsible for the extinction of about 170 spot groups; that is to say, of more than one-sixth of the whole number that came into view round the eastern limb. Whether this effect was produced by bringing about the premature dissolution of old groups, or by hindering the generation of new groups, or by a combination of the two, there is no doubt as to the magnitude of the resulting effect.

5. *Birth-rate and Death-rate of Spot Groups in the Different Lunes.*

It will be interesting to examine whether there is any tendency for spots to take their origin in one lune rather than another, and also whether certain lunes are especially favourable for their dissolution. It is manifest that the fact that a spot is first seen in lunes 1 and 2 affords very small evidence that it actually arose in those lunes; the strong probability is that it arose in the unseen hemisphere. Similarly, the fact that a spot is last seen in either lune 13 or 14, affords no presumption that its actual dissolution took place there. But when we are dealing with the ten central lunes we have a certain amount of positive evidence that the groups first seen or last seen in these regions of the disc were not visible earlier or later. The effect of foreshortening will no doubt be to hide a few of the smaller groups, whilst in the two lunes nearest to either limb, and in a few quite accidental cases, this may even extend to the third lune from each limb; but the record of appearances and disappearances for at least the central lunes must be substantially correct.

Table VIII., which may be considered as a supplement to Table VII., gives the number of groups first seen in each lune from the 3rd to the 12th inclusive, and also the number of those last seen.

Ephemeral spots, that is those only seen on a single day, are not included in this table.

TABLE VIII.

Numbers of Groups first seen or last seen in the Ten Central Lunes.

Lune.	First Seen.			Last Seen.		
	North.	South.	All Spots.	North.	South.	All Spots.
3	113	139	252	10	22	32
4	72	91	163	33	36	69
5	50	78	128	39	55	94
6	54	77	131	45	57	102
7	70	81	151	62	74	136
8	47	65	112	61	75	136
9	41	44	85	66	116	182
10	39	40	79	80	82	162
11	27	28	55	100	92	192
12	17	22	39	116	146	262
Total,	530	665	1195	612	755	1367

Since the 3rd lune is symmetrical with the 12th, the 4th with the 11th, and so on, any error, from such effects of foreshortening, in the number of spots first seen in the 3rd lune, will be of the same amount as that in the number of spots last seen in the 12th lune; and so with the other pairs of corresponding lunes. The difference, therefore, ought to be free from any such effect of foreshortening, and to represent not an apparent effect but a real difference.

Table IX. is derived from Table VIII., and exhibits the differences in the number of groups first seen in any particular lune, with the number last seen in the corresponding lune.

TABLE IX.

Comparison of Numbers of Groups first seen in any Lune with those last seen in the Corresponding Lune.

Seen					
first in Lune.		last in Lune.	North.	South.	All Spots.
3	–	12	– 3	– 7	– 10
4	–	11	– 28	– 1	– 29
5	–	10	– 30	– 4	– 34
6	–	9	– 12	– 39	– 51
7	–	8	+ 9	+ 6	+ 15
8	–	7	– 15	– 9	– 24
9	–	6	– 4	– 13	– 17
10	–	5	– 0	– 15	– 15
11	–	4	– 6	– 8	– 14
12	–	3	+ 7	– 0	+ 7
Totals,			– 82	– 90	– 172

From Table IX., therefore, we see that there is an average excess per lune of 17 dissolutions over formations; this being made up of 8 in the northern hemisphere and 9 in the southern, which is very nearly in the proportion of the total number of groups of either hemisphere. For every 7 groups that form on the visible disc, 8 dissolve, or at least one-seventh more groups form on the invisible hemisphere than on the visible. The death-rate on the visible hemisphere is systematically higher than the birth-rate.

When we deal with the distribution of the ephemeral spots (see Table V.), we find a steady increase in number per lune from the east to the central meridian, and a decrease thence to the west limb. But the ratio of those generated on the eastern side to those generated on the western is about 11 to 10, being an excess of 37 in a total number of 755.

In Table III. a comparison was made between the total spotted area shown by the eastern and western hemispheres, the area of the former being 1347282 and of the latter 1310606 millionths of the Sun's visible hemisphere; the excess of the eastern hemisphere was thus 2·8 per cent. In Table VII., in which the number of groups in the two most easterly lunes were compared with those in the two most westerly, the excess of the easterly amounted to 21·9 per cent. In Table V., where the groups seen in each lune were summed up together, the preponderance of the eastern hemisphere was 9·4 per cent. In Table V. the ephemeral groups show a preponderance in the eastern hemisphere of 10·3 per cent. over the western; whilst the number of dissolutions given in Table VIII. shows a predominance over the formations of 14·3 per cent. It will be seen that, though the predominance of the eastern hemisphere over the western in the matter of area is well marked and consistent, yet that it is by no means so great in proportion as is the predominance in the number of groups. This would appear to indicate that, whatever was the cause of the predominance of the eastern hemisphere, it was one which affected small groups to a greater degree than large ones. But if we group the areas in the same way as we have already grouped the numbers, we shall see the introduction of another factor which will render the last inference more clear.

6. "*Spot Phase.*"

In Tables X. and XI. the total areas are given for each lune, but the spots of the different classes are exhibited separately. Both tables are for the whole disc of the Sun; but in Table XI. and in fig. 2 the areas are expressed, not in millionths of the Sun's visible hemisphere, but each class of spot has been taken separately, and the area of each lune has been expressed in thousandths of the total area for that class. These present the remarkable feature that the maximum area for any class of spot tends to shift over from the east towards the west as we go from the spots of long duration to those of short.

TABLE X.* TABLE XI.

Areas for each Lune for All Spots of Different Durations.

Table X. — Total.

No. of Days.	1	2	3	4		6	7	8	9	10	11	12	13	14	East.	West.
14	10381	18467	20883	21621	21781	21302	19590	19180	17880	16592	16705	15560	12628	7465	134025	106610
13	31116	65321	74909	81254	82086	81353	80459	77608	76474	73221	66456	61456	53382	22061	496498	430658
12	7699	43536	53858	56202	56298	57587	58412	57634	56075	53523	49667	44913	34832	4770	333592	301414
11	3367	12654	15503	18508	22917	25568	25599	24859	24167	22255	18438	16013	10858	1607	124116	118197
10	2007	7142	8652	11634	13786	15441	16412	16487	16214	14597	13549	11611	9414	2008	75074	83880
9	1429	5936	6538	7984	9917	11488	11627	10768	9952	9990	8049	7272	5523	2254	54919	53808
8	1898	6169	7024	6985	6942	7558	7962	10046	10638	10494	9878	8837	6428	1360	44538	57681
7	946	2927	3596	3650	3905	4047	5264	7278	8491	8900	8638	8022	5826	719	24335	47874
6	163	2112	2630	2845	2629	2155	2362	3189	4889	5415	5514	6178	5150	486	14896	30821
5	469	1584	2125	1994	1835	1654	1773	2323	2816	3496	4646	5756	5609	1841	11434	26487
4	417	1844	2234	2562	2311	1731	1769	1466	1533	2988	4688	5149	5097	1797	12868	22718
3	64	984	1469	1278	1089	722	956	1008	1032	1151	2075	3199	3042	1071	6562	12578
2	0	592	888	791	569	813	823	858	543	861	1071	1368	2787	624	4476	8112
1	13	427	719	727	610	737	916	727	681	673	611	583	909	503	4149	4687

Table XI. — Expressed in Thousandths.

No. of Days.	1	2	3	4		6	7	8	9	10	11	12	13	14	East.	West.
14	43	77	87	90	91	89	82	80	74	69	70	65	53	31	558	442
13	34	70	81	88	89	88	87	84	82	79	72	66	58	24	536	464
12	12	69	85	88	89	91	92	91	88	84	78	71	55	8	525	475
11	14	52	64	76	95	106	106	103	100	92	76	66	45	7	512	488
10	13	45	54	73	87	97	103	104	102	92	85	73	59	13	472	528
9	13	54	60	73	91	105	107	99	91	92	74	67	51	21	505	495
8	19	61	69	68	68	74	78	99	104	103	97	87	63	13	435	565
7	13	41	50	51	54	56	73	101	118	123	120	111	81	10	337	663
6	4	46	58	62	58	47	52	70	107	118	121	135	113	11	326	674
5	12	42	56	53	48	44	47	61	74	92	122	152	148	49	301	699
4	12	52	63	72	65	49	50	41	43	84	132	145	143	51	362	638
3	3	52	77	67	57	48	50	53	54	60	109	167	159	56	343	657
2	0	47	71	63	45	65	65	68	43	68	85	109	221	50	356	644
1	3	48	81	82	69	83	104	82	77	76	69	66	103	57	470	530
Total,	23	64	76	82	86	88	88	88	87	85	79	74	61	18	507	493

* *There is a slight difference in the material employed in Tables III. and X. Table III.* ..*s all spots of the cycle 1889–1901, whilst Table X. does not extend beyond the end*

The curves given by the 14- and 13-day spots are somewhat flat, and come to a maximum on the fifth day. The curve continues to be very flat for the 12-day spots, but the maximum is shifted to the seventh lune. The 11-day spots show a more decided curve, and by the time the 8-day spots are reached the maximum is

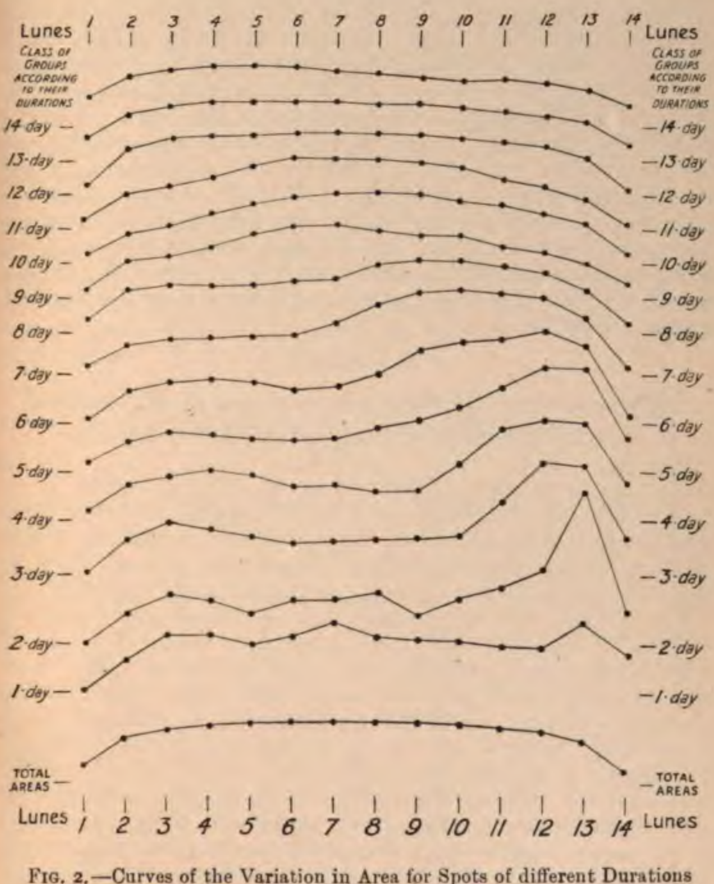


FIG. 2.—Curves of the Variation in Area for Spots of different Durations as seen in the different Lunes.
(The areas are expressed in thousandths of the total area for each class.)

found in the ninth lune. The 6-day spots show two maxima, a small one in the fourth lune and a much greater in the twelfth. The spots of 5-, 4-, and 3-day duration give curves of the same form, but with the two maxima more and more accentuated. The 2-day groups show a tremendous peak in the thirteenth lune, beside two small maxima, the one in the third lune and the other at the centre of the disc. The ephemeral spots also have three maxima,

one in the fourth lune, one in the seventh, and one in the thirteenth. The total areas of all spots, irrespective of duration, give very nearly a smooth curve, but the eastern lunes systematically show a slight preponderance over the western.

It is thus seen that the preponderance in area of the eastern hemisphere over the western, as shown in Table III., is one which is entirely confined to the classes of spots which have long duration. Table XII. brings this out yet more clearly, for it exhibits in tabular form the differences between the corresponding lunes east and west; the areas for each class being expressed as in Table XI., in thousandths of the total area of that class. It shows that in the long-lived groups there is a steady preponderance of the east over the west, not only for the two hemispheres of the Sun, but also for each pair of lunes. It shows also how, contrariwise, for the short-lived groups, the west preponderates still more emphatically over the east. Just the same result is obtained if the northern and southern hemispheres are treated separately, though the figures naturally show greater irregularities, since the numbers treated are smaller.

TABLE XII.

Differences between the Corresponding Lunes, E.–W., expressed in thousandths of Total Area for Whole Sun.

No. of Days.	1–14	2–13	3–12	4–11	5–10	6–9	7–8	E.-W.
14	+12	+24	+22	+20	+22	+15	+2	+116
13	+10	+12	+15	+16	+10	+6	+3	+72
12	+4	+14	+14	+10	+5	+3	+1	+50
11	+7	+7	−2	+0	+3	+6	+3	+24
10	−0	−14	−19	−12	−5	−5	−1	−56
9	−8	+3	−7	−1	−1	+14	+8	+10
8	+6	−2	−18	−29	−35	−30	−21	−130
7	+3	−40	−61	−69	−69	−62	−28	−326
6	−7	−67	−77	−59	−60	−60	−18	−348
5	−37	−106	−96	−69	−44	−30	−14	−396
4	−39	−91	−82	−60	−19	+6	+9	27[illegible]
3	−53	−107	−90	−42	−3	−16	−3	−314
2	−50	−174	−38	−22	−23	+22	−3	−288
1	−54	−55	+15	+13	−7	+6	+22	−6[illegible]
Total,	+5	+3	+2	+3	+1	+1	+0	+1[illegible]

What, then, can be the cause of this striking difference in the behaviour of groups of long duration and of short duration? Why should the former show their greatest areas in the eastern hemisphere, and the latter so markedly in the western?

The expl[illegible] Three circumstances comb[illegible] to bring it [illegible]

First, whilst groups observed for twelve, thirteen, or fourteen days are undoubtedly groups of long duration, groups observed for only three or four days are by no means necessarily of short duration. They may have existed in the unseen hemisphere before they came under observation, or they may have continued to exist there after they had passed beyond the range of observation. The time during which we observed them may in many cases represent simply the beginning or the close of the life-history of long-duration groups, so that the spots classed as of short duration must really include a great number of long duration.

Next, it has been already pointed out in Table IV., that the longer the life-history of a spot, the greater on the average will be its mean area.

Third, just exactly as the spotted area of the Sun increases much more quickly as the cycle runs up from minimum to maximum than it decreases when the cycle runs down from maximum to minimum, so is it with the great majority of individual spot groups—the growth in area is more rapid than is the decline. This characteristic we may denote by the term "spot phase."

7. *Analysis of the Spot Groups of 8-days apparent Duration.*

It follows, therefore, if we are dealing with one of the classes of short-duration spots, say, for example, with those of eight days, that beside the true 8-day spots, we have a number of spots really of long duration but classed as short because they formed in the invisible hemisphere, and only the last eight days of their existence were passed on this side of the Sun. Whilst these groups are under observation they are decaying, and their chief energy is already passed, so that their areas are not of a much higher order than those of true 8-day groups. The areas, therefore, in the eastern hemisphere will be but little affected. But it will not be so in the western hemisphere. Beside the true 8-day groups there will also be a number of long-duration groups just forming, and classed with them because observed for eight days before they passed to the other side of the Sun. Being spots of long duration they will on the average be of great mean area, and being observed during the period of rapid growth, their areas will be of quite a different order from those of the true 8-day groups with which they have been classified.

Tables XIII. and XIV. bring out this effect very clearly. In Table XIII., the 8-day groups, whether they genuinely existed for only eight days or simply appeared to do so, have been divided into three categories:—Eastern groups, that is to say, groups seen first close to the east limb, *i.e.* in the first or second lunes; Central groups, first seen in the third, fourth, or fifth lunes, and in the tenth, eleventh, or twelfth lunes, thus clearly origin and dissolution in the visible hemisphere; and

Western groups, those last seen close to the west limb, *i.e.* in the thirteenth or fourteenth lunes. The first category will, therefore, be largely made up of groups which formed in the unseen hemisphere, but dissipated on the visible disc. The last category will

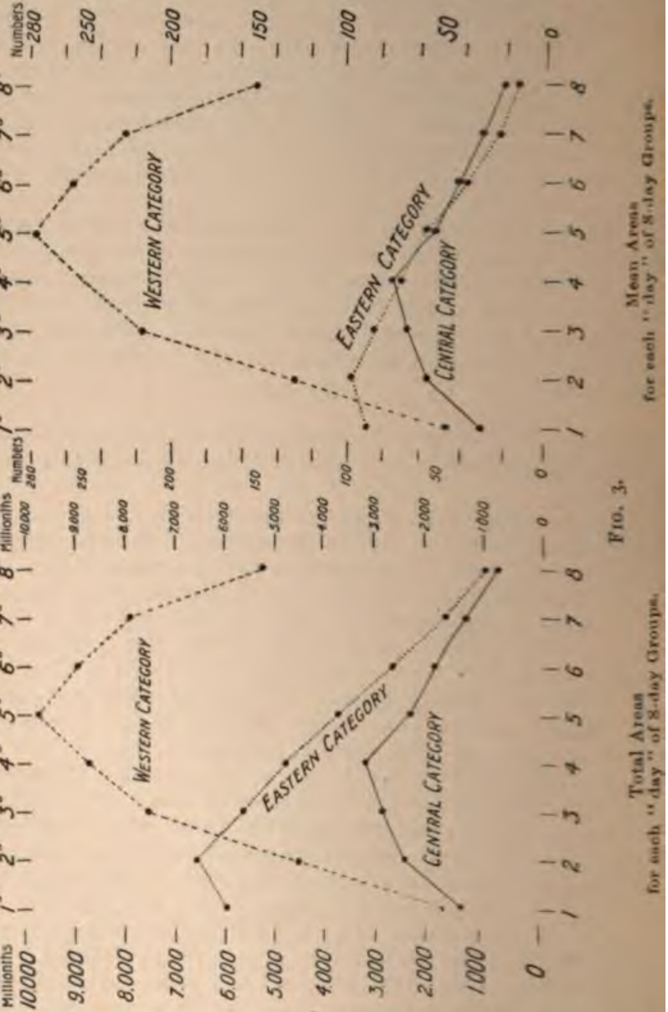


FIG. 3.

include many which, though they formed on the visible hemisphere, did not dissipate until after they had passed out of view at the west limb. Many therefore of both these classes are not true 8-day groups; they only appeared to be so, but really had a longer duration, and this would mean that on the average they were groups of greater area than true 8-day groups.

TABLE XIII.

Total Areas for each Day of Life of Eastern, Western, and Central Groups of the 8-Day Spots, for Northern, Southern, and All Spots.

Class of Group.	1st Day.	2nd Day.	3rd Day.	4th Day.	5th Day.	6th Day.	7th Day.	8th Day.	Number of Groups.
				Northern.					
Eastern,	2587	2637	2085	1845	1409	1124	671	444	26
Western,	967	2593	3921	3916	4058	3606	3183	2832	14
Central,	725	1437	1296	1439	970	768	600	379	21
				Southern.					
Eastern,	3348	3908	3541	2933	2292	1495	941	541	41
Western,	691	1916	3635	4803	5666	5337	4723	2460	21
Central,	597	956	1531	1712	1309	1049	715	527	22
				All Spots.					
Eastern,	5935	6545	5626	4778	3701	2619	1612	985	67
Western,	1658	4509	7556	8719	9724	8943	7906	5292	35
Central,	1322	2393	2827	3151	2279	1817	1315	906	43

In the first place, it should be noted that the number of groups in the Eastern category is almost double the number of groups in the Western, and yet the total area in the Eastern is much less than in the Western, so that the mean area of an Eastern group is very much less than the mean area of a Western. These mean areas are given in Table XIV. Fig. 3 presents both Tables XIII. and XIV. in graphical form.

TABLE XIV.

Mean Area for each Day of Life of Eastern, Western, and Central Groups of the 8–Day Groups, for Northern, Southern, and All Spots.

Class of Group.	1st Day.	2nd Day.	3rd Day.	4th Day.	5th Day.	6th Day.	7th Day.	8th Day.
				Northern.				
Eastern,	100	101	80	71	54	43	26	17
Western,	69	185	280	280	290	258	227	202
Central,	35	68	62	69	46	37	29	18
				Southern.				
Eastern,	82	95	86	72	56	34	23	13
Western,	33	91	173	229	270	254	225	117
Central,	27	44	70	78	60	48	33	24
				All Spots.				
Eastern,	89	98	84	71	55	39	24	15
Western,	47	129	216	249	278	256	226	151
Central,	31	56	66	73	53	42	31	21

The distinction between the "lunes," into which the visible disc of the Sun is divided, and the "days" in the life-history of a spot group, should be clearly borne in mind. The first *day* of the life of an Eastern group may fall in the first or in the second *lune*; its eighth *day* will fall in the eighth or ninth *lune*. The first *day* of a Central group may fall in the third, in the fourth, or in the fifth *lune*. The first *day* of a Western group may fall in the sixth or in the seventh *lune*. But in Tables XIII. and XIV. the areas for each day of the life of a group, in the several categories, are summed irrespective of the lunes in which they may actually fall; and since the first and the fourteenth lunes have really but half the area of any of the remaining twelve lunes, the areas of the *first day* of the Eastern category, and of the *eighth day* of the Western category, should be each multiplied by a factor greater than unity.

There is a very marked difference, not only in the total and mean areas of the three categories, but also in their several progressions. The Eastern category has a steady decay from the second day of its life—or indeed from its first day, if we allow for the fact that the first lune is smaller in area than other lunes,—and the eastern "day" has a greater area than the corresponding western "day"; the fourth day is greater than the fifth, the third than the sixth, etc. The Western category increases in area until its fifth day, after which it again steadily declines; and here it is the western "day" that is always greater than the corresponding eastern. In the Central category, the culmination is on the fourth day, and again the areas for the eastern "days" are greater than for the corresponding western.

And, considering the three categories, we find very strong evidence of the action of "spot phase." The groups of the Central category may take their rise over a range of the third, fourth, and fifth lunes; and as on the whole disc of the Sun there are 43 such groups, there is an average of 14 groups arising in each lune, the mean area of each group being 47. We may assume that this is the average number of true 8-day groups, arising in each lune all over the Sun, and that true 8-day groups average 47 in area. But we find, on comparing these numbers and mean area with those of the Eastern and Western categories, though these have but a range of one and a half lunes (half, that is to say, the range of the Central category), that these latter rank much greater both in numbers and mean areas. In both the Eastern and Western categories the true 8-day groups are swamped by the greater numbers and areas of the longer-lived groups classed as 8-day groups, because they are only *observed* for eight days in the visible hemisphere; the true 8-day groups seem to exercise little perceptible modification of the course run by either the Eastern or the Western category of groups.

The Eastern category must then be composed—overwhelmingly, both in numbers and areas—of the last eight days of decaying groups, and these final eight days show a steady decline. The Western category must be composed—overwhelmingly in areas, if not in the numbers of the groups—of the first eight days of activ

and long-lived groups, and must include in the mean their period of greatest activity, for the mean area culminates on the fifth day of their apparent life. Comparing the Eastern and Western categories, we find that the Eastern has very nearly double the number of spot groups of the Western, but that the Western has nearly double the total area and very nearly four times the mean area of the Eastern.

This detailed examination of the 8-day groups, therefore, renders two things clear. First, that for groups of this class—observed for eight days—many more come round at the east limb from the unseen hemisphere and die on the visible disc, than rise on the visible hemisphere and pass out of sight at the west limb.

Next, we have a clear explanation of the gradual shift of the maxima towards the west, as shown in Tables X. and XI. It is simply a function of "spot phase."

It is not possible to treat in the same manner groups that have a longer visible duration than 10 days. We cannot discriminate between those groups which existed for only from 12 to 14 days, and those which rose or dispersed in the unseen hemisphere. We should reasonably expect the former to be but a small fraction of the latter; but for them the influence of "spot phase" would be to throw the maximum somewhat to the east of the central meridian, since the rise of spot groups is more rapid on the average than their decline. But for groups that are observed as of shorter duration than 10 days, the curves in fig. 2 show the influence of the three categories of Eastern, Central, and Western. Already in the 6-day class, the preponderance in area and numbers of the Eastern category over the Central has produced a secondary maximum in the fourth lune, the greater maximum due to the Western falling in the twelfth. For the remaining classes, we see the effect of the Eastern categories in the swift run up in the early lunes, and subsequent decline. In the 2-day and 1-day spots, the central maximum is without doubt due to the better presentation there which enables small spots to be seen which might have been passed over altogether when foreshortened.

8. *Preponderance as to Area of the Eastern Hemisphere in Long-lived and Recurrent Groups.*

But there is a class of groups which must be free from any tendency of "spot phase" to link their greatest development with any particular lune. If we consider only those groups which have been seen in three or more distinct apparitions, and, rejecting the first and last, take only the intermediate appearances, then we are clearly dealing with the most stable portion of the life-history of the most permanent and important groups. We are not confusing together short-lived and unimportant groups with those which are greater and more durable, but are accidentally under observation for only a short time. Nor are we in any way introducing the condition that they must ei[illegible] their rise or dissipate in any particular lune. In Tabl[illegible]re for the northern hemisphere

the total areas in each lune of the intermediate appearances of the groups which have been under observation, in three or more distinct rotations, the first and last appearances not being used. The total areas are also exhibited in the same table, for each lune, for all the 13- and 14-day groups in the northern hemisphere. It will be seen that there is an unfailing predominance of the eastern hemisphere

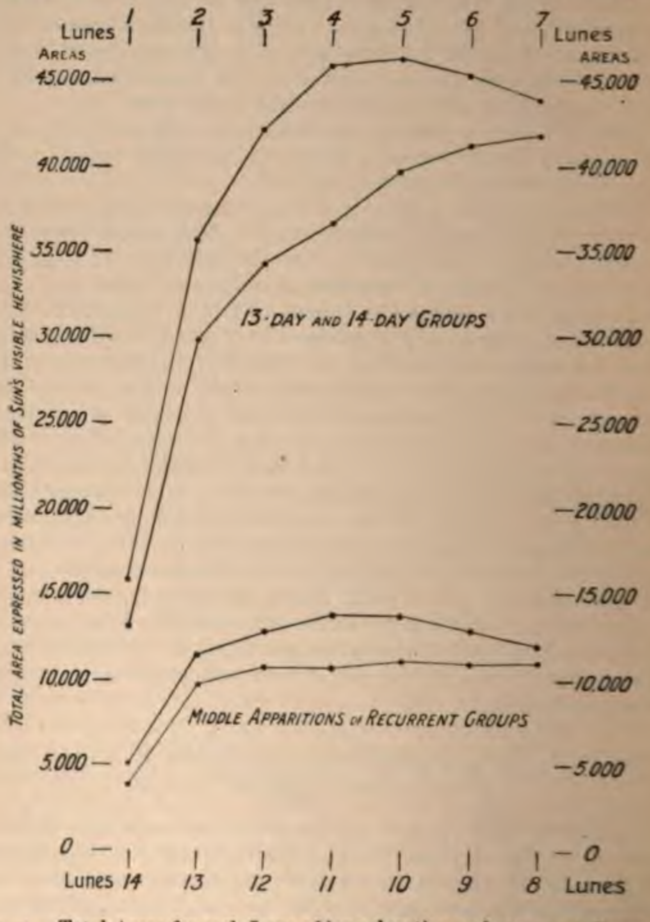


FIG. 4.—Total Areas for each Lune of long-duration and recurrent Groups.

over the western, but that the recurrent groups give a more regular curve than the 13- and 14-day groups. A comparison of the tw sets of figures shows that there is a slight heaping up of areas central lunes for the 13- and 14-day groups as compared recurrent groups, which is due no doubt to the influence phase." But nevertheless, in both cases the predomina

eastern areas over the western culminates in the central lune of each hemisphere. Fig. 4 represents Table XV. graphically.

Table XV.

Areas in each Lune of Middle Apparitions of Recurrent Groups in Northern Hemisphere.

Lunes.	1	2	3	4	5	6	7	East.
	14	13	12	11	10	9	8	West.
	5035	11401	12665	13800	13660	12934	12047	81542
	3824	9847	10815	10734	11092	11022	10953	68287
Diff. E. – W.	+1211	+1554	+1850	+3066	+2568	+1912	+1094	+13255

Areas in each Lune for all 13-Day and 14-Day Groups in Northern Hemisphere from Table VII.

	15677	35601	42280	45949	46129	45325	43888	274849
	13052	29913	34277	36616	39894	41311	41929	236992
Diff. E. – W.	+2625	+5688	+8003	+9333	+6235	+4014	+1959	+37857

9. *Summary.*

The foregoing tables show that there is a well-marked and steady preponderance of the eastern half of the Sun's disc over the western half, both as to the areas of the spots and as to the numbers of the separate groups. This excess in area of the eastern hemisphere over the western amounts to 17 per cent. for spots observed consecutively for 13 or 14 days; to more than 19 per cent. for recurrent spots during their time of greatest stability; and to 3 per cent. for the totality of areas of spots of all sizes and durations. As to the numbers of groups, the excess of the eastern hemisphere is 10 per cent. for ephemeral spots; is more than 9 per cent. if the groups are summed up as seen in the different lunes; and is 22 per cent. if we compare the groups seen in the two lunes nearest to the east limb with those seen in the two lunes nearest to the west limb. This last relation is the most striking which the inquiry has brought out. No fewer than 947 groups came into view round the east limb, or formed close to it, whilst only 777 disappeared round the west limb, or dissolved close to it. So that the difference of 170, or 22 per cent. of the number of disappearances, must be laid to the account of some influence exercised by the Earth. And these disproportions either in area or in number cannot be put down to any cause connected with the history or growth of the spots themselves, or to any solar cause whatever. East is east, and west is west, solely from an earthly point of view. From a solar standpoint, the "east limb," "central meridian," and "west limb" are purely conv [illegible] behind which every portion of the Sun's surface n [illegible] d landmarks upon the Sun. The

cause of the disproportion must then be terrestrial, and terrestrial only.

It must be terrestrial only, but it may also be apparent only, being due in some way or other to an effect of presentation.

Some error must creep in from the effect of foreshortening. Foreshortening pure and simple is, of course, corrected for in the ordinary process of reduction, but it is probable that a few minute corrections may be further required on grounds already alluded to. Further, some spots, large enough to be seen and measured when near the centre of the disc, will wholly escape notice if they happen to be near the limb. Thus foreshortening may affect to some slight degree not only the areas, but also the numbers of the spot groups.

But, from the nature of the case, the effect of foreshortening must be symmetrical, with respect to the central meridian, and no systematic difference between the eastern and western hemispheres can arise from this cause. "Spot phase" (as has been already shown) has a distinct asymmetrical effect, when the spots of any particular class are considered by themselves; but when the totality of all spots is considered, this asymmetrical effect must go out, since (unless we admit that the Earth has a real effect upon the Sun) it is not possible that any particular phase in the life-history of a spot should be associated with any particular meridian of the Sun as seen from the Earth.

It does not appear, therefore, that it can be a mere effect of presentation.

I have called this paper, therefore, "An *Apparent* Influence of the Earth on the Numbers and Areas of Sun-spots," because it is quite conceivable that there may be some physical characteristic of sun-spots, purely solar in its nature, which renders them more easily visible when they are approaching us than when they are receding. Thus if we suppose that sun-spots are holes in the photosphere of which the axes are generally sloped downward from west to east, or if we suppose that there is a great heaping up of the surface behind a sun-spot, we might have some explanation of the predominance of the eastern hemisphere both in the numbers and areas of sun-spots. Neither of these suggestions would, in my opinion, account for the relations which I have put in evidence, and there is another class of phenomenon which seems to show the same predominance of the eastern hemisphere over the western which could hardly be affected in the same way.

10. *Preponderance of the Numbers of Prominences in the Eastern Hemisphere.*

I had completed the foregoing statistics when it occurred to me to ascertain whether prominences were more frequently on the east or on the west limb, and in making this inquiry I came across a note by M. Sykora in *della Società degli Spettroscopisti Italiani* for 18 which he drew attention to the circumstance t

slight but unmistakable predominance of eastern prominences. I carried on the summatic of the prominences as given from the Catania Observatory in Sicily to the end of 1905, and found that from 1892–1900 inclusive the eastern hemisphere was the more prolific in every year. The tables were turned, however, with the year of minimum, 1901, and for four years the west was slightly the more favoured. In 1905 the east again predominated. It

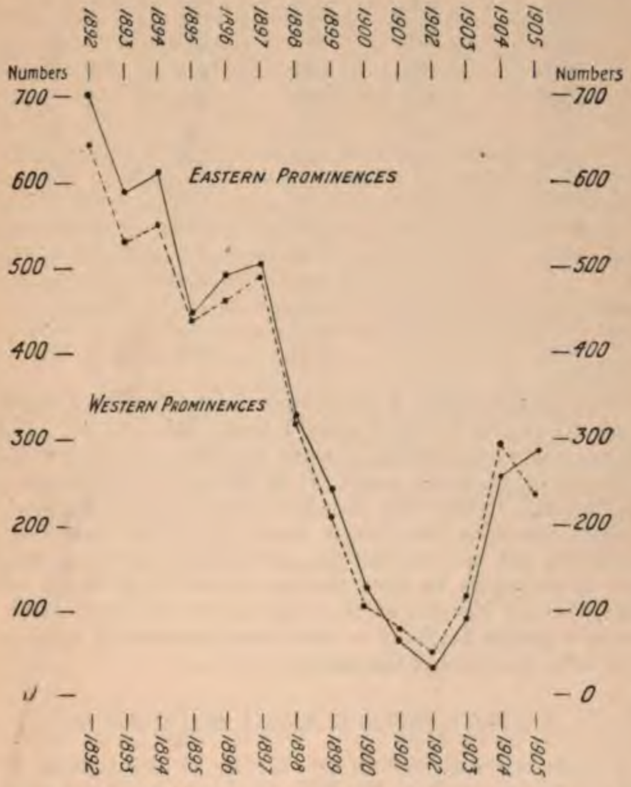


Fig. 5.—Numbers of Prominences as observed in different Years on the East and West limbs of the Sun at the Royal Observatory, Catania.

will be seen, therefore, that for the period with which my paper is concerned, namely, the cycle 1889–1901, the prominences showed the same feature as the sun-spots; and it seems very difficult to suggest any function of presentation, either connected with the solar constitution or with the conditions of observation, which [illegible] both phenomena in the same way. Table X[illegible] [illegible]it the comparison of the numbers of pr[illegible] [illegible]east and west limbs at the Catania O[illegible]

TABLE XVI.

Prominences observed at Catania in Sicily.

Year.	E.	W.	Total.	E.-W.
1892	700	641	1341	+ 59
1893	589	530	1119	+ 59
1894	613	552	1165	+ 61
1895	446	439	885	+ 7
1896	491	462	953	+ 29
1897	504	493	997	+ 11
1898	324	319	643	+ 5
1899	242	213	455	+ 29
1900	134	107	241	+ 27
1901	64	80	144	− 16
1902	33	49	82	− 16
1903	91	118	209	− 27
1904	256	295	551	− 39
1905	282	238	520	+ 44
Total, 1892–1905	4769	4536	9305	+ 233

The results to which I have come in the foregoing paper are not what I expected to find when I began this inquiry. Indeed, they are the very opposite of what has been supposed would be the result of planetary action, if it had any effect on the solar disturbances. It has been usually assumed that if a planet had any influence upon the Sun, it would be in the nature of spot production, not spot extinction. But the evidence of the foregoing tables seems to show that spots tend to diminish in area rather than to increase as they pass under the Earth, and that there is a decided tendency to check the generation of spots on the hemisphere presented to the Earth.

11. *Preponderance in Spotted Area at Apogee.*

I have no speculations to offer as to the nature of this terrestrial influence upon the solar spot groups. Were it in any way analogous to a tidal effect, we might expect to find some slight difference in the mean spotted area of the Sun in perigee and in apogee. In Table XVII., therefore, I have given the mean daily spotted area per month for two periods of eleven years each, namely, 1880–1890 and 1891–1901, and also for the twenty-two years taken together. The area in each month of any year has been expressed in thousandths of the sum of the twelve monthly mean areas for that year. On the whole, the summer months, when the Earth is most distant from the Sun, show the larger spotted area; May to August giving a mean of 86, whilst November to February give a mean of 79. There are, however, big

irregularities, as the comparatively short period of twenty-two years is insufficient to eliminate accidental irregularities.

TABLE XVII.

Mean Daily Spotted Area per Month, expressed in thousandths of the Total Area, for the Periods 1880–1890, 1891–1901, *and* 1880–1901.

Period.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1880–1890	74	76	78	95	84	89	93	73	89	89	93	67
1891–1901	87	93	67	75	84	88	93	95	96	78	62	82
1880–1901	81	85	72	84	84	89	93	85	93	83	76	75

This table I drew up several years ago, but it seemed to me so contrary to what one would naturally expect, that the spotted area should be greatest at apogee, that I did not then pursue the inquiry further. But in the light of the statistics that I have given in this present paper, it suggests that the inquiry is well worth pursuing with regard to the other planets.

The Earth is of such insignificant size, not merely as compared with the Sun, but even as compared with very many individual sun-spots, that its influence (if any such exists) on their number and size might well have been expected to be imperceptible. I commenced this investigation fully expecting to obtain a purely negative result; I was certainly not prepared to find that the effects of the Earth's apparent influence (of whatever nature this may be), would be so great, so striking, so consistent, and seen in so many directions.

69 *Tyrwhitt Road, St John's, Brockley, S.E.:*
1907 *May* 6.

Appendix added 1907 *May* 28.

It has been suggested to me that I might supplement the above tables by some statistics as to the numbers of groups that are born or die on the visible or invisible hemisphere respectively. This is done in Table XVIII., in which each group has been counted only once, no matter in how many rotations it has been seen. In other words, the numbers have been corrected for the recurrence of long-lived groups.

TABLE XVIII.

Born and dying on Visible Hemisphere.

	N.	S.	Whole Disc.
Short-lived groups	700	829	1529
Groups seen in more than one apparition .	39	55	94
Total . .	739	884	1623

TABLE XVIII.—*continued.*

	N.	S.	Whole Disc.
Born on Visible and dying on Invisible Hemisphere.			
Groups seen only in one apparition . .	107	129	236
Groups seen in more than one apparition .	25	39	64
Total . .	132	168	300
Born on Invisible and dying on Visible Hemisphere.			
Groups seen only in one apparition . .	166	225	391
Groups seen in more than one apparition .	43	36	79
Total . .	209	261	470
Born and dying on Invisible Hemisphere.			
Groups seen only in one apparition . .	28	31	59
Groups seen in more than one apparition .	21	22	43
Total . .	49	53	102
Grand Total . .	1129	1366	2495
Groups seen twice	128	152	280
Groups seen three or more times . . .	43	52	95
Total . .	1300	1570	2870

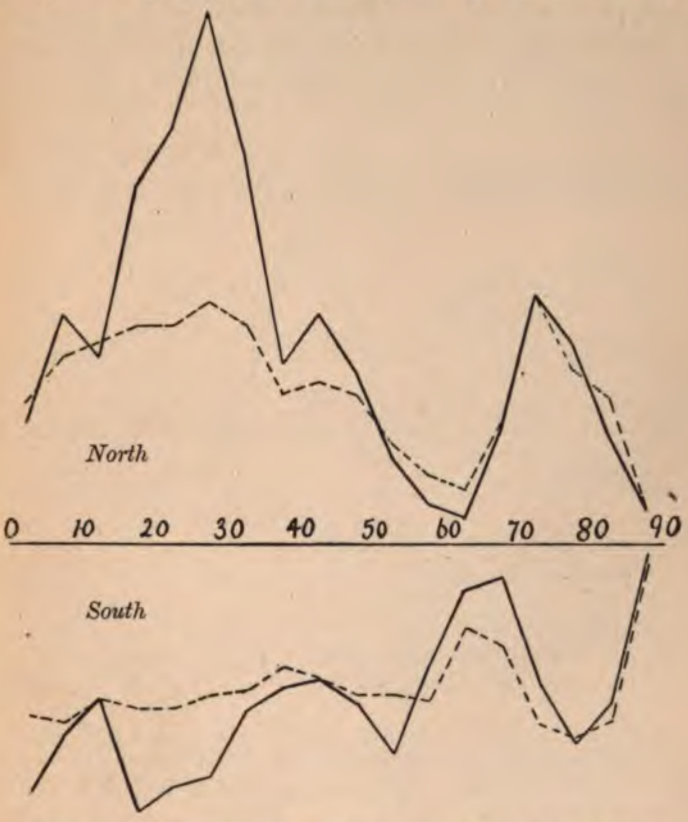
Neglecting therefore the short-lived spots that are born and die on the visible hemisphere of the Sun, we find

TABLE XIX.

	N.	S.	Whole Disc.
Born on visible hemisphere	171	223	394
,, invisible ,,	258	314	572
Dying on visible ,,	248	316	564
,, invisible ,,	181	221	402

Distribution of Prominences in Latitude in the Year 1906, *from observations made at Kodaikánal on* 156 *days in the first half of the year, and* 105 *days in the second half.* By J. Evershed.

The distribution of prominences in latitude for each half of the year 1906 is represented in the diagrams by two curves. The



Distribution of prominences in latitude—1906 January 1 to June 30.

Broken line = mean numbers } for each 5 degrees.
Continuous line = mean activity }

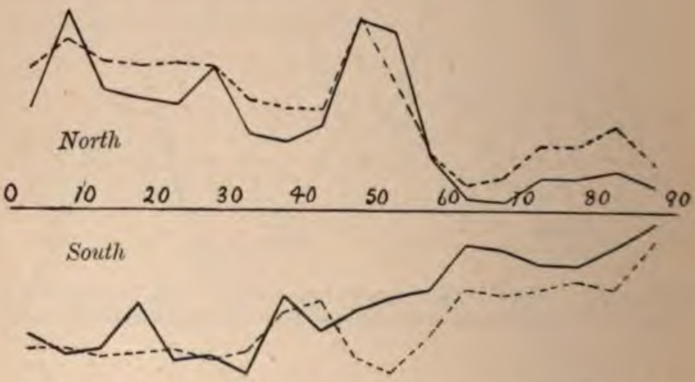
broken line gives the mean numbers of prominences observed in each zone of 5° from the equator to the poles. The continuous line gives the mean "activity" for the same zones. The activity is obtained by taking account of the height and extent of each prominence, the unit adopted being a prominence of 1° in extent and 10″ in height.

The average size of the prominences in each zone is indicated by the difference of the ordinates of the two curves: thus, at latitudes where the continuous line rises above the broken line in the northern hemisphere (or falls below it in the southern), the prominences were larger than the average for the whole year, and at the crossing points they were of average size.

General Remarks.

The distribution curve for the first half of the year has the same form as that of the previous year, with a maximum in the zone 25°–30°, and a well-defined zone of activity in high latitudes.

On both sides of the equator there is a very pronounced minimum of activity in the zones 55°–65°.



Distribution of prominences in latitude—1906 July 1 to December 31.

Broken line = mean numbers } for each 5 degrees.
Continuous line = mean activity }

About the end of June a considerable change took place in the distribution, and in the second half of the year the curve takes a very different form. There is also a great falling off in activity in both hemispheres.

The year as a whole is interesting, as showing the culminating point in the prominence period, when the high-latitude zones of activity finally envelope the poles. This took place last at about the epoch 1894·7. At that time, as in the past year, the zones of maximum activity were at latitudes 25°–30° on both sides of the equator, and during the past 12 years these regions of greatest activity have been slowly advancing towards the poles. In the years 1896 to 1901 they remained practically stationary between 45° and 55°, but since that date the advance polewards has been at the average rate of 5° per annum. New maxima appeared

between latitudes 20° and 30° in 1902, and the shifting of the old maxima towards the poles appears to be correlated with the development of these.

An interesting feature in these high-latitude prominences is the narrow limits of latitude within which at any one time the zone is defined, and the tendency to form extended chains of prominences all approximately in the same latitude, but often exceeding 100° in longitude. When this occurs one may get an apparently stationary prominence, remaining at the same position angle for 10 or 14 days, after which it appears on the opposite side of the pole for another series of days.

Observations of Jupiter's Sixth and Seventh Satellites from Photographs taken with the 30-inch Reflector at the Royal Observatory, Greenwich, in 1906–7.

(*Communicated by the Astronomer Royal.*)

The Sixth and Seventh satellites of Jupiter have been under observation at Greenwich from 1906 August 28 to 1907 April 6, and photographs have been taken at every available opportunity. In all, 55 photographs of J. VI. have been obtained on 28 nights, the observations extending over a period of 222 days, and 12 photographs of J. VII. on 7 nights extending over a period of 87 days. All these have been measured and reduced and the results are given in the present paper.

In addition to the satellites, eight reference stars, whose positions were taken from the Astronomische Gesellschaft Catalogue (Berlin B. Zone), were measured on each photograph and the constants determined in the usual manner. Right ascensions and declinations of the satellites were then determined, and by comparison with the tabular place of Jupiter, position angles and distances deduced.

Observations of Satellite VI.

Date and G.M.T. 1906.	Apparent R.A.	Apparent Dec.	Position Angle.	Distance.	Exp.
d h m s	h m s	° ′ ″	°	″	m
Aug. 28 15 23 29	6 22 42·60	+22 37 48·1	208·053	1712·8	28
31 15 24 32	6 24 58·69	+22 36 25·1	203·733	1672·8	45
Sept. 25 13 43 0	6 40 52·04	+22 26 21·9	158·992	1662·5	40
25 14 28 50	6 40 53·01	+22 26 21·4	158·935	1662·5	40
26 14 16 1	6 41 23·20	+22 26 5·7	157·130	1678·1	41
Oct. 13 14 2 11	6 48 14·10	+22 24 21·6	129·782	2137·5	54
16 13 44 50	6 49 3·42	+22 24 45·1	125·897	2245·7	
16 14 35 51	6 49 3·74	+22 24 44·9	125·910	2244·9	
19 16 28 9	6 49 46·76	+22 25 23·0	122·183	2363·6	

Observations of Satellite VI.—continued.

Date and G.M.T. 1906.			Apparent R.A.	Apparent Dec.	Position Angle.	Distance.	Exp.
	d	h m s	h m s	° ′ ″	°	″	m
Oct.	27	12 59 35(*a*)	6 50 58·60	+22 28 14·3	113·838	2670·0	13
	27	13 36 28	6 50 58·76	+22 28 14·8	113·817	2671·2	40
	27	14 38 59	6 50 58·90	+22 28 15·6	113·793	2671·8	60
	27	15 49 32	6 50 59·17	+22 28 16·6	113·755	2673·9	57
Nov.	17	11 3 37	6 49 29·92	+22 44 44·1	96·457	3336·5	30
	17	11 59 59	6 49 29·40	+22 44 46·7	96·425	3337·8	57
	22	12 44 17	6 48 4·59	+22 50 30·5	92·685	3409·5	20
	22	13 17 17	6 48 4·23	+22 50 32·8	92·655	3410·8	25
Dec.	6	9 20 42	6 42 9·44	+23 8 18·9	81·815	3294·6	30
	6	9 54 21	6 42 8·75	+23 8 21·1	81·793	3294·9	20
	6	11 3 46	6 42 7·21	+23 8 24·6	81·755	3293·2	10
	10	11 15 35(*b*)	6 39 53·79	+23 13 43·5	78·103	3159·1	20
	10	11 47 22	6 39 53·08	+23 13 45·6	78·080	3159·2	15
	10	12 45 49	6 39 51·69	+23 13 49·6	78·028	3157·9	80
	12	11 32 38	6 38 42·36	+23 16 21·6	76·103	3074·8	60
	12	12 27 24	6 38 40·96	+23 16 26·0	76·035	3073·1	30
	12	12 57 9	6 38 40·25	+23 16 27·7	76·017	3072·9	10
	12	13 13 32	6 38 39·82	+23 16 28·9	75·997	3072·0	10
	14	10 17 2	6 37 30·57	+23 18 53·6	74·009	2982·9	21
	14	10 50 10	6 37 29·66	+23 18 54·7	73·992	2980·9	31
	21	11 6 37	6 32 52·53	+23 27 20·9	64·962	2564·9	30
	21	11 31 13	6 32 51·81	+23 27 22·3	64·927	2563·7	15
	1907.						
Jan.	17	7 20 27	6 13 56·30	+23 43 16·7	329·545	1525·4	15
	17	8 36 17	6 13 54·33	+23 43 16·7	329·351	1527·2	120
	17	12 4 12	6 13 48·78	+23 43 16·3	328·750	1533·2	44
	17	12 56 35	6 13 47·42	+23 43 16·4	328·620	1534·6	32
	22	8 18 23(*c*)	6 10 50·63	+23 42 50·0	311·757	1799·9	30
	22	8 55 6	6 10 49·70	+23 42 49·0	311·657	1801·6	30
	22	9 33 19	6 10 48·78	+23 42 48·4	311·570	1803·0	15
Feb.	1	10 40 41(*d*)	6 5 41·74	+23 39 30·8	290·539	2444·0	15
	1	11 0 38	6 5 41·43	+23 39 29·9	290·511	2444·1	10
	2	10 6 30(*e*)	6 5 16·97	+23 39 5·0	289·135	2501·9	16
	7	7 41 50	6 3 27·65	+23 36 34·0	282·900	2757·6	60
	7	8 55 56	6 3 26·65	+23 36 31·4	282·825	2759·6	60
	7	9 44 6	6 3 25·93	+23 36 31·5	282·807	2762·2	5
	7	9 51 34	6 3 25·87	+23 36 30·3	282·782	2761·7	7

(*a*) Faint and diffused.
(*b*) Very faint. On the réseau line.
(*c*) Bad image; a little out of focus.
(*d*) Very faint.
(*e*) Nearly coincident with a faint star.
(*f*) Very diffused.

Observations of Satellite VI.—continued.

Date and G.M.T. 1907.					Apparent R.A.			Apparent Dec.			Position Angle.	Distance.	Exp.
	d	h	m	s	h	m	s	°	′	″	°	″	m
Feb.	8	10	15	13	6	3	6·36	+23	35	56·1	281·637	2809·0	120
	8	11	37	27(*d*)	6	3	5·36	+23	35	56·3	281·620	2811·1	15
	11	9	10	1	6	2	16·20	+23	34	17·5	278·632	2929·8	86
	15	10	6	39(*d*)	6	1	22·76	+23	31	57·9	274·947	3064·0	24
Mar.	4	8	6	52(*f*)	6	0	48·89	+23	22	33·6	262·460	3238·6	120
	4	9	36	14	6	0	49·25	+23	22	31·6	262·417	3239·1	30
	10	9	55	30	6	1	47·39	+23	19	37·3	258·460	3181·2	106
	11	8	33	1	6	1	59·59	+23	19	9·0	257·800	3168·5	147
	11	10	22	0(*d*)	6	2	0·62	+23	19	9·2	257·792	3166·9	30
	18	8	57	3	6	3	55·59	+23	16	14·2	253·305	3042·7	65
Apr.	6	8	19	43	6	12	28·97	+23	9	30·2	239·872	2528·5	60

(*d*) Very faint.
(*f*) Very diffused.

Observations of Satellite VII.

Date and G.M.T. 1906.					Apparent R.A.			Apparent Dec.			Position Angle.	Distance.	Exp.
	d	h	m	s	h	m	s	°	′	″	°	″	m
Nov.	17	11	59	59	6	47	58·03	+22	53	51·8	85·408	2057·2	57
	22	12	44	17	6	46	6·94	+22	59	28·8	78·242	1815·5	20
	22	13	17	17(*a*)	6	46	6·29	+22	59	31·4	78·165	1813·3	25
Dec.	12	11	32	38	6	35	32·88	+23	24	12·6	17·282	1257·2	60
	12	12	27	24(*b*)	6	35	31·48	+23	24	15·7	17·150	1258·1	30
1907.													
Jan.	17	8	36	17	6	11	58·73	+23	55	37·9	311·093	3134·5	120
	17	12	4	12	6	11	53·68	+23	55	43·1	311·010	3143·7	44
	17	12	56	35(*b*)	6	11	52·39	+23	55	42·8	310·965	3145·2	32
Feb.	7	7	41	50	6	2	44·02	+24	0	14·6	301·897	3860·6	60
	7	8	55	56(*c*)	6	2	43·04	+24	0	17·1	301·903	3863·8	60
	8	10	15	13	6	2	25·27	+24	0	15·0	301·542	3878·8	120
	11	9	10	1	6	1	41·17	+24	0	13·5	300·695	3915·7	86

(*a*) Faint and diffused.
(*b*) Faint.
(*c*) Almost coincident with a faint star.

In deducing the position angle and distance giv error of the tabular place of Jupiter has been neglec

To determine this error a number of photogra were taken with the 26-inch Refractor, using the oc By this means good measureable images of Jupite together with sufficiently exposed images of the The field of the 26-inch, on a 16 c.m. plate, is o and all the stars in this area given in tl

Gesellschaft Catalogue were measured together with Jupiter, and the position of Jupiter deduced with reference to them.

It will be observed that the deduced place of Jupiter will be affected by the systematic error of the Catalogue, and the error due to unknown proper motions, and for that reason it is directly comparable with the positions of the satellites deduced in the same manner.

Five photographs were selected for measurement, and the results are given below.

Errors of Tabular Place (T – O).

1907.	R.A.	Dec.
	s	″
Feb. 28	– ·06	+0·2
Mar. 1	– ·06	+0·4
1	– ·05	–0·2
20	– ·05	–0·7
21	– ·07	+0·1
Mean	– ·058	–0·04

The photograph on March 20 is unsatisfactory.
The tabular place is taken from the *Nautical Almanac.*

Royal Observatory, Greenwich:
1907 *May* 10.

Note on the Spectrum of α Orionis. By H. F. Newall and B. Cookson.

Titanium Flutings in the Red End of the Spectrum of α Orionis.

A photographic study of the red end of the spectrum of some of the brighter stars has been made during the past two months at the Cambridge Observatory with a four-prism spectrograph attached to the 25-inch refractor. It has led to the discovery, amongst other things, of a marked feature in that part of the spectrum of *α Orionis*, viz. three strong flutings, which in this note are interpreted as absorption flutings, with heads towards the blue end. Only a preliminary determination of the wave-lengths of these heads has been made; for, on account of the paucity of lines in the spark spectrum of iron, which has been used as a comparison spectrum, some uncertainty—to the extent of about 2 tenth metres—is attached to the identification of a red line near 7155, which has been used as one of the reference lines.

This line 7155 and the stellar flutings are in truth outside of the range of spectrum, for which the four-prism spectrograph had

been adjusted, and the definition is not what it ought to be for a careful determination of wave-length.

However, the interest of the matter lies in the facts (i) that the provisional values deduced for the wave-lengths of the heads of the flutings are—

7053
7087
7124

and (ii) that Hale and Adams have in their paper (*Astroph. Jour.*, xxv., 77) given the wave-lengths of three absorption flutings which they have found in the spectrum of Sun-spots, and have identified with bright flutings found by them in the spectrum of the flame of the titanium arc, with heads at wave-lengths—

7054·6
7088·0
7125·9

Thus a chain of evidence seems to be completed :—

(*a*) Professor Fowler discovers many flutings in the flame of the titanium arc and points out the close resemblance, in position and intensity, between these bright flutings and the absorption flutings seen in stellar spectra of type III. (like *α Orionis*).—(*Proc. R.S.* lxxiii. 219.)

(*b*) Messrs Hale and Adams have proved the close resemblance between those features which distinguish both Sun-spot spectra and the spectrum of *α Orionis* from the solar spectrum, viz. certain lines which are present in both Sun-spots and *α Orionis*, with intensity accentuated relatively to the corresponding solar lines (*Astroph. Journ.*, xxiii. 400). (Some further evidence on this point is given in the later paragraphs of the present note; it tends to support the contention of Messrs Hale and Adams.)

(*c*) Messrs Hale and Adams, in their latest note (*Astroph. Jour.*, xxx. 77) on Sun-spot spectra, announce that they have been able to identify flutings in the Sun-spot spectrum as reversals of the bright flutings at the red end of the spectrum of the flame of the titanium arc.

(*d*) The object of the present note is to announce that these dark flutings have been detected in photographs of the red end of *α Orionis*.

Attempts have been made to detect these red flutings in *β Geminorum* and *α Boötis*; but the material so far collected is not quite good enough to establish their presence or absence with complete certainty.

Sun-spot Lines in the Green Region of the Spectrum of α Orionis.

Three photographs of the green region of the spectrum of α *Orionis* were taken by Mr H. J. Bellamy in January 1905 with four-prism spectrograph. They have been recently measured

Mr W. H. Manning, and the measures have been discussed with a view to seeing whether the lines which were relatively stronger in *a Orionis* than in the Sun were Sun-spot lines. The regions covered are as follows :—

Plate No.	Date 1905.	Limits of Spectrum studied.
Fmg. 680	Jan. 13	5098–5328
Fmg. 684	Jan. 14	5108–5324
Fmg. 696	Jan. 27	5341–5521

Within these regions many dark (absorption) lines were measured, and also some apparently bright (emission) lines, though it was difficult to say whether these latter were really bright lines or only bright interstices of continuous background between dark absorption lines.

The wave-lengths of all the lines measured are given in the table below. They have been corrected for velocity: up to and including 5328·28 the tabulated values are in general the means of the two plates Fmg. 680 and Fmg. 684; the rest are from Fmg. 696. The scale of the original photographs is 1 mm. = 25 Ångström units.

Photographs of this region of the spectrum of the Sun and of Sun-spots were taken last summer, and it was thus possible to compare the star lines with solar lines and see whether they were lines that were intensified in spots. The results of comparison with three different photographs of Sun-spot spectra (Nos. R.S. 187, 188, 189) are given in the table. A cross (×) indicates that the line was relatively stronger in the spot than in the Sun; o that it was not; ? that intensification in the spot was doubtful. Unfortunately there is a gap in our series of photographs of Sun-spot spectra between 5341 and 5600.

A list of Sun-spot lines was published in *Astroph. Jour.*, xxiii. 15–27, by Hale and Adams, and the fourth column shows those lines which are included in their list, and are considered by them as lines affected in spots.

This table shows clearly that a large number of the star lines are also spot lines. The following lines are almost certainly not spot lines:

All the bright lines marked B.
Lines 5133·61, 5145·44, 5273·21.

The so-called bright lines cannot be identified for certain: it was thought they might be reversals of the Sun-spot fluting of which the wave-lengths are given in *M.N.*, Dec. 1906, but this view has not been upheld. It seems probable that they are not really bright lines.

The other three are not well-marked lines:

5133·61 is very faint and hazy.
5145·44 is fairly strong but very hazy on the red side.
5273·21 is extremely faint.

The last four lines in the table are not very well seen on the

Wave-Lengths of Lines in α Orionis.

α Orionis λ in star.	Sun-spot Lines: R.S. 187. / R.S. 188.	Sun-spot Lines: R.S. 189.	Sun-spot Lines: Hale and Adams.
5099·10 *	×		o
5107·72	×	×	o
5110·56	?	×	×
5123·71	?	×	×
5127·75	o	×	?
5133·61 *	o	o	o
5135·66 *B			
5137·20	o	o	×
5138·23 B			
5139·44 *	?	?	×
5140·86 *B			
5143·03	×	?	o
5144·32 B			
5145·44	?	?	o
5147·82	×	×	×
5169·12 *	?	?	×
5190·29 *B			
5203·62	o	×	o
5208·59	×	?	×
5210·52	×	×	×
5219·93	×	×	o
5227·30	?	×	×
5228·93 B			
5230·25	×	×	×
5231·64 *B			
5247·31	×	×	×
5250·40	o	×	×
5255·27	×	×	×
5264·22	×	×	×
5265·21 *	o	×	×
5266·28	×	?	×
5267·91 B			
5269·92	?	?	×
5273·21 *	?	?	o
5275·56 *	×	×	×
5300·75	×		×
5324·32 *	?	o	×
5328·28	?		o

α Orionis λ in star.	Hale and Adams.
5341·17	×
5371·80	×
5395·01	×
5397·23	×
5406·04	×
5409·94	×
5429·92	×
5434·91	o
5447·08	×
5455·95	o
5490·92	×
5497·84	×
5501·61	o
5506·83	×
5512·49	
5514·68	×
5517·51	
5520·97	

* denotes that the line was measured on one plate only.
B denotes hypothetical bright lines.

only plate on which they can be measured, and their wave-lengths may be liable to errors amounting to 0·25 λ. Only one of them seems to be a Sun-spot line.

In their paper, *Astroph. Jour.*, xxiii. 400–405, Hale and Adams give the wave-lengths of lines in the spectrum of *α Orionis* from λ = 5393–5703. Out of 25 lines given by Hale and Adams, all but three can be identified; and three lines appear which are not given by them. The three lines which cannot be identified are 5393·36, 5407·62, 5418·99. The first is a very doubtful spot line, the second is a line much intensified in spots, the third is not intensified in spots. According to Hale and Adams, the first is a strong line in the star spectrum, the second is a fairly strong line, and the third is weak. The absence of the first two lines is remarkable, and suggests the possibility of change in the spectrum of the star.

Of the three lines not given by Hale, 5520·97 is the strongest, though it is only just measurable: the other two are very weak and not measurable; their approximate wave-lengths are 5449 and 5474.

Flutings in the Green Region of the Spectrum of α Orionis.

Measures have been made of the wave-lengths of two absorption flutings sharp on the violet side and fading off towards the red; the heads of these two are at 5166·8 and 5447·1. The first of these flutings is the more marked, possibly on account of the fact that the lines b_4 and b_3 are involved near the head. The second also has a line at its head, whose wave-length is given in the table. According to Professor Fowler, the wave-lengths of the heads of the two strongest titanium flutings are 5167·5 and 5447·0, and their intensities he gives as 10. There seems little doubt then that the two flutings in *α Orionis* are titanium flutings. There are in this region 5 other titanium flutings, whose wave-lengths and intensities are given by Fowler, namely:

5241·0	intensity 5
5308·0	3
5356·6	4
5407·0	1

All of these are to be found in the spectrum of *o Ceti*, but there is no certain indication of their presence in *α Orionis*.

On the Presence of Tin in Stellar Atmospheres. By Joseph Lunt, B.Sc., F.I.C., Assistant at the Royal Observatory, Cape of Good Hope.

During the course of measurements of stellar radial velocities, Mr A. W. Goatcher happened to measure a line at $\pm\lambda$ 4525 on 14 negatives of the spectrum of α Scorpii. He pointed out that the line in question was markedly discordant, giving a velocity which was more than 6 kilometres per second too low (positive) in the mean, when the wave-length of the stellar line was assumed to be λ 4525·285, corresponding to a blend of the two lines in Rowland's tables at λ 4525·110 Int 0, origin unknown, and λ 4525·314 Int 5, due to iron.

On searching for some cause for this pronounced discrepancy, I found that a strong tin line occurred on the ultra-violet side of the iron line.

Exner and Haschek place its wave-length at λ 4525·00 Int 30, and they give another line at λ 4585·80 Int 20, but they record no other lines brighter than intensity 1 in the region covered by the Cape 4 prism spectrograph.

On taking photographs of the spectrum of tin, with electrodes of the pure tin "fuse wire" ordinarily used in electrical circuits, it was found that whilst the λ 4525·00 line came out strongly, the λ 4585·80 line was entirely absent, and in fact no other line appeared except a faint line at λ 4227 due to a trace of calcium.

This was due to the fact that self-induction was used in the secondary circuit of the 18-inch spark induction coil, as is usual in producing the iron spark as a source of the comparison spectrum for our stellar spectra.

On eliminating the self-induction, the λ 4585·80 line came out much more strongly than the λ 4525·00 line, differing in this respect from Exner and Haschek's relative intensities; but with the exception of air lines no other lines of importance appeared.

The line of lower refrangibility is, in fact, the strongest enhanced line of tin recorded by Lockyer, whilst the λ 4525 line is absent from his list of enhanced lines.

It is evident that the λ 4585 line is not likely to occur in the cooler stars such as α Scorpii, and we are reduced to a singl[illegible] in our 4-prism region in searching for evidence of the pre[illegible] tin in these stars.

The tin line from the "fuse wire" was carefully m[illegible] gave the wave-length λ 4525·01, which shows that it i[illegible] to coincide with Rowland's solar line λ 4525·009 Int [illegible] the line closer to the iron line, viz. λ 4525·110 Int 0.

If we assume that the Sn and Fe lines have [illegible] 3 and 4 respectively in α Scorpii instead of 00 [illegible] Sun, and wave-lengths λ 4525·009 and λ 4525·[illegible] Rowland's solar wave-lengths, we can well represe[illegible] which as a blend would then have the mean wave-l[illegible]

Photographs of the tin and iron spectra taken on the same plate show the blended lines as a single unresolved broad line with the dispersion employed, just as is shown in the stellar spectra.

We must therefore conclude that, so far as our limited range of spectrum is concerned, a line corresponding to the one and only line of tin which seems likely to occur in the cooler stars is certainly present, and further confirmation of the existence of this metal in stars should be looked for outside the region of spectrum here considered, particularly in the region of longer wave-lengths.

Professors Hale and Adams * have secured with the Snow telescope a spectrum of α Orionis which extends to λ 5703·73. It will be of much interest to ascertain whether the arc line of tin in the yellow region at λ 5631·91 (Kayser and Runge) is shown in their photograph of the spectrum of this star. It occurs in a region of the solar spectrum which is free from strong lines or groups. It is shown as a strong line in Hagenbach and Konen's Atlas, and has also been recorded by Huggins, Thalén and Lecoq de Boisbaudran.

In the Harvard Annals a line is recorded at λ 5632·0 as being stronger in α Orionis than in the Sun, α Aurigæ and α Boötis.

The star line at λ 4525 has been used in radial velocity determinations, once by Frost † in β Leporis and once by Belopolsky ‡ in γ Cephei, and in each case the line gives a lower result than any other on the plate. These two stars belong to classes G and K respectively, according to the Harvard classification, and the discordance of the tin-iron line is therefore likely to be less marked than in α Scorpii.

The recorded evidence of the presence of tin in the Sun appears to be very meagre and doubtful.

In "Rowland's table of solar elements," § arranged according to number of lines associated with them, tin appears at the end of the list amongst elements showing only one or two coincidences between the lines of their spectra and solar lines.

In Rowland's "Preliminary table of solar spectrum wavelengths," the only references to Sn seem to be the following three lines :—

Rowland's.			Kayser and Runge (arc).		Exner and Haschek (spark).	
λ	Int.	Origin.	λ	Int.	λ	Int.
3262·409	3	Fe-Sn	3262·44	2	3262·48	30
3330·745	000	Sn?	3330·71	3	3330·83	6
3801·163	000	Sn?	3801·16	3	3801·32	20

These are outside the 4-prism region of the Cape Spectrograph, and no photographs of star spectra are here available to search for these lines in the spectra of the cooler stars.

* *Ap. J.*, vol. xxiii. p. 400.
† *Ap. J.*, vol. xviii. p. 257.
‡ *Ap. J.*, vol. xix. p. 101.
§ *Problems in Astrophysics*, Clerke, p. 27.

Royal Observatory, Cape of Good Hope:
1907 *March* 27.

Note on the Range in Brightness at Maximum of Long-period Variables. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In an interesting letter dated April 16, commenting upon the paper "On the Classification of Long-period Variable Stars, etc." (*Mon. Not.*, lxvii. p. 332), Mr C. L. Brook draws attention to the possibility of explaining the well-known differences between separate maxima in terms of the hypothesis put forward in the paper. His words are as follows:—

"Not only does the amount of solar disturbance vary greatly at different Sunspot maxima, but also the two hemispheres differ to a considerable extent. . . . Now if the Sun were viewed poleward instead of equatorially, the effect of this would be to further increase the difference that already exists in the intensity of the various maxima: we should either see the whole of the then more disturbed hemisphere or the whole of the less disturbed hemisphere, and not as now half (about) of each. If then the analogy between the Sun and the long-period variables holds in this particular, then the maxima of those variables which we view polewards should differ to a greater extent in intensity than the maxima of those which are viewed equatorially."

2. A rough test of the interesting corollary indicated by Mr Brook is simply made. On pages 351 and 352 of the paper above quoted are two lists of twenty stars from Chandler's Revised 3rd Catalogue which should be (on the hypothesis put forward) stars viewed polewards and stars viewed equatorially. We need only take from Chandler's 3rd Catalogue (*A.J.*, 379) his tabulated range of magnitude at maximum (not repeated in the Revision, *A.J.*, 553). Thus for X Ophiuchi (No. 6682), Chandler gives the magnitude at maximum as 6·8–7·0, or a range of 0·2 magnitude; and in most cases he gives figures of this kind; while the range is often much larger, as for *o* Ceti (No. 806), where he gives 1·7–5·0, a range of 3·3 magnitudes. In a few cases he gives a single value only, as for RS Libræ (No. 5511), where he sets down 8·2 simply: and in the subjoined table the range has in such cases been set down as 0·0; for though further observations may show an appreciable range, it will no doubt remain small.

3. The following table will be understood without explanation; means have been taken for each ten star will be seen that for the extreme groups of ten the diff range is quite marked, but is *in the opposite sense* to that by Mr Brook.

4. It is not, however, impossible to frame an exp this table in terms of the hypothesis put forward. If th in the two hemispheres are related by some system of a if, for instance, the total quantity of bright, flocculous star at maximum were always the same, but distri greater part sometimes in one hemisphere and som other—then the view enunciated by Mr Brook is th

Range at Maximum of the Forty Variables tabulated on pages 351–2.

Large Values of α or "Polar" Stars.			Small Values of α or "Equatorial" Stars.		
α	Star's No.	Range at Max.	α	Star's No.	Range at Max.
+·25	6682	0·2	−·63	1582	0·7
+·22	8591	0·2	−·42	112	3·0
+·19	5511	0·0	−·35	2528	1·2
+·18	5795	1·2	−·34	5504	1·7
+·17	5887	0·5	−·34	4315	0·6
+·16	3637	0·0	−·31	2946	2·3
+·13	715	0·7	−·27	3825	2·2
+·12	107	1·0	−·27	6871	0·0
+·11	7659	0·9	−·27	7045	2·1
+·10	7779	1·8	−·27	243	0·8
	Mean .	. 0·65		Mean .	. 1·46
+·08	7609	1·6	−·26	5095	0·3
+·08	1717	1·1	−·25	806	3·3
+·07	7577	1·0	−·23	2539	0·7
+·07	7754	1·3	−·22	4492	1·4
+·07	5338	0·9	−·22	3170	1·2
+·05	7428	2·7	−·22	6849	1·5
+·05	5190	1·4	−·20	2100	1·1
+·03	1855	1·3	−·20	5194	0·7
+·03	294	0·0	−·19	434	1·1
+·02	6905	1·0	−·19	976	0·7
	Mean .	. 1·23		Mean .	. 1·20

Viewing the star equatorially, we should get the combined brightness of both hemispheres, which would be nearly the same; viewing the star polewards, we should get the full effect of the alternation between hemispheres.

5. Suppose, however, that the variations in the two hemispheres are independent of each other: or, if related, related on the principle of sympathy rather than that of alternation. In either case we might have them both bright (at maximum) or both faint (at maximum) together; and the range for equatorial view would not be any smaller than for polar view. But it would not, in the long run, be greater. Within a limited period it might seem to be greater, because one hemisphere might be regular for a limited period: and if the star were seen poleward the maxima

would resemble each other, whereas if seen equatorially the irregularities in the other hemisphere would come in. The problem is very similar to the well-known problem of bimetallism.

6. There is, however, in our problem a factor not represented in that of bimetallism. Increased activity may mean the production of increased clouds of absorbing Corona, and the effect of this on the maximum might be different in different aspects. It would be unprofitable to speculate on the importance of this unknown factor in the present state of our knowledge. It seems better simply to call attention to the above fact. It is, on the face of it, unfavourable to the hypothesis put forward: but a direct contradiction is more satisfactory than a fact which goes neither one way nor the other, since there is always the possibility that a simple inversion of some kind has been overlooked.

Elements of the Variable Star RV Andromedæ. R.A. = $2^h\ 4^m\ 34^s$, Decl. = $+48°\ 27'{\cdot}6$ (1900). By A. Stanley Williams.

A period of 182 days was suggested for this variable star in 1904 by the writer, from a comparison of a single visually-observed minimum in January of that year with some earlier photographic observations.* From the observations made here last year, combined with two maxima and two minima observed by Professor A. A. Nijland at Utrecht,† the following much more satisfactory elements of variation have been derived:

$$\text{Maximum} = \left\{ \begin{array}{l} \text{1904 April 26} \\ \text{J.D. 2416597} \end{array} \right\} + 170^{d}{\cdot}2\ \text{E}$$

the interval M − *m* being 82 days.

The above period was derived from five observations of four maxima and seven observations of six minima, the periods resulting from the maxima and minima being respectively 172·2 days and 166·2 days. The observations of the minima were more numerous than those of the maxima, and extended over a longer interval of time; but the maxima were more sharply defined than the minima, and the observations also more accordant, so that double weight was given to the determination from the maxima.

Quite recently the dates of two additional maxima and additional minima have been published by Nijland;‡ and of two maxima and a minimum by Mr F. H. Seare observations made by Mr E. S. Haynes at the Laws Obse Table I. below contains the particulars of all the observed together with the computed dates according to the elements, and the differences O − C. Table II. gives

* *A.N.*, No. 3944. † *A.N.*, No. 4116. ‡ *A.N.*,
§ *Bulletin No.* 10 *of the Laws Observatory*, p. 147. Se
period of 169 days for the star.

sponding particulars for the minima. In both tables the observations published subsequently to the derivation of the elements are marked with an asterisk.

TABLE I.

Observed and Computed Maxima.

E	Obs. Max.	Comp. Max.	O – C.	Obs. Mag.	Observer.
			d		
1	2416767	2416767·2	– 0·2	8·3	Williams
2	6934	6937·4	– 3·4	9·0	Nijland
3	7103	7107·6	– 4·6	8·0	Williams
3	7105	7107·6	– 2·6	8·9	Nijland
4	7289*	7277·8	+11·2	8·9	Nijland
5	7445*	7448·0	– 3·0	...	Haynes
5	7453*	7448·0	+ 5·0	9·1	Nijland
5	7455	7448·0	+ 7·0	8·2	Williams
6	7609*	7618·2	– 9·2	...	Haynes

TABLE II.

Observed and Computed Minima.

E	Obs. Min.	Comp. Min.	O – C.	Obs. Mag.	Observer.
			d		
0	2416509	2416515·0	– 6·0	10·1	Williams
1	6694	6685·2	+ 8·8	10·2	Williams
2	6875	6855·4	+19·6	10·4	Williams
3	7045	7025·6	+19·4	11·1	Nijland
4	7174	7195·8	–21·8	10·8	Nijland
4	7177	7195·8	–18·8	10·3	Williams
5	7341*	7366·0	–25·0	10·8	Nijland*
6	7514*	7536·2	–22·2	...	Haynes
6	7518*	7536·2	–18·2	11·3	Nijland
6	7518	7536·2	–18·2	10·6	Williams

The residuals in the fourth columns of the preceding tables are somewhat large, particularly those in Table II., but having regard to all the circumstances, it does not seem that any important correction to the foregoing elements of variation is indicated by the subsequently published observations. One somewhat interesting feature is, however, pointed out. The progressive nature of the change in the residuals of Table II. seems to show that the interval from minimum to maximum is not constant in the case of this star, but varies to a very appreciable extent from time to

* Nijland also observed a secondary minimum on J.D. 2417405 ($O - C = +61^d$), with an intervening secondary maximum, about half a magnitude brighter.

time. In other words, this would mean that the form of the light curve is, doubtless, within limits, in a state of more or less constant change. It is possible that the change here referred to may be due, in part at least, to the existence of a periodical irregularity in the length of the period of variation, but the observations hardly favour this, though they have not yet been continued long enough to settle the question. It should be noted that the largest and most important change indicated by the residuals of Table II. is supported by the accordant observations of two different observers, so that there cannot be much doubt as to its reality.

With regard to the magnitudes in the penultimate columns of the tables, it should be remarked that the systematic difference between Nijland and the writer is probably due to a difference of scale. The comparison stars made use of at Utrecht were different from those used here. In particular, Nijland used the star BD. $+47°\cdot573$ ($8^{m}\cdot3$) when the variable was bright, whereas at such times the star BD. $+47°\cdot571$ ($8^{m}\cdot2$) was employed by the writer. At the Laws Observatory the comparisons were made with another different star, BD. $+48°\cdot612$ ($9^{m}\cdot3$), Seares stating that Haynes's observations show a range of $2^{m}\cdot8$, with a maximum $1^{m}\cdot0$ brighter than the last mentioned star. Probably we shall not be far wrong if we adopt a range of variation from $8\frac{1}{2}$ mag. to 11 mag.

The light curve of this variable, according to the observations made here, is very acute at the time of maximum, and the decline very quick. But after the minimum there is sometimes at first only a slight increase in brightness, followed by a long standstill; or even by a secondary minimum, with an intervening secondary maximum, as observed by Nijland in 1906.

The spectrum of RV Andromedæ appears to be peculiar. Thus the Rev. T. E. Espin recorded it as III !!! in 1893,* whilst Mr F. Krüger described it as being III !! (?) in 1902, the bands appearing broad and deep, and the blue and violet wanting.†

On the Variable Star RX Andromedæ. R.A. = $0^{h}\ 58^{m}\ 55^{s}$, Decl. = $+40°\ 46'\cdot2$ (1900). By A. Stanley Williams.

The observations of this variable star, which were m[illegible] in the year 1905, have already been published in full in [illegible] Notices, vol. lxvi. pp. 336, 337. These showed that the [illegible] that year underwent many sudden and irregular variations in [illegible], of such a character that, taken in connection [illegible] observations, it seemed as though the variable [illegible] as belonging to the highly-remarkable type of [illegible]

Between July 24 and December 21 of last [illegible] of the brightness of the star were made on [illegible]

* *A.N.*, No. 3231. † *A*[illegible]

history during the period of time covered by the observations proved of a most uneventful and quiet character. With the exception of the few occasions hereafter more particularly referred to, when a comparatively slight increase in brightness was manifested, the variable remained almost unchanged at about 9 of the provisional light scale,* corresponding to 11·6 mag., that is, at about its minimum brightness.

It seems unnecessary, under the circumstances, to set out all these observations in full, and I therefore give below, in Table I., simply the dates when the star was observed faint or near its minimum brightness.

TABLE I.—*Dates when RX Andromedæ was observed faint.*

1906 Aug. 19, 21, 22, 25, 27, 28, 29, 30, 31; Sept. 1, 6, 9, 10, 15, 16, 19, 20, 22, 24, 25, 26, 27, 29; Oct. 23, 25, 27, 28; Nov. 11, 15, 17, 19, 22; Dec. 21.

TABLE II.

Observations made when RX Andromedæ was bright.

Date.	G.M.T.	Observations.	Sky.	Brightness.
1906 July 24	13^h 10^m	d 3 v 8 e, c 3 v	IV.	19·5 }
25	13 34	d 7 v 5 e, c 8 v	IV.	17·6 }
27	13 25	d 7 v 3 e, c 9 v	IV.	16·7 }
Aug. 16	11 46	d 8 v 1 e	IV.	15·6
Sept. 11	11 33	d 3 v 4 e, c 2 v†	IV.	18·8 }
,,	13 47	d 2 v 5 e, c 2 v	IV. ☽	19·4 }
12	16 03	d 5 v 0 e, c 6 v	IV. ☽	15·3 }
Oct. 5	15 47	d 5 v 0 e‡	III. ☽ ☽ ☽	15·2 }
6	12 57	d 4 v 5 e	III. ☽ ☽	18·1 }
10	9 38	v 2 d, v 9 e, v 2 c	III.	23·4 }
13	10 07	d 6 v 0 e	III.	14·9 }
14	8 46	d 3 v 3 e	IV.	17·8 }
Dec. 21	10 58	d 5 v 2 e	III.	16·6

The last column gives the brightness according to the provisional light scale. It will be seen that none of the observations of Table II. make the star very bright, and although it is possible that in one or two cases the actual maximum may have happened at a time when there are no observations available, yet this certainly could not have been the case as regards the temporary increase in brightness that occurred on September 11 and 12, since there are observations both before and after these dates showing that the

* *Monthly Notices*, vol. lxvi. p. 336, Table I.
† Light of variable very unsteady.
‡ Sky bright and stars faint but variable, certainly fainter than *d*.

star was then faint. It would seem, therefore, that we are justified in concluding that during the latter half of 1906 the variable remained persistently faint, and nearly at its minimum brightness, during the greater part of the time, and only underwent occasional and comparatively feeble augmentations of brightness, or abortive maxima. The *U Geminorum* type of variation of the star is confirmed by the recent observations, though there seem to be minor differences between all the stars of this class. The range of variation of *RX Andromedæ* is less than that of *U Geminorum*, but the former appears to be more irregular even in its light changes than the latter.

New Double Stars. By the Rev. T. E. Espin, M.A.

The following pairs have been found with the 17¼ in. Reflector during the spring. The weather has been persistently unfavourable since the end of March, and few measures have been obtained.

No.	B.D.	R.A. 1900		Decl.		P.	D.	Mags.		Nights.	Date.	
	°	h	m	°	′	°	″				1900.	
404	+56,114	0	37·2	+57	5	68·1	2·90	9·3	12·5	1	7·063	
405	+57,171		49·8	57	15	114·8	4·17	9·0	9·0	2	7·043	
406	+56,153		50·9	56	52	149·1	3·67	9·2	9·5	2	7·073	
407						90·1	3·55	9·4	11·0	2	7·073	
408	+57,251	1	13·3	57	45	160·1	2·70	9·3	11·2	2	7·073	
	408 407					345·2	89·98			3	7·094	
409	+55,506	2	0·6	55	39	108·7	1·95	9·5	11·0	2	7·106	
410	+34,769	3	50·2	34	10	102·8	2·88	9·4	9·8	3	7·073	
411	+33,752		52·1	33	40	46·2	4·47	9·2	9·3	2	7·073	
412	+32,876	4	57·4	32	14	267·6	4·77	8·5	12·0	3	7·108	
413	+32,880		58·6	32	13	5·4	5·93	9·0	13·5	3	7·108	
414	+33,975	5	6·3	33	25	183·8	2·38	9·0	9·1	3	7·129	
415	+32,1109		44·8	32	6	16·1	15·00	6·9	12·5	2	7·089	
416	+31,1219	6	4·1	31	33	197·7	7·65	8·9	11·5	2	7·166	
417	+33,1265		4·2	33	1	330·0	13·55	8·0	12·5	2	7·206	AB
						237·0	14·50		14·5	2	7·206	AC
						275·5	37·55		11·0	2	7·206	AD
418	+35,1600	7	16·8	35	12	15·7	13·20	8·1	13·3	3	7·166	
419	+34,1641		28·2	33	56	125·6	3·58	10·0	10·7	3	7·073	
						44·0	58·30	A = 9·4		2	7·[illegible]	
420	+29,1597		39·5	28	59	273·4	2·65	12·5	13·0	1	[illegible]	
						118·1	60·79	A = 8·7		1		
421	+29,1632		46·1	28	57	217·8	6·20	9·0	12·0	2		

No.	B.D.	R.A. 1900 h m	Decl. ° ′	P. °	D. ″	Mags.		Nights.	Date. 1900.	
422	+29,1645	49·2	+29 25	181·3	14·86	8·1	10·5	1	7·246	
423	+34,1741	59·4	34 53	298·9	3·21	8·5	10·4	3	7·073	
424	+35,1756	8 0·6	35 15	73·7	5·77	9·1	10·5	3	7·073	
425	+25,1854	2·3	25 50	257·2	4·52	8·1	12·5	3	7·223	
426	+28,1597	18·8	28 47	284·0	4·85	9·0	9·2	1	7·246	
427	+29,1821	39·9	29 22	72·4	2·50	9·0	10·2	2	7·183	
428	+28,1774	9 30·7	28 47	20·0	12·55	8·6	9·1	3	7·223	
429	+31,2017	32·6	31 18	10·8	3·82	9·5	9·5	2	7·239	
430	+29,1992	10 0·0	29 47	170·8	1·45	9·4	9·7	4	7·230	
431	+27,1852	2·5	27 17	198·7	3·95	10·5	10·6	2	7·282	BC
				347·3	41·87	A =	8·0	1	7·246	AC
				351·6	44·20			1	7·317	AB
432	+33,1988	20·1	33 8	160·4	2·52	9·3	9·6	2	7·232	
433	+30,2087	51·7	30 16	220·0	5·75	9·3	10·5	2	7·282	
434	+35,2233	11 13·4	35 32	247·8	7·76	9·3	14·5	2	7·216	
435	+29,2207	39·6	28 59	347·2	6·80	9·2	9·4	2	7·227	
436	+30,2277	12 23·5	30 26	316·5	1·85	9·2	9·2	2	7·269	
437	+30,2281	24·5	30 4	222·0	2·50	8·8	9·4	2	7·242	BC
				2·0	69·63	A =	8·5	1	7·246	AB
438	+26,2382	38·0	26 27	281·0	5·00	8·2	13·0	2	7·232	
439	+28,2155	46·6	27 46	63·8	1·83	8·9	9·4	3	7·237	
440	+29,2387	13 12·7	29 12	361·5	2·55	9·5	9·5	1	7·301	
441	+28,2211	13·0	28 39	77·5	4·80	8·6	13·2	2	7·238	
442	+28,2251	37·9	28 43	243·2	7·45	8·4	13·0	2	7·234	

Notes.

405.—This pair was found in 1892. The only other measures are mine:

1892·805, P. 116°·6 D. 4″·86 2 nights.

407, 408.—Two pairs a little N of ϕ Cassiopeiæ.
409.—Measures discordant.
417.—The *comes* C is very difficult and the measures are discordant.

Erratum in Pogson's Observations of U Geminorum.

Vol. lxvii. page 129, footnote:—The inverted commas should come at the end of the footnote, and not before the last sentence. The whole note is Pogson's. But Mr C. L. Brook points out that his suspicions are probably unnecessary. "Knott makes U bright on January 4 and 6, but fading: on January 17 he makes it $<13·3$. Baxendell also makes it bright on January 4–6 and <13 on January 20. Pogson's observation falls in with these, and the rapid descent in 5 days from 10·2 to $<12·8$ is not unusual: see April 12 to 17, 10·3 to 13·8."—(Letter of April 16 to H. H. T.)

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXVII. JUNE 14, 1907. No. 8

H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Major-General Henry Herbert Lee (late R.E.), The Mount, Dinas Powis, near Cardiff, South Wales;
Augustus Edward Hough Love, D.Sc., F.R.S., Sedleian Professor of Natural Philosophy, Oxford;
Rev. Reginald William Bickerton Moore, M.A., Vicar of St James's, Bath, 11 Devonshire Buildings, Bath; and
Herbert Gerard Tomkins, Government of India Finance Department, 86 Old Dover Road, Canterbury,

were balloted for and duly elected Fellows of the Society.

Anders Donner, Director of the Observatory, Helsingfors, Finland, was balloted for and duly elected an Associate of the Society.

The following candidates were proposed for electio[illegible] [illegible]llows of the Society, the names of the proposers from perso[illegible] [illegible]dge being appended :—

Burdett Mason, Larondouette, Ba[illegible], France [illegible] by W. E. Wilson); and
Arthur William Meers, F.R.G.S., [illegible], Lu[illegible] [illegible]ham Park, Beckenham, Kent (pro[illegible] S. A. S[illegible]

Ninety-three presents were anno[illegible] having [illegible]d since the last meeting, including, am[illegible] [illegible]ers :—

The Oxford Astrographic Catal[illegible] [illegible], pr[illegible]

University Observatory, Oxford; Potsdam Observatory, Photographische Himmelskarte, Band 4, presented by the Observatory; Dr Backlund's Reports on the Russian Mission for the measurement of an Arc of Meridian in Spitzbergen, presented by the Pulkowa Observatory.

Twenty charts of the Astrographic Chart of the heavens, presented by the Royal Observatory, Greenwich; six positive enlargements on glass of photographs of portions of the Solar Surface, taken at the Royal Observatory, Greenwich, and photograph of the Solar Eclipse of 1905 August, taken at Sfax, Tunis, presented by the Astronomer Royal; nine photographs of Mount Wilson Observatory, and photographs of the Spectra of Sun-spots, presented by Prof. G. E. Hale.

On the Inclinations of Binary Star Orbits to the Galaxy.

By T. Lewis and H. H. Turner.

Summary.

§ 1. It was suggested that the axes of rotation of variable stars tend to be parallel to the Galaxy. Do double stars tend to revolve in corresponding fashion?

§ 2. List of 59 orbits.

§§ 3–5. Application of method used for variables: with the result that orbits nearly at right angles to the line of sight seem to avoid the Galactic Poles, as was noticed for stars which present their poles to us.

§§ 6–12. Discussion of distribution of the spurious poles of double-star orbits, and conclusion that at least 75 per cent. are scattered uniformly over the sphere.

§ 13. If they were uniformly scattered there would remain $38\frac{1}{2}$ real poles within 30° of the Galaxy and $20\frac{1}{2}$ in the rest of the sphere: whereas, on the hypothesis of uniform distribution, both numbers should be $29\frac{1}{2}$.

§ 14. If the small orbits ($a < 0''\cdot 5$) be excluded, there are 33 poles within 30° of the Galaxy as against 15 in the remainder of the sphere.

§§ 15–16. There is evidence of systematic error in very small distances, which would specially affect small orbits.

§ 17. If orbits of large eccentricity be excluded ($e > 0\cdot 51$) there remain 21 poles within 30° of the Galaxy, and 5 in the rest of the sphere.

§§ 18 and 19. Remarks on high eccentricities.

1. In a paper by one of the present writers on the "Classification of Long-period Variables" (*Mon. Not.*, lxvii. p. 332), it was suggested that the type of light curve might indicate the inclination of the axis of rotation of the star to the line of sight; and attention was drawn to the fact that, in terms of this hypothesis, there were no variables near the poles of the Milky Way with their axes turned towards us. After the meeting at which the above paper was read, Mr Inwards suggested that the orbits of double stars might show a similar peculiarity of distribution. Various attempts have already been made to detect any such peculiarity, without success; see, for example, pp. 244–6 of Dr T. J. J. See's *Evolution of Stellar Systems*, and Miss Everett's paper in *Mon. Not.*, lvi. pp. 464–5. On examining these lists, it will be seen that there is no very well-marked feature of distribution; but at the same time there appear to be certain tendencies which have perhaps not yet received sufficient attention, and the following notes may serve to promote further inquiry.

2. There were some discrepancies in the two lists above mentioned; and other orbits have since been computed. Hence the first step was to form a revised list of orbits as follows:—

TABLE I.

List of 59 Orbits in Order of R.A. of Star.

No.	Name.	R.A.	Dec.	Gal. Lat.	e	a	i	Γ	Γ
		h. m.	°	°		"	°	°	°
1	Σ 3062	0 1	+58	− 4	0·45	1·37	44	64	51
2	Σ 2	0 4	+79	+16	·40	0·55	70	84	81
3	13 Ceti	0 30	− 4	−67	·74	0·21	48	69	30
4	β 395	0 32	−25	−87	·15	0·66	77	78	56
5	η Cassiop.	0 43	+57	− 6	·34	8·51	43	67	54
6	36 Androm.	0 50	+23	−40	·75	1·01	45	87	10
7	Σ 186	1 51	+ 1	−57	·67	1·15	74	84	78
8	γ^2 Androm.	1 58	+42	−19	·82	0·35	77	83	33
9	Σ 228	2 8	+47	−13	·38	0·90	66	66	40
10	20 Persei	2 48	+39	−19	·48	0·24	74	[illegible]	[illegible]
11	40 Eridani	4 11	− 8	−37	·14	6·25	[illegible]	[illegible]	[illegible]
12	β 883	4 46	+11	−20	·48	0·24	[illegible]	[illegible]	[illegible]
13	Sirius	6 40	−17	− 8	·59	7·59	[illegible]	[illegible]	[illegible]
14	9 Argus	7 41	−14	+ 8	·70	0·65	[illegible]	[illegible]	[illegible]
15	ζ Cancri	8 6	+18	+26	·38	0·86	[illegible]	[illegible]	[illegible]
16	ε Hydræ	8 41	+ 7	+38	·68	[illegible]	[illegible]	[illegible]	[illegible]
17	Σ 3121	9 12	+29	+44	·33	[illegible]	[illegible]	[illegible]	[illegible]

TABLE I.—*continued*.

No.	Name.	R.A.	Dec.	Gal. Lat.	e	a	i	Γ	Γ'
		h. m.	°	°		"	°	°	°
18	ω Leonis	9 23	+ 9	+40	·54	0·88	63	67	14
19	φ Ursæ Maj.	9 45	+55	+48	·44	0·34	30	64	26
20	ξ Ursæ Maj.	11 13	+31	+70	·40	2·51	56	59	57
21	ι Leonis	11 19	+11	+64	·76	2·49	66	71	65
22	OΣ 234	11 25	+42	+68	·30	0·35	51	65	42
23	OΣ 235	11 27	+62	+53	·32	0·87	49	89	19
24	Σ 1639	12 20	+26	+86	·70	0·71	58	61	55
25	γ Centauri	12 36	−48	+14	·80	1·02	62	86	81
26	γ Virginis	12 37	− 1	+62	·88	3·90	30	43	36
27	42 Comæ	13 5	+18	+82	·48	0·66	90	83	83
28	OΣ 269	13 28	+36	+77	·36	0·32	71	69	61
29	25 Can. Ven.	13 33	+37	+75	·87	1·12	36	43	26
30	β 612	13 35	+11	+70	·27	0·31	64	83	46
31	α Centauri	14 32	−60	− 1	·53	17·70	79	88	43
32	Σ 1879	14 41	+10	+57	·65	0·84	64	87	36
33	ξ Boötis	14 47	+20	+61	·59	5·33	51	81	35
34	η Cor. Bor.	15 19	+31	+56	·27	0·92	52	88	30
35	μ^2 Boötis	15 21	+38	+54	·54	1·27	44	77	11
36	OΣ 298	15 32	+40	+51	·58	0·80	61	83	25
37	γ Cor. Bor.	15 39	+27	+50	·42	0·73	84	79	67
38	ξ Scorpii	15 59	−11	+29	·77	0·70	72	77	50
39	σ Cor. Bor.	16 11	+34	+45	·54	3·82	47	90	18
40	λ Ophiuchi	16 26	+ 2	+30	·68	0·99	30	61	46
41	ζ Herculis	16 38	+32	+39	·56	1·40	50	85	24
42	De 15	16 41	+44	+40	·36	0·77	75	81	44
43	Σ 2107	16 48	+29	+38	·48	0·73	14	67	38
44	β 416	17 12	−35	+ 1	·51	1·22	37	86	72
45	Σ 2173	17 25	− 1	+17	·20	1·14	81	61	54
46	μ^1 Herculis	17 43	+28	+25	·22	1·39	64	83	48
47	τ Ophiuchi	17 58	− 8	+ 7	·59	1·25	58	72	50
48	70 Ophiuchi	18 0	+ 3	+10	·50	4·55	58	88	81
49	99 Herculis	18 3	+31	+21	·78	1·01	0	68	68
50	ζ Sagittarii	18 56	−30	−16	·28	0·69	67	60	45

TABLE I.—*continued.*

No.	Name.	R.A.	Dec.	Gal. Lat.	e	a	i	Γ	Γ′
		h. m.	°	°		″	°	°	°
51	γ Cor. Austr.	19 0	−37	−19	·42	2·45	34	87	53
52	Σ 2525	19 23	+27	+ 4	·95	1·41	57	38	28
53	OΣ 400	20 7	+47	+10	·40	0·57	69	83	73
54	β Delphini	20 33	+14	−17	·37	0·67	61	54	31
55	4 Aquarii	20 46	− 6	−30	·51	0·73	73	61	30
56	δ Equulei	21 10	+10	−27	·36	0·31	77	42	21
57	τ Cygni	21 11	+38	− 8	·37	1·16	47	75	63
58	κ Pegasi	21 40	+25	−22	·49	0·42	81	74	68
59	85 Pegasi	23 57	+27	−35	·39	0·89	56	80	66

3. In the last two columns of the above table, under the notation Γ and Γ′ adopted by Dr See, are given the two possible inclinations of the orbital planes to the plane of the Galaxy. This ambiguity has hitherto somewhat embarrassed the discussion of the distribution of the inclinations in space, and we shall presently return to the consideration of it. But first it may be remarked that it need not trouble us if we adopt the method used in the case of variable stars: for we are at least as well off in dealing with double-star orbits as in dealing with variable stars. We know the inclination of the plane to the line of sight, though we do not know on which side it lies. In the case of the variables, we similarly suppose that we have indications of the inclination of the equator (or of the axis of rotation) to the line of sight, but as regards its orientation round the line of sight we are quite ignorant—our alternatives are infinite in number instead of being two only. Hence we can at any rate apply to the double-star orbits the same test as to the variables; and we proceed to do this in the first instance.

4. On p. 351 of the present volume of *Mon. Not.* is given a list of twenty stars with large values of a, the suggestion being that these are rotating suns which present their polar regions to us, their equators lying at right angles to the line of sight analogy in the case of a double star is an orbit for which i so that the apparent orbit is well open. The values of i in the 8th column of the above table, and we can accor out the small values of i; or, what is more complete, whole series according to the value of i as in Table II.

TABLE II.

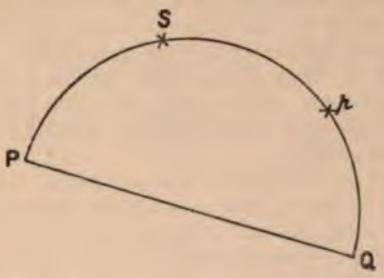
Orbits in Order of Inclination to Plane of Projection.

i	Star's No.	Gal. Lat.	i	Star's No.	Gal. Lat.	i	Star's No.	Gal. Lat.
0°	49	$+21^\circ$	50°	41	$+39^\circ$	69°	53	$+10^\circ$
11	15	+26	51	22	+68	69	11	−37
14	43	+38	51	33	+61	71	28	+77
28	12	−20	52	34	+56	72	2	+16
30	40	+30	56	59	−35	72	38	+29
30	19	+48	56	20	+70	73	55	−30
33	26	+62	57	52	+ 4	74	10	−19
34	51	−19	58	47	+ 7	74	7	−57
36	16	+38	58	24	+86	75	17	+44
36	29	+75	58	48	+10	75	42	+40
37	44	+ 1	61	36	+53	77	4	−87
43	5	− 6	61	54	−17	77	8	−19
44	1	− 4	62	25	+14	77	56	−27
44	35	+54	63	18	+40	78	14	+ 8
45	6	−40	64	30	+70	79	31	− 1
45	13	− 8	64	32	+57	81	45	+17
47	39	+45	64	46	+25	81	58	−22
47	57	− 8	66	9	−13	84	37	+50
48	3	−67	66	21	+64	90	27	+82
49	23	+53	67	50	−16			

5. It will be seen from Table II. that the orbits with small values of i do show some tendency to lie near the Galaxy. There are exceptions such as No. 29 which, with an inclination of 36°, has a Galactic Latitude of 75°. This star is 25 Canum Ven. It has not yet completed a revolution (period 220 years) or been observed near apastron, while the orbit is so eccentric that it cannot be observed near periastron; consequently the elements are somewhat uncertain: possibly further knowledge will sensibly increase the value of i and the eccentricity, which seem (from inspection of the three orbits computed) to go together. But to enter upon criticism of orbits computed would take us too far: it is sufficient to remark here that there may be considerable uncertainty in the determination of i, as a few examples will illustrate. Thus in the case of γ Virginis (No. 26) there will be found on p. 339 of *Mem. R.A.S.*, vol. lvi., a list of 22 orbits with values of i ranging from 0° to 68°. They are not all of equal merit, but all of them are *bonâ fide* attempts to satisfy the observations. Again, for λ Ophiuchi the value 58° rather than 30° has been assigned to i; for ξ Scorpii the value of i has recently been changed from 72° to 29°; for ξ Boötis

the values range from 35° to 80°, and so on. Of course this uncertainty cuts both ways, and cannot be used to bring the above table into better accord with a hypothesis; but it is to be remembered as a possible explanation of discrepancies.

6. Before proceeding to analyse the columns Γ and Γ′, it is desirable to consider the general effect of the ambiguity. Let us represent each orbit by its pole, *i.e.* the point in which a line perpendicular to it cuts the celestial sphere. For every real pole P there is a corresponding spurious pole p on the opposite side of the



star S, and equidistant from it. We cannot tell without independent evidence such as may ultimately be afforded by the spectroscope,* which is the right one. But we can study this question:—Suppose P tends to lie near the Milky Way for all positions of S, will p have a tendency to lie in any particular region or regions? and, if so, which?

7. A preliminary question must first be asked with regard to the distribution of the star S. Does it appear from the above list that double stars congregate towards the Milky Way? The numbers for every 10° of Galactic Latitude are as follows:—

TABLE III.

Distribution of the Stars of Table I. in Galactic Latitude.

Galactic Lat.	Observed No. of Stars.	Calcd.	O – C.
0° – 9°	9	10	– 1
10 – 19	13	10	+ 3
20 – 29	6	9	– 3
30 – 39	7	9	– 2
40 – 49	6	7	– 1
50 – 59	7	6	+ 1
60 – 69	4	4	0
70 – 90	7	4	+ 3
Total	59	59	0

* It is strange to think that we do not yet know which is the real even of α Centauri. Could not some spectroscopic observer determine th us? There are also η Cassiopeiæ, p Eridani, and others.

In the column "Calc." the theoretical distribution of 59 stars, uniformly scattered over the sphere, is given. The column O – C shows that the actual stars are in defect near the Galaxy and in excess near its poles. But the differences are small and the scattering may be taken as nearly uniform.

8. Returning now to the figure, let us suppose in the first instance that, the pole P being given and remaining therefore fixed, the star S is scattered at random over the sphere. What will be the consequent distribution of the spurious pole p? One feature of it is especially noteworthy. The most common value of PS will be 90°, since the equator of P is the largest circle with centre P. Now all the stars which lie in the equator of P have corresponding poles at 180° from P (since $Pp = 2PS$): that is, in Q, the point opposite to P on the sphere. There is thus a great concentration of spurious poles in and near this point Q. Similarly the point Q will, as a real pole, give rise to a concentration of spurious poles near P.

9. It is easy to write down the symbolical expressions for the density of the spurious poles, but it will be more directly useful to us to work with numbers. Starting with the numbers of stars in each zone of 10° outwards from P up to PS = 90°, the spurious poles of these will fall in zones of 20°, up to $Pp = 180°$. The stars in the hemisphere for which PS > 90° will be referred to pole Q, and will give a series of numbers in the reverse order; and adding together the two, we get a distribution of spurious poles (corresponding to a uniform scattering of stars and a given pair of real poles PQ) as follows:—

Table IV.

PS.	Number of Stars: see Table III.	Pp.	Number of Spurious Poles.	Uniform Dist.	Excess over Uniformity.
° ° 0–10	1	° ° 0– 20	1 + 10 = 11	3	+ 8
10–20	3	20– 40	3 + 10 = 13	8	+ 5
20–30	4	40– 60	4 + 9 = 13	12	+ 1
30–40	6	60– 80	6 + 9 = 15	14	+ 1
40–50	7	80–100	7 + 7 = 14	15	– 1
50–60	9	100–120	9 + 6 = 15	14	+ 1
60–70	9	120–140	9 + 4 = 13	12	+ 1
70–80	10	140–160	10 + 3 = 13	8	+ 5
80–90	10	160–180	10 + 1 = 11	3	+ 8
Total			118	89	29

10. Thus for uniformly scattered stars we have, out of 118 spurious poles, 89 of them scattered uniformly over the sphere and 29 tending to collect round the real poles, the density near the poles being quite considerable.

11. If then the 59 real poles lie near the Milky Way, while the stars are scattered uniformly over the sphere, we shall have

44 of the spurious poles scattered uniformly over the sphere and 15 tending to coincide with the real poles near the Milky Way.

Now we may proceed in the following manner. We may assume in the first instance that the spurious poles are all uniformly scattered, and then deduce the distribution of the real poles. If any peculiarity of distribution is found, we must then reduce it by some 20 per cent. to allow for the spurious poles.

13. Let us therefore consider the distribution of real and spurious poles together, and then make allowance for the spurious poles on the principle of uniform scattering; this is done in Table V.

TABLE V.

Values of Γ or Γ′.	Observed Total Number.	Subtract Uniform Distribution of Γ′.	Γ alone.	Observed Excess over Unif. Distr.
$0^\circ - 19^\circ$	7	4	3	− 1
20 − 29	6	4	2	− 2
30 − 39	11	6	5	− 1
40 − 49	12	7	5	− 2
50 − 59	14	8½	5½	− 3
60 − 69	28	9½	18½	+ 9
70 − 79	13	10	3	− 7
80 − 90	27	10	17	+ 7
Total	118	59	59	0

In the second column are given the number of cases (from Table I.) where either Γ or Γ′ lies within the limits indicated in the first column. In the third, an allowance is made for Γ′ on the hypothesis of uniform distribution. The remaining numbers in the fourth column should represent the distribution of the real poles: and in the fifth column this is compared with a random or uniform distribution. It will be seen that an excess towards the Galax exists, though it is not very large. Still it is large enough to m it impossible, for instance, to maintain the contrary propositi that the orbits of double stars tend to lie parallel to the Gal

14. The above result is obtained from the whole list of without any selection. But it is noteworthy that if we inquiry to the larger orbits, the tendency of the pole the Galaxy is more marked. It seems worth while t record the result, for example, of limiting the material of which the semi-major axis exceeds 0″·50. Recurring t there are 48 of these orbits: and for them we shou Table V. by Table VI. as follows :—

TABLE VI.

Larger Orbits only.

Values of Γ or Γ'.	Observed Total Number.	Subtract Uniform Distribution of Γ'.	Γ alone.	Observed Excess over Unif. Distr.
0°–19°	6	3	3	0
20–29	4	3	1	–2
30–39	8	5	3	–2
40–49	8	6	2	–4
50–59	13	7	6	–1
60–69	22	8	14	+6
70–79	11	8	3	–5
80–90	24	8	16	+8
Total	96	48	48	0

15. The exclusion of the smaller orbits is reasonable since there is systematic error affecting observations of very close doubles. The following figures will illustrate the nature of this error. In Dr T. J. J. See's *Evolution of Stellar Systems*, tables are given showing the residuals $(\rho_0 - \rho_c)$ of the observed distances over the calculated. These were collected according to the value of ρ_c in all cases where ρ_c falls below 0″·60 after the year 1870. [The measurement of very small distances has become more feasible in recent years, and it was thought better to limit the inquiry to modern observations.] The following table gives the results for all orbits represented in all three of the groups selected :—

TABLE VII.

Evolution of Stellar Systems. $\rho_o - \rho_c$ *for small values of* ρ_c.

Star.	ρ_c = 0″·60 to 0″·45.		0″·44 to 0″·25.		Under 0″·25.	
	$\rho_o - \rho_c$ (Sum).	No. Obs.	$\rho_o - \rho_c$ (Sum).	No. Obs.	$\rho_o - \rho_c$ (Sum).	No. Obs.
γ Androm.	+ ″·02	6	+ ″·10	4	+ ″·14	7
9 Argus	– ·01	3	+ ·03	7	+ ·06	1
Σ 3121	– ·20	7	+ ·21	11	+ ·08	4
42 Com. Ber.	+ ·21	8	+ ·25	8	– ·02	3
OΣ 298	+ ·09	3	– ·11	10	+ ·14	2
γ Cor. Bor.	– ·63	6	– ·11	3	+ ·60	2
Σ 2173	– ·07	3	– ·10	4	+ ·01	2
β Delphini	– ·10	4	– ·04	9	+ ·21	4
δ Equulei	– ·57	4	– ·20	6	+ ·10	3
Sums	– 1″·26	44	+ ″·03	62	+ 1″·32	28
Means	– 0″·029	...	+ 0″·000	...	+ 0″·047	...

16. It thus appears that double-star observers are liable to measure very small distances too large. There is nothing unreasonable or even unlikely in this, and the above figures may even underestimate the amount, since some concession to the error is probably made in determining the orbit. Now, if the existence of such an error is admitted, the elements of the small orbits will require systematic revision. It is not easy to say what the general effect on the inclinations would be, and such an inquiry cannot in any case be undertaken here. But it seems desirable to suspend judgment at present, with regard to any distribution of orbital inclinations, until we know more of the possible systematic errors of the smaller orbits.

17. If, now, we further limit the material to orbits of moderate eccentricity, the figures become still more striking. Excluding from the 48 above, 22 orbits for which e exceeds 0·51, we get—

TABLE VIII.

Values of Γ or Γ′.	Observed Total Number.	Subtract for Γ′.	Γ alone.	Excess over Uniform.
0°–19°	2	2	0	−2
20–29	0	2	(−2)	−4
30–39	4	2	2	0
40–49	4	3	1	−2
50–59	8	4	4	0
60–69	15	4	11	+7
70–79	5	4	1	−3
80–90	14	5	9	+4
Total	52	26	26	0

18. The limit $e = 0·51$ was selected in order to make the excess near the Galaxy as striking as possible. The general effect of including other orbits can be seen from the following figures for the 48 orbits :—

Value of e . . .	< ·40	·40 to ·51	·52 to ·59	·60 to ·79	> ·80	Total
Values of Γ and Γ′ < 60°	12	6	8	7	6	39
,, > 60°	18	16	10	11	2	57

or if we assume uniform distribution of Γ′ the numbers would be

Value of e . .	< ·40	·40 to ·51	·52 to ·59	·60 to ·79	·80 and
Values of Γ < 60°	4½	½	3½	2½	
,, Γ > 60°	10½	10½	5½	6½	

The difference between the results for large values those for small may be due either to observational or to causes. When the stars approach very close together, ol are difficult and likely to be affected by systematic err the determinations of inclination will be affected ; ɛ

other hand, just as in the solar system the planetary orbits are at the same time of small eccentricity and systematically disposed near one plane (which is at a high inclination to the Galaxy), while comets have orbits of high eccentricity with planes in miscellaneous orientations,—so there may be a real difference between double star orbits of low and of high eccentricity.

19. It seems worth while to form a new table corresponding to Table II. for the 26 orbits retained in Table VIII.

Table IX.

Similar to Table II., but excluding orbits of small size ($a < 0''\cdot5$) and high eccentricity ($e > 0\cdot51$).

i	Star's No.	Gal. Lat.	i	Star's No.	Gal. Lat.	i	Star's No.	Gal. Lat.
11	15	+26°	56	20	+70°	75	17	+44°
14	43	+38	58	48	+10	75	42	+40
34	51	−19	61	54	−17	77	4	−87
37	44	+ 1	64	46	+25	81	45	+17
43	5	− 6	66	9	−13	84	37	+50
44	1	− 4	67	50	−16	90	27	+82
47	57	− 8	69	53	+10			
49	23	+53	69	11	−37			
52	34	+56	72	2	+16			
56	59	−35	73	55	−30			

Ancient Eclipses. By P. H. Cowell, F.R.S.

As my whole argument from the ancient lunar eclipses is—"If I have placed a wrong interpretation upon the records of solar eclipses, how is it that the results so obtained agree so well with the lunar eclipses?" I am, of course, perfectly satisfied with such sentences as these in Mr Nevill's recent papers:—

"During past years in my researches I have repeatedly found evidence of an unexplained apparent secular acceleration in the motion of the Moon's argument of latitude . . . ." (*M.N.*, lxvii. p. 17).

"These early eclipse observations . . . . are certainly not inconsistent with Mr Cowell's conclusion that the Moon's argument of latitude requires an increased secular acceleration" (p. 13).

In my view the lunar eclipses by themselves do not amount to a proof, and I do not ask Mr Nevill to say more than he has said.

Mr Nevill, however, attaches more weight than I do to the times of lunar eclipses. From the times the mean elongation is

deduced, whereas the argument of latitude is found from the magnitudes. For the mean elongation, Mr Nevill deduces a secular term $7''\cdot4$ from the Ptolemaic eclipses and $6''\cdot2$ from the Arabian eclipses. I regard the agreement of these numbers with the $6''\cdot8$ that I obtained from solar eclipses as most satisfactory. Mr Nevill (p. 10) regards $6''\cdot8$ as inconsistent with both $6''\cdot2$ and $7''\cdot4$. Apparently, upon the inconsistency of $6''\cdot2$ with $7''\cdot4$, he bases his supposition of unknown long-period terms. But he has not shown that the $6''\cdot2$ and $7''\cdot4$ are subject to such small probable errors as to negative the possibility that the discordance is accidental. My view, of course, is that $6''\cdot8$ is nearly the true value.

It is on the solar eclipses that I rely. The divergence between Mr Nevill's views and my own in this case arises largely from differences of numerical calculation, and the first thing to be done is to remove those differences. The following table gives for several solar eclipses (1) my residual, which is approximately the least distance between the centres of the Sun and Moon as seen from the place assumed in the calculations; (2) the quantity which Mr Nevill calls "Cowell's $d\phi$."

Date.	Residual. See *M.N.*, lxvi. 531.	$d\phi$ See *M.N.*, lxvi. 414, 416.
− 309	$-177''$	$-0^\circ\cdot9$
− 399	+ 14	+3·1
− 403	− 525	− 2·4
− 430	+ 34	+0·1
− 602	+ 46	− 0·6
− 647	+ 8*	+0·5
− 660	+ 163	+6·0
− 678	+ 115	+4·8
− 762	− 42	− 0·8
− 1062	+ 29	+0·8
− 1123	− 697	− 2·1
− 600	+ 429	+2·0
− 708	+ 211	− 3·0
− 180	+ 27	+0·9
− 187	+ 456	+1·9
− 299	− 157	− 3·3

The relationship between my residual and Mr Nevill's $d\phi$ is a complicated one; the following test will, however, serve to detect large errors:—"Divide $d\phi$ by 60 and compare with the residual, allowing $100''$, partly because the divisor 60 is only approximate

* From *M.N.*, lxv. p. 867.

and partly on account of approximations used in obtaining the residual, and possibly other approximations used in obtaining $d\phi$."

It will be noted that this is a very rough test. My point is that certain formulæ will give residuals of less than 50″ for several eclipses, and in applying the above test I allow 100″; hence discrepancies in arithmetic that are important for my argument may slip through, undiscovered by the test, which only detects the still larger discrepancies.

The test, however, brings to light discrepancies in the eclipses of −309, −399, −403, −660, −678, −1123, −600, −708, −187. To this list −1116 would also have been added if Mr Nevill (*M.N.*, lxvii. p. 15) had not corrected his result as given on *M.N.*, lxvi. p. 414.

Assuming for the moment that my figures are correct, it is evident that Mr Nevill has not appreciated the superior precision of solar eclipses. Lunar eclipses are liable to errors of a tenth of a magnitude or 200″, but solar eclipses appear to be only liable to uncertainties of 50″, a little greater than the maximum difference of semidiameter.

If, on the other hand, Mr Nevill's figures are correct, solar eclipses exhibit residuals as large as lunar eclipses, and the case for an unexplained acceleration in the Moon's argument of latitude ceases to amount to anything like certainty.

Now it would be only natural that Mr Nevill and I should each of us believe that the discrepancies exhibited above are due to the blunders in computation of the other. What I now ask of Mr Nevill is that he should put me into the same position with regard to his computations as that into which I have already put him with regard to mine. I have set down every formula that I have used. In my papers on the eclipses of +1030, +1239, and +1241, it can be seen to what extent the formulæ used for ancient eclipses are approximate. On *M.N.*, lxvi. pp. 533–536, a full calculation is given for one eclipse. In all cases numerical results for a great number of intermediate stages are given.

But, in Mr Nevill's computations, I do not know whether he has calculated his own places for the Sun and Moon, or used Oppolzer's, with suitable modifications. In either case I do not know what inequalities he has retained. Further, I should like to see the numerical places obtained in the various cases, so that I can tell whether the discrepancies between our results arise in the geocentric places or parallactic part of the calculations. Again, on *M.N.*, lxvii. p. 15, Mr Nevill says: "The changed path of the curve of central totality can be deduced from the comparison of the new positions of the beginning, middle, and the end of eclipse with those given by Oppolzer." What path of the curve of central totality underlies the changes deduced by Mr Nevill? The paths published by Oppolzer are not sufficiently accurate, but Mr Nevill has not stated by what formulæ he has calculated his own paths. All the information for which I ask could be supplied if Mr Nevill, in addition to some intermediate results for all eclipses, would

publish at full length and with suitable explanations the entire calculation for one eclipse. If he will do this, I should be glad if he will select the eclipse of -1123. This is a very important one, for his figures on *M.N.*, lxvi. p. 414, indicate that the theoretical accelerations (Mr Nevill's B) make this eclipse total near Babylon. My figures on *M.N.*, lxvi. p. 532, contradict this conclusion.

As the foregoing remarks have brought me to the eclipse of Babylon, I may add—

1. Mr King originally indicated the second half of the eleventh century B.C. as the probable date. He has recently discussed this eclipse very fully in the first volume of *Chronicles concerning Early Babylonian Kings* (Luzac & Co., 1907).

2. According to my calculations, at no date in the summer months of either the eleventh or twelfth centuries B.C. except -1062 July 31 could the Babylonians have seen any phenomenon answering to the words of the inscription quoted on *M.N.*, lxv. p. 861.

3. Mr King, though he dislikes the equation, Sivan 26 = July 31, does not reject it. I see no alternative to accepting it.

The Perturbations of Halley's Comet. By P. H. Cowell and A. C. D. Crommelin.

The action of Saturn is here considered. The calculations are similar in character to those for the action of Jupiter published on pp. 386–411.

The differential equations given on p. 386 are again applicable. In the value of e_x on that page the square root should be omitted, so that we have

$$e_x = -(1-e^2)\sin u \cos u.$$

Table I., pp. 387–390, is again applicable. The tabulated values of e_x are correct.

The $V = V_3 + V_4$ method explained on p. 391 is again introduced at $u = 90^\circ$ and dropped at $u = 270^\circ$. V_4 would be the disturbing function if the motion were referred to the centre of gravity of the Sun and disturbing planet. Hence $V_3 = V - V_4$ is the change in the disturbing function that would arise from a displacement of the origin of co-ordinates. This consideration gave us the assurance, *a priori*, that the differential equations depending on V_3 would be integrable.

The positions of Saturn have been taken from a table with argument g' mean anomaly of Saturn similar to Table II., pp. 394–401. The table is not printed here, but it is based upon the following assumptions and formulæ.

For Saturn $e' = \cdot 05607$, hence

$$\frac{\xi'}{a'} = -\frac{3}{2}e' + \left(1 - \frac{3}{8}e'^2\right)\cos g' + \left(\frac{1}{2}e' - \frac{1}{3}e'^3\right)\cos 2g' + \frac{3}{8}e'^2 \cos 3g' + \frac{1}{3}e'^3 \cos 4g'$$

$$= \cos g' - \cdot 084\ 105 - \cdot 001\ 179 \cos g' + \cdot 027\ 976 \cos 2g' + \cdot 001\ 179 \cos 3g' + \cdot 000\ 059 \cos 4g'$$

$$\frac{\eta'}{a'} = \left(1 - \frac{5}{8}e'^2\right)\sin g' + \left(\frac{1}{2}e' - \frac{5}{12}e'^3\right)\sin 2g' + \frac{3}{8}e'^2 \sin 3g' + \frac{1}{3}e'^3 \sin 4g'$$

$$= \sin g' - \cdot 001\ 965 \sin g' + \cdot 027\ 962 \sin 2g' + \cdot 001\ 179 \sin 3g' + \cdot 000\ 059 \sin 4g'$$

Also

$$g' = 112^\circ \cdot 09 + 144^\circ \cdot 8437\ nt$$

$$\frac{a'}{a} = \cdot 539\ 842$$

Also let NCϖ be the comet's orbit, ♈CS the ecliptic of 1850, Nϖ'S the orbit of Saturn,

♈C $= 55^\circ\ 50'$ C$\varpi = 111^\circ\ 0'$ NCS $= 17^\circ\ 46'$
♈S $= 112^\circ\ 21'$ S$\varpi' = -22^\circ\ 14'$ CSN $= 2^\circ\ 30'$

Hence NC $= 6^\circ\ 20'$ N$\varpi = 117^\circ\ 20'$
NS $= 50^\circ\ 31'$ N$\varpi' = 28^\circ\ 17'$
CNS $= 180^\circ - 19^\circ\ 15'$,

and hence

$$\frac{x'}{a} = -\cdot 432\ 815\frac{\xi'}{a'} - \cdot 281\ 251\frac{\eta'}{a'}$$

$$\frac{y'}{a} = -\cdot 311\ 431\frac{\xi'}{a'} + \cdot 433\ 315\frac{\eta'}{a'}$$

$$\frac{z'}{a} = +\cdot 084\ 333\frac{\xi'}{a'} + \cdot 156\ 733\frac{\eta'}{a'}$$

We have performed the mechanical quadratures with intervals $du = 2^\circ$. With this value of du

$$\left(\frac{a}{r'}\right)^3 du = \cdot 222\ 920 + \cdot 037\ 453 \cos g' + \cdot 003\ 139 \cos 2g' + \cdot 000\ 259 \cos 3g'$$

Table III. (the enumeration being the same as on pp. 402–404) gives the disturbing forces.

TABLE III.

First Quadrant. Unit 0·0001. | *Second Quadrant.* Unit 0·0001.

g'	a^2Xdu	a^2Ydu	a^2Zdu	u	g'	a^2X_4du	a^2Y_4du	a^2Z_4du
112°·17	− 68	− 6	+ 1	91°	202°·07	+ 104	− 94	+ 33
112·34	74	+ 31	11	93	207·29	105	77	31
112·52	76	70	22	95	212·69	104	62	30
112·71	76	111	34	97	218·26	100	47	29
112·93	71	157	47	99	223·99	95	34	27
113·17	62	205	61	101	229·90	88	23	25
113·44	48	255	75	103	235·97	80	12	23
113·75	− 28	308	90	105	242·21	72	− 3	21
114·11	+ 2	360	106	107	248·61	61	+ 5	19
114·51	38	414	120	109	255·18	52	11	17
114·97	86	466	135	111	261·91	41	16	15
115·50	142	512	149	113	268·80	30	21	13
116·08	209	548	160	115	275·84	19	22	10
116·75	286	574	168	117	283·04	10	23	8
117·48	369	579	172	119	290·39	+ 1	21	6
118·30	457	566	171	121	297·89	− 7	20	4
119·21	545	531	165	123	305·54	13	16	2
120·22	625	470	153	125	313·33	18	11	+ 1
121·32	694	384	136	127	321·26	21	+ 6	0
122·52	743	279	114	129	329·33	21	− 1	0
123·83	770	156	91	131	337·53	18	9	0
125·25	772	+ 25	67	133	345·86	12	15	+ 1
126·79	751	− 107	43	135	354·31	− 3	20	3
128·45	709	236	22	137	2·88	+ 8	23	6
130·23	648	356	+ 4	139	11·57	19	22	10
132·15	572	461	− 10	141	20·37	28	17	12
134·19	488	553	19	143	29·28	32	10	14
136·37	398	629	24	145	38·30	34	− 1	15
138·69	305	691	26	147	47·41	28	+ 7	14
141·16	210	739	22	149	56·61	19	12	12
143·77	118	775	17	151	65·90	10	14	9
146·53	+ 28	797	− 8	153	75·27	+ 1	13	7
149·44	− 59	810	+ 4	155	84·72	− 6	10	4
152·51	143	813	17	157	94·25	10	8	2
155·73	223	807	32	159	103·84	12	+ 3	+ 1
159·12	300	793	48	161	113·49	13	− 1	0
162·66	374	774	66	163	123·20	10	3	− 1
166·37	444	746	85	165	132·95	8	7	− 1
170·24	511	712	104	167	142·75	4	8	0
174·28	572	671	125	169	152·59	− 1	9	0
178·49	630	624	145	171	162·46	+ 4	9	+ 1
182·86	686	572	167	173	172·36	6	8	1
187·41	736	514	188	175	182·28	8	8	2
192·12	781	449	210	177	192·19	12	5	3
197·01	− 824	− 379	+ 231	179	202·16	+ 14	− 4	+ 3

TABLE III.—*continued.*

	Third Quadrant.	Unit 0·0001.				*Fourth Quadrant.*	Unit 0·0001.	
u	g'	a^2X_4du	a^2Y_4du	a^2Z_4du	u	g'	a^2X_4du	a^2Y_4du
269°	212°·19	+ 152	+ 57	+ 38	359°	302°·09	− 77	− 39
267	206·97	143	40	32	357	301·92	70	78
265	201·57	132	26	27	355	301·74	59	126
263	196·00	119	13	22	353	301·55	42	171
261	190·27	106	+ 1	18	351	301·33	− 20	216
259	184·36	93	− 8	14	349	301·09	+ 8	261
257	178·29	80	15	11	347	300·82	43	303
255	172·05	65	21	8	345	300·51	86	344
253	165·65	52	26	5	343	300·15	136	377
251	159·08	40	28	3	341	299·75	194	404
249	152·35	27	30	+ 1	339	299·29	259	418
247	145·46	14	29	− 1	337	298·76	329	420
245	138·42	+ 4	29	2	335	298·18	400	404
243	131·22	− 5	27	2	333	297·51	471	369
241	123·87	13	22	3	331	296·78	537	316
239	116·37	19	18	3	329	295·96	593	243
237	108·72	23	12	2	327	295·05	634	152
235	100·93	26	− 5	− 1	325	294·04	658	− 47
233	93·00	25	+ 1	+ 1	323	292·94	659	+ 68
231	84·93	22	9	4	321	291·74	640	185
229	76·73	15	15	6	319	290·43	601	299
227	68·40	− 6	20	10	317	289·01	544	406
225	59·95	+ 5	23	13	315	287·47	473	501
223	51·38	17	21	15	313	285·81	390	584
221	42·69	28	18	16	311	285·03	302	652
219	33·89	35	+ 11	16	309	282·11	208	706
217	24·98	36	0	14	307	280·07	113	744
215	15·96	33	− 8	11	305	277·89	+ 18	770
213	6·85	24	13	8	303	275·57	− 72	783
211	357·65	14	16	4	301	273·10	161	785
209	348·36	+ 4	15	+ 2	299	270·49	247	777
207	338·99	− 3	13	0	297	267·73	327	760
205	329·54	8	8	0	295	264·82	402	734
203	320·01	11	− 3	0	293	261·75	474	700
201	310·42	10	+ 1	0	291	258·53	541	660
199	300·77	9	3	+ 1	289	255·14	602	613
197	291·06	6	6	2	287	251·60	657	560
195	281·31	− 2	7	2	285	247·89	709	503
193	271·51	0	7	3	283	244·02	755	440
191	261·67	+ 5	7	4	281	239·98	795	374
189	251·80	9	6	4	279	235·77	828	302
187	241·90	12	5	4	277	231·39	856	228
185	231·98	13	+ 3	4	275	226·85	877	149
183	222·07	14	0	4	273	222·14	892	+ 69
181	212·10	+ 14	− 2	+ 4	271	217·25	− 900	− 14

Table IV. gives for argument u

$$\frac{1}{m'}\frac{dn}{n},\ \frac{1}{m'}de,\ \frac{1}{m'}ed\varpi,\ \frac{de}{m'}-\frac{d\varpi}{m'}\left\{1-\sqrt{(1-e^2)}\right\} \text{ and } \frac{1}{m'}\frac{dn}{n}(2\pi-nt)$$

The mechanical quadratures for the second and third quadrants require to be supplemented by the definite integrals arising from V_2.

TABLE IV.

First Quadrant.

u	$\frac{1}{m'}\frac{dn}{n}$	$\frac{1}{m'}de$	$\frac{1}{m'}ed\varpi$	$\frac{de}{m'}-\frac{d\varpi}{m'}\{1-\sqrt{(1-e^2)}\}$	$\frac{1}{m'}\frac{dn}{n}\times(2\pi-nt)$
1°	+ ·0001	·0000	+ ·0001	·0000	+ ·001
3	− ·0036	+ ·0001	·0001	·0000	− ·023
5	·0073	·0001	·0001	·0000	·046
7	·0112	·0003	·0001	·0000	·070
9	·0151	·0004	·0001	− ·0001	·095
11	·0188	·0004	·0002	·0001	·118
13	·0221	·0005	·0002	·0002	·139
15	·0249	·0005	+ ·0001	·0003	·156
17	·0260	·0006	− ·0001	·0004	·163
19	·0261	·0006	·0005	·0006	·164
21	·0239	·0006	·0011	·0007	·150
23	·0193	·0006	·0019	·0010	·121
25	− ·0114	·0004	·0030	·0012	− ·071
27	·0000	·0003	·0045	·0012	·000
29	+ ·0151	+ ·0001	·0063	·0011	+ ·094
31	·0336	− ·0002	·0083	·0008	·210
33	·0552	·0005	·0104	− ·0001	·344
35	·0783	·0009	·0125	+ ·0010	·488
37	·1019	·0013	·0143	·0026	·634
39	·1238	·0017	·0155	·0048	·769
41	·1425	·0021	·0159	·0075	·884
43	·1566	·0024	·0152	·0104	·969
45	·1651	·0028	·0135	·0135	1·022
47	·1679	·0032	·0108	·0168	1·036
49	·1645	·0036	·0070	·0197	1·013
51	·1555	·0039	− ·0024	·0222	·955
53	·1423	·0043	+ ·0030	·0243	·872
55	·1253	·0047	·0090	·0256	·767
57	·1054	·0052	·0154	·0262	·643
59	·0830	·0056	·0221	·0257	·505
61	·0596	·0061	·0288	·0243	·361
63	·0351	·0067	·0355	·0218	·212
65	+ ·0101	·0074	·0421	·0182	+ ·061
67	− ·0153	·0079	·0485	·0134	− ·092
69	·0404	·0085	·0542	+ ·0072	·24[illegible]
71	·0654	·0090	·0601	− ·0003	
73	·0900	·0095	·0654	·0093	
75	·1140	·0100	·0700	·0197	
77	·1372	·0104	·0740	·0318	
79	·1588	·0106	·0769	·0451	
81	·1793	·0108	·0792	·0601	
83	·1990	·0108	·0805	·0768	
85	·2166	·0105	·0808	·094	
87	·2322	·0101	·0796	·114	
89	− ·2467	− ·0092	+ ·0775	− ·1	
Sums,	+ ·0163	− ·1744	+ ·8604	−	

TABLE IV.—*continued.*

Second Quadrant.

u	$\frac{1}{m'}\frac{dn}{n}$	$\frac{1}{m'}de$	$\frac{1}{m'}ed\varpi$	$\frac{de}{m'}-\frac{d\varpi}{m'}\{1-\sqrt{(1-e^2)}\}$	$\frac{1}{m'}\frac{dn}{n}\times(2\pi-nt)$
91°	+ ·0311	− ·0025	+ ·0040	+ ·0257	+ ·176
93	·0312	·0022	·0023	·0265	·176
95	·0307	·0018	+ ·0010	·0272	·172
97	·0294	·0014	− ·0002	·0269	·163
99	·0277	·0010	·0014	·0266	·153
101	·0256	·0007	·0022	·0256	·140
103	·0232	− ·0004	·0030	·0239	·126
105	·0208	·0000	·0036	·0223	·112
107	·0176	+ ·0003	·0040	·0194	·094
109	·0151	·0006	·0042	·0170	·080
111	·0119	·0008	·0043	·0136	·062
113	·0089	·0011	·0043	·0098	·046
115	·0059	·0011	·0039	·0060	·030
117	·0035	·0012	·0035	+ ·0026	·018
119	+ ·0011	·0012	·0028	− ·0010	+ ·006
121	− ·0010	·0011	·0021	·0044	− ·005
123	·0026	·0010	·0013	·0070	·013
125	·0039	·0006	− ·0004	·0093	·019
127	·0047	+ ·0003	+ ·0004	·0108	·023
129	·0049	− ·0002	·0013	·0107	·023
131	·0045	·0007	·0021	·0090	·021
133	·0034	·0011	·0025	·0057	·016
135	− ·0017	·0015	·0026	− ·0005	− ·008
137	+ ·0003	·0017	·0023	+ ·0060	+ ·001
139	·0024	·0016	·0015	·0126	·011
141	·0043	·0012	+ ·0004	·0181	·019
143	·0052	− ·0007	− ·0006	·0205	·023
145	·0057	·0000	·0017	·0219	·025
147	·0050	+ ·0007	·0022	·0179	·021
149	·0037	·0011	·0021	·0121	·015
151	·0024	·0011	·0018	·0061	·010
153	+ ·0010	·0012	·0012	+ ·0001	+ ·004
155	− ·0001	·0009	·0005	− ·0046	·000
157	·0006	·0007	− ·0001	·0074	− ·002
159	·0011	+ ·0003	+ ·0004	·0088	·004
161	·0014	− ·0001	·0008	·0095	·005
163	·0011	·0003	·0007	·0073	·004
165	·0011	·0007	·0008	·0058	·004
167	·0009	·0008	·0005	·0028	·003
169	·0008	·0009	·0004	− ·0006	·003
171	·0005	·0009	+ ·0001	+ ·0032	·002
173	·0004	·0008	− ·0001	·0047	·001
175	·0004	·0008	·0003	·0063	·001
177	·0002	·0005	·0005	·0093	·001
179	− ·0002	− ·0004	− ·0007	+ ·0108	− ·001
Sums,	+ ·2782	− ·0096	− ·0289	+ ·3175	+ 1·524

TABLE IV.—*continued.*

Third Quadrant. V_4 only.

u	$\frac{1}{m'}\frac{dn}{n}$	$\frac{1}{m'}de$	$\frac{1}{m'}ed\varpi$	$\frac{de}{m'}-\frac{d\varpi}{m'}\{1-\sqrt{(1-e^2)}\}$	$\frac{1}{m'}\frac{dn}{n}\times(2\pi-nt)$
269°	− ·0455	+ ·0015	− ·0022	+ ·0333	− ·028
267	·0426	·0011	·0033	·0327	·028
265	·0392	·0007	·0043	·0316	·027
263	·0353	+ ·0003	·0050	·0297	·026
261	·0314	− ·0001	·0056	·0275	·024
259	·0275	·0004	·0060	·0250	·022
257	·0237	·0007	·0061	·0223	·020
255	·0192	·0009	·0061	·0186	·018
253	·0155	·0012	·0060	·0152	·015
251	·0120	·0013	·0056	·0118	·012
249	·0084	·0015	·0052	·0077	·009
247	·0048	·0014	·0044	+ ·0033	·005
245	− ·0020	·0015	·0038	− ·0003	− ·002
243	+ ·0004	·0014	·0032	·0038	·000
241	·0026	·0012	·0021	·0069	+ ·003
239	·0042	·0009	·0012	·0096	·005
237	·0053	·0006	− ·0002	·0114	·007
235	·0062	− ·0002	+ ·0009	·0128	·009
233	·0060	+ ·0002	·0015	·0123	·009
231	·0055	·0007	·0023	·0111	·008
229	·0041	·0010	·0026	·0071	·006
227	·0023	·0014	·0027	− ·0021	+ ·004
225	+ ·0001	·0016	·0024	+ ·0042	·000
223	− ·0023	·0015	·0015	·0111	− ·004
221	·0045	·0013	+ ·0005	·0177	·006
219	·0059	+ ·0008	− ·0007	·0220	·011
217	·0065	− ·0001	·0019	·0225	·012
215	·0062	·0008	·0026	·0207	·012
213	·0047	·0012	·0026	·0150	·010
211	·0032	·0014	·0022	·0085	·007
209	·0016	·0013	·0015	+ ·0020	·003
207	− ·0005	·0012	·0009	− ·0027	− ·001
205	+ ·0004	·0007	− ·0002	·0059	+ ·001
203	·0011	− ·0003	+ ·0004	·0080	·003
201	·0012	+ ·0001	·0006	·0072	[illegible]
199	·0011	·0003	·0007	·0065	[illegible]
197	·0009	·0006	·0006	·0042	[illegible]
195	·0007	·0007	·0005	− ·0013	[illegible]
193	·0005	·0007	+ ·0003	+ ·0002	[illegible]
191	·0002	·0007	− ·0001	[illegible]	[illegible]
189	+ ·0001	·0006	·0003	[illegible]	[illegible]
187	·0000	·0005	·0005	[illegible]	[illegible]
185	− ·0001	+ ·0003	·0006	[illegible]	[illegible]
183	·0002	·0000	·0007	[illegible]	[illegible]
181	− ·0003	− ·0002	− ·0007	[illegible]	[illegible]
Sums,	− ·3002	− ·0039	− ·0683	[illegible]	[illegible]

TABLE IV.—*continued.*

Fourth Quadrant.

u	$\frac{1}{m'}\frac{dn}{n}$	$\frac{1}{m'}de$	$\frac{1}{m'}ed\varpi$	$\frac{de}{m'}-\frac{d\varpi}{m'}\{1-\sqrt{(1-e^2)}\}$	$\frac{1}{m'}\frac{dn}{n}\times(2\pi-nt)$
359°	+·0034	−·0001	+·0001	·0000	·000
357	·0070	·0001	·0001	·0000	·000
355	·0111	·0002	·0001	·0000	·000
353	·0144	·0003	·0002	·0000	·000
351	·0172	·0004	·0001	−·0001	·000
349	·0190	·0004	+·0001	·0001	·000
347	·0196	·0004	−·0001	·0002	·000
345	·0186	·0005	·0003	·0003	·000
343	·0156	·0004	·0007	·0004	·000
341	·0102	·0003	·0012	·0005	·000
339	+·0019	−·0001	·0020	·0005	·000
337	−·0091	+·0001	·0030	·0006	·000
335	·0228	·0003	·0041	·0005	−·001
333	·0391	·0005	·0054	−·0002	·001
331	·0570	·0008	·0067	+·0003	·002
329	·0767	·0012	·0080	·0011	·003
327	·0939	·0016	·0089	·0023	·005
325	·1103	·0019	·0094	·0037	·006
323	·1230	·0022	·0092	·0055	·008
321	·1318	·0025	·0083	·0075	·009
319	·1355	·0027	·0065	·0096	·011
317	·1339	·0029	·0040	·0116	·012
315	·1273	·0032	−·0006	·0135	·013
313	·1159	·0034	+·0035	·0150	·013
311	·1010	·0037	·0081	·0160	·013
309	·0824	·0039	·0132	·0164	·011
307	·0612	·0041	·0187	·0160	·009
305	·0380	·0043	·0243	·0149	·006
303	−·0144	·0046	·0299	·0128	−·003
301	+·0106	·0048	·0355	·0099	+·002
299	·0361	·0052	·0409	·0056	·008
297	·0611	·0054	·0461	+·0005	·015
295	·0857	·0057	0506	−·0059	·022
293	·1101	·0060	·0548	·0136	·031
291	·1335	·0061	·0584	·0226	·040
289	·1556	·0062	·0613	·0327	·051
287	·1760	·0062	·0633	·0441	·062
285	·1956	·0061	·0647	·0567	·073
283	·2132	·0058	·0650	·0707	·086
281	·2287	·0053	·0644	·0855	·098
279	·2417	·0047	·0626	·1010	·111
277	·2528	·0038	·0597	·1175	·123
275	·2611	·0027	·0556	·1345	·136
273	·2669	+·0013	·0504	·1517	·147
271	+·2700	−·0004	+·0440	−·1688	+·158
Sums,	+1·3634	+·1156	+·8973	−·8465	+1·037

In order to calculate the definite integrals arising from V_3 for $u=90°$, $180°$, and $270°$ (the middle value is given in addition to what is strictly necessary, in order to give the results for the second and third quadrants separately), the quantities given in the first half of p. 409 are applicable.

We have in addition—

	$u=90°$	$u=180°$	$u=270°$
g'	$199°\cdot52$	$207°\cdot13$	$214°\cdot74$
$\frac{x'}{a}$	$+\ \cdot524$	$+\ \cdot536$	$+\ \cdot540$
$\frac{y'}{a}$	$+\ \cdot175$	$-\ \cdot110$	$+\ \cdot043$
$\frac{1}{na}\frac{dx'}{dt}$	$+\ \cdot310$	$+\ \cdot149$	$+\ \cdot001$
$\frac{1}{na}\frac{dy'}{dt}$	$-1\cdot222$	$-1\cdot263$	$-1\cdot285$
$\frac{1}{m'}\cdot\frac{1}{n}\int dn$	$-0\cdot68$	$-1\cdot598$	$-1\cdot40$
$\frac{1}{m'}\int de$	$-0\cdot387$	$-0\cdot665$	$-0\cdot351$
$\frac{1}{m'}e\int d\varpi$	$+1\cdot16$	$-0\cdot094$	$-1\cdot36$
$\frac{1}{m'}\left[\int d\zeta+\sqrt{(1-e^2)}\int d\varpi-\frac{2\pi-nt}{n}\int dn\right]$	$+3\cdot32$	$+0\cdot600$	$-2\cdot76$

Hence

Quadrant.		$\frac{1}{m'}\cdot\frac{1}{n}\int dn$	$\frac{1}{m'}\int de$	$\frac{1}{m'}\int ed\varpi$
I.	V .	$+0\cdot01\ 63$	$-0\cdot17\ 44$	$+0\cdot86\ 04$
II.	V_4 .	$+0\cdot27\ 82$	$-0\cdot00\ 96$	$-0\cdot02\ 89$
II.	V_3 .	$-0\cdot92$	$-0\cdot27\ 8$	$-1\cdot25$
III.	V_4 .	$-0\cdot30\ 02$	$-0\cdot00\ 39$	$-0\cdot06\ 83$
III.	V_3 .	$+0\cdot18$	$+0\cdot31\ 4$	$-1\cdot27$
IV.	V .	$+1\cdot36\ 34$	$+0\cdot11\ 56$	$+0\cdot80\ 73$
	Sum	$+0\cdot62$	$-0\cdot03\ 6$	

Substituting $\quad m'=\frac{1}{3501\cdot6}$

$$\frac{1}{n}\int dn=+\cdot000\ 177 \qquad \int de=-\cdot000\ 010$$

De Pontécoulant's results are

$$+\cdot000\ 129 \qquad -\cdot001\ 10$$

To calculate $\int d\zeta$

Quadrant	$\frac{1}{m'}\int\frac{dn}{n}(2\pi - nt)$	$\frac{\int de}{m'} - \frac{\int d\varpi}{m'}\left\{1 - \sqrt{(1-e^2)}\right\}$	$\frac{e\int d\varpi}{m'}$
I. .	+0·674	− ·30 95	+ ·86 04
II. .	+1·524	+ ·31 75	− ·02 89
III. .	−0·237	+ ·32 12	− ·06 83
IV. .	+1·037	− ·84 65	+ ·89 73
Sums	+2·998	− ·51 73	+1·66 05

Hence $\frac{1}{m'}\int d\zeta$ (mechanical quadratures only)

$$= +2·998 - 0·517 - ·436 = +2·05$$

Again

$$\frac{1}{m'}\left[\int d\zeta + \sqrt{(1-e^2)}\int d\varpi - \frac{2\pi - nt}{n}\int dn\right]_{u=90°}^{u=270°} \quad . \quad . \quad . \quad = -6·08$$

$$-\frac{\sqrt{(1-e^2)}}{e} \times \frac{e\int d\varpi}{m'} \text{ between limits } \quad . \quad . \quad . \quad . \quad = +0·66$$

$$\frac{1}{m'}\frac{2\pi - nt}{n}\int dn \text{ at upper limit} = -1·40 \times 0·6036 \quad . \quad . \quad = -0·85$$

$$-\text{ditto at lower limit} = -(-0·68) \times 5·6796 \quad . \quad . \quad = +3·86$$

Sum − 2·41

Hence $\frac{1}{m'}\int d\zeta$ (V_3 only) $= -2·41$

$\frac{1}{m'}\int d\zeta = -0·36$ (complete value),

or putting $m' = \frac{1}{3501·6}$

$\int d\zeta = -·000\ 103$ radians.

De Pontécoulant gives −·000628 radians.

Hence his value of the next perihelion passage, considering the action of Saturn only, is two days later than ours. De Pontécoulant's eccentricity is again certainly in error; presumably

the same (or approximately the same) factor has been omitted as in the case of Jupiter.

Errata in previous Paper.

Page 386, $e_x = -(1-e^2)\sin u \cos u$.

Page 393, *read* η' *for* y' in the value of $\frac{x'}{a}$.

Page 411, second line, *read* $-0{\cdot}001129$ *for* $-0{\cdot}000273$; hence, third line, *read* $+0{\cdot}147631$ *for* $+0{\cdot}148487$,

$$\int d\zeta = +{\cdot}15506 \text{ for } +{\cdot}15592 \text{ radians.}$$

Last line, *for* thirteen days *read* nine days.

Observations of Comets d, e, and g 1906, from Photographs taken with the 30-inch Reflector of the Thompson Equatorial at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

The following positions of Comets *d*, *e*, and *g*, 1906, were obtained from photographs taken with the 30-inch Reflector.

The plates were measured with the astrographic micrometer. Six reference stars were, as a rule, measured with the Comet, their positions being derived when possible from the Catalogues of the Astronomische Gesellschaft.

The positions given are not corrected for Parallax.

log Parallax Correction = log Parallax Factor − log Δ.

Comet *d* 1906.

Date and G.M.T. 1906.		Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Dec.
d	h m s	h m s	° ′ ″		
July 25	13 59 38	0 43 30·75	− 8 58 20·3	−9·358	+0·872
31	14 7 57	1 35 5·93	− 4 19 2·5	−9·399	+0·854
Aug. 1	14 5 57	1 44 7·08	− 3 27 42·2	−9·412	+0·850
3	13 59 54	2 2 21·04	− 1 42 58·7	−9·442	+0·843
3	14 21 36	2 2 29·32	− 1 42 11·0	−9·402	+0·843
4	13 53 12	2 11 30·43	− 0 49 56·6	−9·459	+0·839
4	14 8 10	2 11 36·04	− 0 49 19·2	[illegible]	[illegible]
14	14 11 10	3 40 36·13	+ 7 37 57·9	[illegible]	[illegible]
15	14 23 15	3 48 52·33	+ 8 22 15·6	[illegible]	[illegible]
19	14 55 33	4 19 59·80	+11 1 30·2	[illegible]	[illegible]
27	14 17 43	5 12 40·71	+14 56 14·7	[illegible]	[illegible]
28	14 21 16	5 18 30·25	+15 19 1·4	[illegible]	[illegible]
Sept. 1	14 48 29	5 40 16·79	+16 37 [illegible]	[illegible]	[illegible]
25	15 16 08	7 13 11·97	+20 10 [illegible]	[illegible]	[illegible]
26	14 49 5	7 16 1·49	+20 14 [illegible]	[illegible]	[illegible]
Oct. 16	15 33 34	8 1 39·08	+20 49 [illegible]	[illegible]	[illegible]
27	16 39 11	8 17 18·34	+21 5 [illegible]	[illegible]	[illegible]

Comet *e* 1906.

Date and G.M.T. 1906.		Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Dec.
d	h m s	h m s	° ′ ″		
Aug. 28	12 24 28	22 45 5·33	+10 6 34·7	+7·909	+0·762
29	13 22 15	22 44 17·63	+10 2 20·7	+9·036	+0·766
Sept. 11	10 51 47	22 35 1·48	+ 8 54 23·6	−8·570	+0·773
25	10 24 13	22 28 7·13	+ 7 17 52·0	+8·248	+0·785
26	11 5 37	22 27 47·95	+ 7 10 40·3	+8·958	+0·788
27	10 26 39	22 27 31·62	+ 7 3 53·5	+8·549	+0·787
27	10 56 57	22 27 31·15	+ 7 3 44·1	+8·927	+0·789

Comet *g* 1906.

Date and G.M.T.		Apparent R.A.	Apparent Dec.	Log. Parallax Factor. R.A.	Dec.
Nov. 17	13 5 42	9 47 58·07	+21 6 24·8	−9·578	+0·768
22	14 9 10	10 15 26·06	28 3 29·9	−9·558	+0·679
Dec. 10	13 50 55	12 25 54·32	50 4 6·1	−9·749	+0·562
12	13 55 23	12 42 49·47	51 43 23·4	−9·766	+0·553
21	12 59 28	13 57 37·79	56 51 2·3	−9·811	+0·692
1907. Jan. 17	11 49 51	16 31 15·30	60 46 4·7	−9·730	+0·826

Observations of Jupiter during the apparition of 1906–7.

By Rev. T. E. R. Phillips.

During the past apparition I was able to observe the planet on 106 occasions. The instrument used was a 9¼-inch equatorial reflector, the large mirror of which (originally by With) has been recently refigured by Mr Calver. Somewhat curiously—considering the exceedingly favourable position of the planet—atmospheric troubles proved unusually annoying, especially when observations were made before midnight. No doubt the somewhat low position of the instrument is partly accountable for this state of things, but there can be no question that the atmosphere was commonly much disturbed, and during the spring months the prevalence of cloud was almost abnormal. Owing to this latter cause observations had practically to be discontinued at the beginning of May.

The following are the particulars of the planet's position, etc. on the date of "opposition."

Date of "Opposition."	R. A.	Dec.	Equatorial Diameter.	Latitude of Centre of Disc.
	h m s	° ′ ″	″	°
1906 Dec. 28	6 26 14	+23 12 50	47·8	+1·97

General Remarks on the Appearance of the Disc.

(1) *Surface Markings.*—The most notable change, as compared with the previous apparition, was the great development of the N. equatorial belt. It may be remembered that at the beginning of

1906 this belt was so feeble as to be little more than the N. edge of a slaty-blue shading, which at that time covered the N. part of the equatorial zone. The first indication of the great change that subsequently occurred was observed by Mr W. F. Denning, who in April found a very dark spot close to the S. edge of the N. temperate belt, and connected with the N. equatorial belt by a slanting streak. This spot seemed to be the centre of an eruption from which a large quantity of dark matter passed *via* the slanting streak into the N. equatorial belt. Unfortunately, the planet was then approaching conjunction with the Sun; but when observations were resumed in August, it was at once seen that a complete transformation had taken place. The N. equatorial belt then obviously exceeded the S. equatorial belt in breadth, and was distinguished by a remarkable series of alternate gaps or white rifts and very dark reddish streaks along its S. edge. These streaks were connected by delicate wisps with dark spots at the N. edge of the S. equatorial belt. Later in the apparition the N. equatorial belt was clearly seen to be triple, but the S. component was much the darkest of the three.

The great S. tropical disturbance was again an interesting feature of the disc. Its f. portion was still in conjunction with the red spot when observations were commenced in August, but the p. end, which had been very vague and ill-defined during the earlier part of the year, had regained all its former sharpness and clearness. The dark matter, as measured along the S. temperate belt, extended over nearly 60° of longitude at the commencement of the observations, but diminished to something like 47° by the end of the apparition. The length, as measured along the S. equatorial belt, was rather less. The p. and f. white spots were still conspicuous objects.

The old red spot was quite plainly seen on several occasions. It commonly presented the aspect of a faint elliptical grey shading which was lighter in the middle. The darkest portion was near the f. end, as has usually been the case in recent years, but in the moments of best seeing the dusky streak at its s.f. border appeared quite separated from the f. "shoulder" of the bay. There was also a wisp connecting the p. end of the spot with the S. temperate belt and the p. "shoulder." The well-known bay or hollow in which the red spot lies was again a well-defined object, with the exception that during the earlier part of the apparition the S. component of the S. equatorial belt was very narrow and very faint at and for some distance in front of the p. "shoulder."

(2) *Colour Changes.*—A conspicuous change of colour, compared with the apparition of 1905–6, was exhibited by [illegible] equatorial belt. Distinctly bluish before the planet's conj[illegible] with the Sun, it was decidedly red when observatio[illegible] recommenced in August. A few months later the S. [illegible] still intensely red, almost brick-red in tone, but the centr[illegible] of the belt had faded to yellowish-red, and the N. edge [illegible] even bluish-grey.

The S. equatorial belt, which during the previous apparition had been purplish, usually seemed wanting in distinctive colouring, but occasionally traces of purple or reddish-purple were seen, especially in the region following the f. "shoulder" of the red spot bay.

The whole of the disc from the N.N. temperate belt to the N. pole was commonly bluish-purple, whereas during the apparition of 1895–6 the N. polar regions had been reddish.

Rotation Periods.

My chief endeavour during the period covered by the observations was to secure as large a number as possible of eye-estimated transits of spots and other markings over the central meridian of the illuminated disc, with a view to determining the mean rotation period in different latitudes. About 1550 such transits were obtained, and on reducing them and charting the longitudes I found that about 90 objects had been sufficiently well observed to leave little doubt as to the identifications. The deduced mean values of the rotation periods of spots situated in the principal surface currents are given in the accompanying table.

Table of Rotation Periods.

Current.	No. of Spots observed.	Mean No. of Observations.	Mean No. of Rotations.	Rotation Period. h	m	s
South South Temperate Belt . .	1	11	171	9	55	10·9
Bright and dark spots at S. edge of South Temperate Belt . . .	20	10	375	9	55	21·8
Great South Tropical Disturbance. *Preceding* white spot . .	1	23	639	9	55	22·9
Great South Tropical Disturbance. *Preceding* end of dark matter (at S. Temp. belt) . .	1	32	644	9	55	22·6
Great South Tropical Disturbance. *Preceding* end of dark matter (at S. Equat. belt) . .	1	21	434	9	55	23·5
Great South Tropical Disturbance. *Following* end of dark matter (at S. Temp. belt) . .	1	26	610	9	55	21·1
Great South Tropical Disturbance. *Following* end of dark matter (at S. Equat. belt) . .	1	16	355	9	55	21·3
Great South Tropical Disturbance. *Following* white spot . .	1	21	478	9	55	21·3
Means for whole S. Tropical disturbance	6	23	527	9	55	22·1
Great Red Spot Hollow	1	34	589	9	55	42·2
White and dark South Equatorial spots	36	15·5	391	9	50	27·1
White and dark North Equatorial spots	18	14	310	9	50	41·8
Spots on North North Temperate belt	7	10	308	9	55	41·7
Spots within N. Polar shading .	2	10	257	9	55	40·6

Remarks on the Rotation Periods and the Movements of Spots.

S.S. Temperate Belt.—The value here given is somewhat longer than that usually exhibited by spots situated in the great southern current. One or two transits were secured earlier in the apparition, which quite possibly belonged to the object observed, and which would make the period about normal ($9^h\ 55^m\ 6^s \pm$). The identification is however uncertain, owing to a rather long gap in the observations, and the transits mentioned have consequently not been used in the reduction.

Spots at S. edge of South Temperate Belt.—The period of this current is also some two or three seconds longer than that usually found, but the number of spots observed is so large that there can be very little doubt as to the reality of the slight retardation indicated.

The Great Red Spot.—The rotation period of this object and of the hollow in which it lies has been notoriously variable during the last few years, especially about the times when the great S. tropical disturbance, which is situated in the same latitude and has a quicker motion, has been in conjunction with it. Conjunctions took place in 1902, 1904, and 1906, and on each occasion the effect of the dark material sweeping past the red spot has been to accelerate temporarily the latter's rotational velocity. This was especially marked last year. Thus in April 1906 the longitude of the red spot was about 29°, but when observations were recommenced in the second week in August it was found that the longitude had diminished to something like 15°·5. So rapid a shift had not previously been observed in connection with the red spot. It is further noteworthy that the very marked rotational acceleration which the shift of longitude denotes had by that time entirely ceased, and was subsequently followed by a decided retardation. By the beginning of May 1907 the longitude of the red spot and hollow had increased to rather over 24°.

The Great S. Tropical Disturbance.—It will be remembered that Major Molesworth, as the result of certain computations, has suggested an interesting hypothesis as regards the manner of the passage of the S. tropical disturbance from the f. to the p. side of the red spot. This passage undoubtedly takes place *via* the S. temperate belt, but the special point in Major Molesworth's contention is that the apparent transference of matter is practically instantaneous. In other words, instead of the p. end of the disturbance occupying between two and three months in passing round the S. edge of the spot, he believes that almost immediate it comes up to the f. "shoulder" of the bay, dark matter app above the p. "shoulder." It is as if the material of t temperate belt were incompressible, so that as soon as t substance of the great disturbance enters it, it displaces quantity at the other end.

An opportunity of partially testing this hypothes

observations seemed to present itself at the time of the 1906 conjunction, but an unexpected difficulty arose through the extreme faintness and indefinite character of the dark matter which first appeared on the p. side of the red spot. Thus the p. end of the disturbance was observed to arrive at the f. "shoulder" ($\lambda = 47^\circ{\cdot}5$ Syst. ii.) about Feb. 25, and the region above the p. "shoulder" ($\lambda = 10^\circ \pm$) was immediately watched. The expected dusky material, however, did not make its appearance, or at any rate it was not sufficiently intense to be observed. But later on, in the early part of April, it became obvious that the S. tropical zone above and in front of the p. "shoulder" had lost something of its white appearance, though it was not as yet possible to feel sure of the existence there of the stream of dark matter. By the middle of the month, however, all doubt had vanished, and the p. end of the disturbance was clearly discerned in about $\lambda\ 355^\circ$. Now, working backwards from the observed rate of motion during the apparition just closed, it appears that this must have been just about the longitude at the middle of April; and further, that the p. end of the disturbance, supposing it to have been moving at a uniform rate, must have been at the p. "shoulder" about March 11.

According to this computation, then, the p. end traversed the distance between the two "shoulders" of the bay—some 37°—in only fourteen days, instead of occupying nearly three months in the transit!

An investigation of the motion of the f. end of the disturbance gives a closely similar result. As above mentioned, the red spot hollow exhibited a quite abnormally large shift in longitude during the time of the planet's invisibility; and at the resumption of observations the longitude of the f. "shoulder" was only about 34°, and that of the p. "shoulder" about 357°. On the occasion of my first observation in the new apparition (1907 August 10) the dusky wisp connecting the end of the red spot with the f. "shoulder" was so dark and broad as to give the impression that it was in reality the f. end of the great S. tropical disturbance just arriving at the f. "shoulder" of the bay, and an entry to this effect was made in my observation-book. This surmise is confirmed by the fact that, assuming a constant rate of motion, the f. end of the disturbance was due in this longitude about August 7, only a very few days before the observation above mentioned. Now, the first few observations I was able to secure of the f. end of the disturbance after its appearance on the p. side of the red spot make it clear that Aug. 22 was about the date of its conjunction with the p. "shoulder." This means that the f. end of the disturbance traversed the length of the red spot bay between approximately August 10 and 22, instead of occupying nearly three months in the process.

On the whole, therefore, it seems that my observations confirm the main point of Major Molesworth's theory, though indicating an interval of about two weeks as needed for the apparent transference of the dark matter from the f. to the p. side of the red spot.

Equatorial Spots.—The motion of both S. and N. equatorial spots varied considerably during the course of the apparition, a decided acceleration of S. equatorial spots between longitudes 0° and 210° ± (System i.), and of N. equatorial spots in almost all longitudes, setting in towards the end of January. There was a very large difference of rate—amounting to over 14 seconds per rotation—between the two portions of the equatorial current.

Description of an Equatorial Reflecting Telescope driven by a Hydraulic Ram. By T. E. Heath.

The drawing shows the optical axis of the telescope pointing to the pole.

A, reflecting telescope ($8\frac{1}{2}''$ mirror) in square teak tube.

B, cast-iron right ascension circle 21″ dia., having a groove formed round periphery.

C C, cast-iron A frames bolted to R.A. circle, carrying declination axis, and allowing telescope to be set at any declination from 40° S. to 90° N.

D, cast-iron circle, carrying steel balls upon which the R.A. circle revolves.

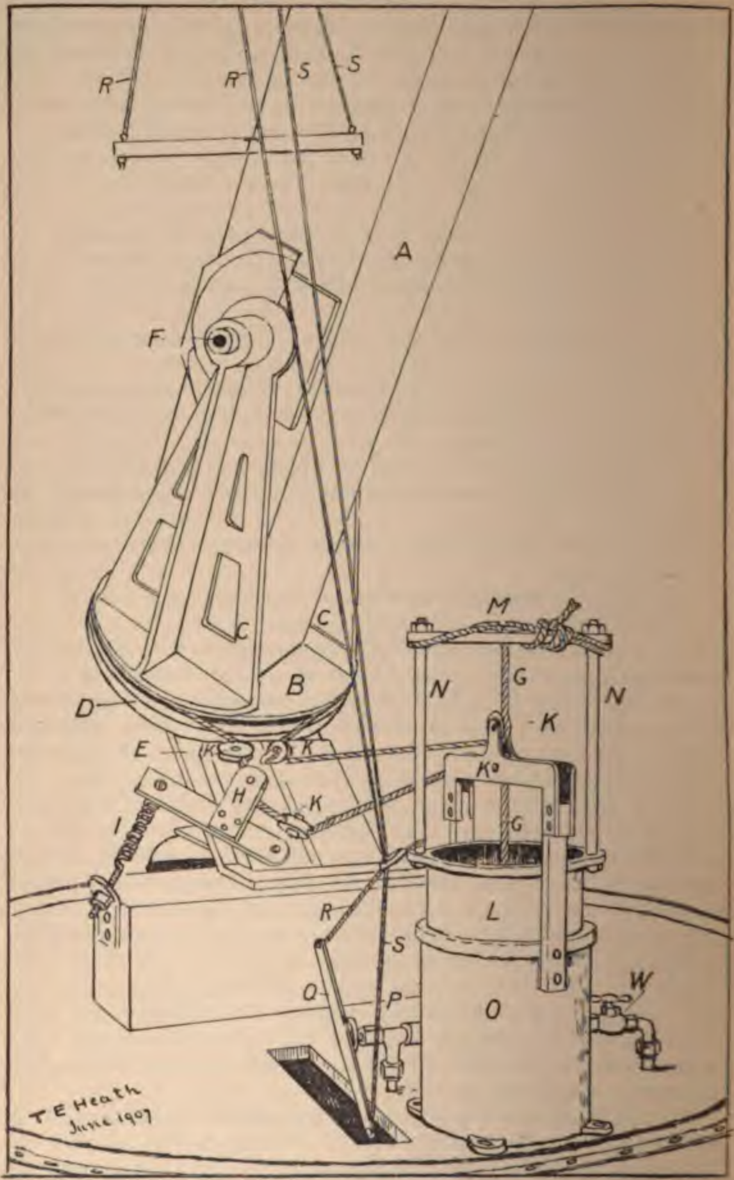
E, cast-iron base-plate, to which D is bolted.

F, declination axis.

G, rope, which fits bottom of the groove in R.A. circle and is kept taut by a tension pulley carried by a lever H which is pulled by a spiral spring I. The rope passes over guide pulleys K, and one end of it is attached to the bottom of a hollow brass hydraulic piston L and the other end to a cross-bar M, which is supported at a sufficient height above the piston L by the pillars N. O is a brass hydraulic cylinder, 8″ dia. and 10″ high. (The piston L has a cup-leather below it.) The supply of liquid to the cylinder is regulated by a valve P (which is one of those sold to regulate the supply of oxygen from a cylinder). The valve is operated by a lever Q, to one end of which is attached a cord R and to the other end a cord S. The cords R and S are led over suitable guides to either end of a bar T, which han[illegible] the roof of the observatory, conveniently near to the obs[illegible]

The telescope is balanced by about [illegible] lead attached to the lower end of the tube, so that it will [illegible] whatever declination it is set, and so that the [illegible] move it in R.A. is everywhere approximately [illegible]

I find that if the rope G [illegible] vers the bottom of the groove in the R.A. [illegible] that it grips the sides, and the spiral sprin[illegible] pull," then the friction will not prevent [illegible] ing the telescope on any required star, [illegible] he let go the telescope to cause the R.A. [illegible]



Equatorial Reflecting Telescope driven by a Hydraulic Ram.

In practice, the liquid is allowed to flow into the hydraulic cylinder all the time the observer is working. He shifts, as desired, from star to star, and the telescope follows automatically. The hydraulic cylinder is large enough to run the telescope for 2½ hours (it could be made longer if desired). A relief valve marked W is opened, and by standing on the piston it is pressed down, ready for a fresh start. This takes about one minute. It takes less than five minutes from the time I leave the house to the time the telescope is following an easily visible star.

At present I use the water direct from the town supply. This does well enough for observing, as it is quite easy, by means of the bar T, to regulate the supply of water to suit the varying pressure. For photographic work, it would be desirable to draw the supply of liquid from a high tank, in which the level is kept constant by a ball valve, and it would be preferable to use glycerine or something not easily frozen. (2½ hours' supply is about one gallon.)

On May 27 I found Jupiter with the circles at 5·50 p.m., and followed it till 9·5 p.m., when it set behind trees. I had several times to vary the supply of water, but sometimes the telescope would run truly for 30 minutes. I had, of course, to empty the cylinder once. The motion is absolutely steady. You can get an even sweep in either direction by turning on more or less water. Then, when a suspected object comes across the field, you can go slow and stop it in the middle.

The observatory runs round upon a circular railway attached to the floor. In each corner, forming part of the structure, is a triangular cupboard. Thus the eyepieces, etc. are always handy.

The cost was approximately as follows :—

Mirror, eyepieces, tube, etc., £35 ; the equatorial stand, £10 ; the hydraulic clock, £6; and the observatory £10. This, of course, does not include my time, but I did not do any of the manual work.

P.S.—I have improved the working of the inlet valve by screwing a wooden sheave, 9 in. diameter, to the outer face of the lever Q. The cords R and S are fastened to the sheave and wound round it before being taken to the observer.

The Origin of certain Bands in the Spectra of Sun-spots.
By A. Fowler.

In the spectra of sun-spots there are numerous short hazy lines, termed "band lines" or "umbra lines," which have not hitherto been traced to their source. They are especially notable in the neighbourhood of *b*, and on the more refrangible side of this group, and the positions of some of them have been estimated by various observers, including Maunder, Cortie, Hale, Newall, and myself. The close resemblance between these lines and the structure lines of a banded spectrum seen under high dispersion led me to make numerous experiments on banded spectra during 1905 and 1906, with the hope of identifying the substance producing them, but the results were then entirely negative.* I have lately discovered, however, that a great number of these lines really form part of a fluted spectrum, and are to be accounted for by the presence of a compound of magnesium and hydrogen in the umbræ of spots. Pending a more complete comparison, which will take a considerable time, the present preliminary note may be useful to those engaged in the investigation of spot spectra.

The magnesium-hydrogen flutings were the subject of an extended research by Liveing and Dewar many years ago,† when it was clearly shown that hydrogen and magnesium were together involved in their production. Independent evidence as to the existence of a compound of these two elements has since been obtained,‡ and there need be no hesitation in attributing the flutings in question to the chemical combination "magnesium hydride."

The brightest of these flutings, or rather groups of flutings, begins in the green near 5211 and fades off towards the violet, while there is another group beginning near 5620, and another on the violet side of H_β. In a series of experiments made by Mr Howard Payn and myself,§ they appeared very strongly in the spectrum of the arc between magnesium poles in a vessel exhausted to a pressure of a few millimetres, the hydrogen necessary for their formation being liberated in sufficient quantity from the heated poles. A re-examination of the photographs obtained in this way was suggested by a second reading of Mr Newall's account ¶ of his observations of a fluting in spot spectra beginning at 5210·2 or

* *Trans. Int. Solar Union*, vol. i. p. 217 (1906).
† *Proc. Roy. Soc.*, vol. xxxii. p. 189 (1881).
‡ Winkler, *Ber. Deutsch. Chem. Gesell.*, vol. xxiv. p. 1973 (1891).
§ *Proc. Roy. Soc.*, vol. lxxii. p. 254 (1903).
¶ *Monthly Notices*, vol. lxvii. p. 170 (1906).

5211·0 *; and although the dispersion was comparatively small, the general resemblance to the spot bands was so striking as to encourage further inquiry.

The 5211 group has accordingly been photographed with a more powerful spectrograph, giving about five-tenth metres to the millimetre in the region of *b*, and new determinations of wave-lengths have been made. Many of the components of the flutings which appeared as single lines with the smaller dispersion are now shown to be double or triple, exactly as they appear in the spot spectrum at corresponding wave-lengths.

The identification might be considered an extremely probable one by comparison with the various visual observations of the spots, but it is rendered quite certain by reference to the magnificent photographs of the spot spectrum which have been obtained at Mount Wilson.† The correspondence would be best shown by photographs of the two spectra on the same scale, but a grating was not available for the magnesium hydride spectrum, and an exact superposition cannot yet be made. Nevertheless, through the medium of a solar spectrum taken with the same instrument as the flutings, the relation can be closely followed.

The first part of the 5211 group agrees remarkably well with Mr Newall's description of the corresponding spot fluting and with the photographic map, the fluting in each case fading off rather rapidly in the direction of *b*. A second fluting, with its head near the more refrangible edge of b_1, is also clearly indicated in the spot photographs, while the well-known double umbra lines about 5163, 5160, and 5157 are also well represented by components of the flutings. Compared in this way, the apparent coincidences are so numerous as to make the identification almost independent of precise determinations of wave-lengths.

A comparison of wave-lengths, however, so far as it can at present be made, leaves no doubt as to the magnesium hydride origin of most of the spot bands in the region considered. A complete catalogue of the band lines in spots is not yet available; many of them are doubtless masked by superposition upon the stronger Fraunhofer lines, and numerous others which appear in the photographs are not yet included in the Mount Wilson tables. Still, as will appear from the following abridged table, giving *provisional* wave-lengths for some of the brighter components of the magnesium hydride flutin[illegible] references to the spot spectrum, there is abundant justificati[illegible] identification in question. It will be seen that som[illegible] uting components are not represented in the list of [illegible] hs, but it is believed that all of them can be traced [illegible] [illegible]hic map, unless hidden by Fraunhofer [illegible]

* Umbra [illegible] own catalogue, but I had not pre[illegible] th the head of a fluting.

† *Astrop*[illegible] cent unpublished map which I[illegible]

Magnesium Hydride in Sun-spots.

Magnesium Hydride.		Spots.		Remarks.
λ (Approx.)	Intensity.	Fowler.*	Hale.†	
Triple Head. 5211·00	4	11·11 / 11·02	11·02	Newall 11·00
10·47	5			Falls on edge of Ti 10·56
09·90	4	10·06 / 09·95		Newall 10·2
09·24	2	09·2		
08·43	2			On edge of strong Cr 08·60
07·30	2	07·26		
05·97	2			On edge of strong Cr 06·21
04·37	1–2		Fringe on violet edge of Cr 04·68	
03·61	1–2	03·66	03·66	
03·08	2	03·12 / 02·95		
01·82	1–2	01·77	01·80	
01·08	1–2			Combined in spot with Ti 01·2
5200·68	1–2			
5198·90	1–2			Masked by Fe 98·89
98·45	1–2	98·51	98·52	
96·58	1–2			Masked by Cr and Mn
96·24	1–2		Fringe on violet edge of Fe	
85·90	1–2	85·9		
85·14	1–2	85·20	85·21	
82·37	2			
82·08	2	82·12 / 82·01		
81·37	3	81·50		
80·70	3	80·75	80·66	
80·12	3			On Fe 80·23
79·18	3	79·29		
78·63	3	78·64	78·67	
77·41	3			On Fe 77·41
5176·94	3	76·95		Spot line in part due to V

* Visual Estimates, *Trans. Int. Sol. Union*, vol. i. p. 208, and *Monthly Notices*, vol. lxv. p. 218.

† Photographic Determinations, *Astrophys. Journal*, vol. xxiii. p. 26.

Magnesium Hydride in Sun-spots—*continued*.

Magnesium Hydride.		Spots.		Remarks.
λ (Approx.)	Intensity.	Fowler.*	Hale.†	
5175·50	3	75·58 75·42	75·56	
75·06	3	75·10	75·11	
71·21	3	71·19	71·20	
70·74	3	70·77	70·78	
68·67	2–3			Close to Ni 68·83
68·26	2–3	68·36	68·35	
66·09	2–3	66·13		
65·69	2–3			Close to Fe 65·69
63·25	3–4	63·0*d*	63·10 very broad	
62·89	3–4			
60·30	2–3	60·2	60·39	
59·91	2–3	59·8	59·96	
57·07	2–3	57·2	57·20	
56·79	3	56·8		
55·89	1–2			Falls on Ni 55·94
54·84	3			Conspicuous dark place in photographic map
53·85	2			
53·41	3			
50·31	4	50·4	50·36	
49·64	3	49·9	49·67	
46·54	1–2			Falls on Ni 46·66
46·34	3			Conspicuous in photographic map
45·35	1–2			
44·82	1–2			
44·21	1–2		44·21	
43·76	1–2		43·83	
42·91	2			Falls on Ni 42·96
42·57	2			Close to Fe 42·69
41·41	1–2		41·40 (broad)	
40·43	1–2		40·44 (broad)	
38·92	2		38·96 (broad)	
36·55	2		36·65	Conspi[illegible]graphic map
5136·23	2			

* Visual Estimates, *Trans. Int.* [illegible] vol. [illegible] *Monthly Notices*, vol. lxv. p. 218.

† Photographic Determinations, [illegible]

Additional photographs, taken with longer exposures, and covering a greater range of the spectrum, are required before it can be ascertained exactly how much of the detail of the spot spectrum is due to magnesium hydride, but there are indications that several hundreds of the umbra lines will be accounted for. The interpretation of the spot spectrum will evidently be greatly simplified by the elimination of the part due to the flutings, which can be made with greater certainty now that their origin has been traced.

The identification is also of interest as supporting the view that the vapours in spots are at a relatively low temperature. The special strengthening of the flame lines of metals in spots determined independently by Professor Hale and myself, and the subsequent detection by Professor Hale of the extreme red flutings of titanium oxide, however, had left little doubt on this point.

As a great number of the umbra lines under consideration were found by Professor Hale to correspond with very faint solar lines tabulated by Rowland, it must be inferred that magnesium hydride not only occurs in spots, but in a less degree in the general reversing layer of the Sun.

I have great pleasure in expressing my obligations to Messrs H. Shaw and E. J. Evans for able and unstinted assistance in carrying out this preliminary investigation under somewhat difficult conditions.

The Spectrum of Mira Ceti in December 1906, *as photographed at Stonyhurst College Observatory.*

Rev. Walter Sidgreaves, S.J.

The short series of photographs of the spectrum of ο Ceti obtained during the unfavourable weather of December last consists of 18 plates on eight nights between December 1 and the following January 3. Of these, 13 plates are by the Thorp objective prism and 5 by the Hilger compound prism. Both prisms give only short spectra. Those by the objective extend well beyond the head of the hydrogen series of a star of the 3rd magnitude, with a length of 22 mm. between H_γ and the red end limit. This limit is a little beyond H_β, for the camera cannot conveniently be adapted to the focal curve of the green and yellow rays. The compound prism stops the violet light by absorption, and the spectrum reaches H_ϵ only of the brighter stars. On the other hand, it extends to the yellow sodium lines on an Edwards isochromatic plate, all in good definition, but covering only about 16 mm. on the plate. This prism was the only one in use at the maxima of 1897 and 1898, and the five plates exposed during the recent exceptionally bright maximum serve for comparisons with those of the earlier dates. They are all that can be desired in definition; but unfortunately no one of them was sufficiently over-exposed (slowly trailed) for

a comparison with the hydrogen spectrum given by the objective prism, as will be seen later.

The line-absorption-spectrum of the star's light in 1906 is substantially the same as in 1897 and 1898; but the bands are very much weaker—quite enough to account for the increase of light at this bright maximum. The bands in the neighbourhood of H_β and H_γ are nearly, perhaps quite, absent on our plates of 1906, and will be referred to immediately in connection with the bright hydrogen lines.

The remarkable change in the hydrogen spectrum, noted in the intensity of H_β by all spectroscopic observers of Mira, is accompanied by a decided change in the relative intensities of H_γ and H_δ. On the plates of 1897 and 1898 the intensities of these lines are quite equal, without allowance for the prismatic absorption at H_δ, or the sensibility curve of the film; and on the same quality of plate (Edwards) in 1906 H_γ is distinctly the stronger.

But whether these changes are to be attributed to an altered condition of the star's hydrogen envelope, or to a diminished absorption in the regions of the lines, cannot yet be determined with certainty. There is, however, some evidence on our spectrograms that the changed appearance of H_β is mostly due to the changed condition of an absorbing atmosphere. The bands in the neighbourhood of H_β [numbered 12, 13, 14 in our tabulations of 1897, *M.N.*, lviii. 348], including the one covering the line, were very strong in 1897, and do not appear on our plates of 1906, but in their stead their sharp edges on the more refractive sides are strong, stout lines. Also the same absorption-bands are weaker, and H_β is stronger on the plates of 1898 than on those of 1897.

The change in the relative intensities of H_γ and H_δ may be accounted for on the same lines, but evidence is wanting on our plates, owing to the weak condition of the absorption-spectrum near H_δ through loss of light in the prism. The objective prism shows a weak absorption-band about H_δ; and if this was, like the rest, stronger in 1897, we should conclude that the change must be attributed to the radiation of the origin rather than to external absorption. The same prism has shown that though apparently weaker than H_γ, H_δ still claims to be the strongest of the series. For on December 1 plates were simultaneously exposed on both spectrographs; and while H_δ appears weaker than H_γ by the compound prism, it is quite its equal by the obj[illegible] prism, without allowing for the less sensibility of the plate [illegible] radiation.

Other peculiarities of the hydrogen spec[illegible] been brought out at this maximum by the objective [illegible] have now a photographic record of the series from H_β [illegible] there is no sign of H_ϵ, while H_ζ is quite strong. [illegible] lines H_η H_θ H_ι, although they [illegible] the stronger photographs, are out [illegible] their intensities following the natural [illegible] with shorter wave-lengths. The [illegible] of proportion weaker than the [illegible] ly

feeble, and are seen only on the strongest photographs, numbered 1983, 1990, and 1992, one of December 12 and two of the following January 3. But they are all easily pointed with a low-power micrometer eyepiece, especially on plate 1992; and they are all bright lines.

The following lines have also been noted as more probably fine bright radiations than thin separations of very broad absorptions, at λ 3843, 3853, 3862, 3872, 3893, 3905.

A special search has been made for the bright lines quoted by Plaskett,* but without any satisfactory result. Our spectra are probably too small. But three lines found at λ 4106, 4119, 4137, agreeing with three of Plaskett's lines, might be called possible bright lines.

But the most remarkable feature brought out by the objective prism is the character of the hydrogen lines. Mr Plaskett, of the Dominion Observatory, has observed on his plates an asymmetrical widening of the lines on the red side, which was more marked on the denser photographs: "No trace could be found of Campbell's triple formation. . . . The lines were, however, in the majority of the plates, unsymmetrically broadened with respect to the actual centre of intensity determined from the tips of the emission lines. . . . . And this asymmetry became more evident the more intense became the line." †

On all our plates by the Thorp objective prism, excepting two of weaker photographic intensity, H^{δ} appears greatly widened on the red side, shading down, but terminating in a comparatively strong bright line; and the same feature is seen in H_{γ} but with less dispersive separation; also on three extra-dense impressions the same is seen in H_{ζ}.

In all cases the features are greatly exaggerated by over-exposure, and there is some evidence of their being more pronounced when the definition is less perfect. On this account, and still more on account of the total absence of apparent asymmetry of the lines on the series of plates by the Hilger compound prism, the writer has long hesitated to put the facts on record. But, after many repeated examinations of both series of photographs, he has come to the conclusion that they cannot be attributed to any photographic freak. And in favour of this conclusion it must be observed, first, that the absence of asymmetry on the photographs by the Hilger prism is quite explained by the great prismatic absorption of the H_{ζ} region, and our unfortunate want of a sufficiently over-exposed plate to bring out the weaker extension of the line; while the dispersion at H_{γ} would be too small to be effective. Secondly, that the features cannot be attributed to inferior definition is satisfactorily established by two photographs of December 10, although at first examination these were the plates that threw doubt upon the reality of a stellar origin of the phenomena. On the first of these

* *Spectrum of Mira Ceti*, by J. S. Plaskett, Roy. Astr. Soc. of Canada, Jan.–Feb. 1907.

† *L.c.*, p. 52.

the photographic intensity is very great with poor definition, and the red-side wing terminating in a bright line is seen very clearly in H_γ H_δ and H_ζ. On the second, the impression is weak with perfect definition, and there is little or no sign of the wing even in H_δ; but alongside of this line there is a feeble but quite distinct bright line in precisely the position of the bright line termination of the wing on the preceding plate: the exposure was just enough to bring out the stronger edge of this extension, leaving the weaker interval untouched; and the same condition of things was found on the other excepted plate of December 1. Again, the measured intervals between the strong centres and the bright line terminations of the wings in H_γ H_δ and H_ζ agreed in showing the same spectral interval of 5-tenth metres.

Our photographs therefore go to confirm Campbell's observation of the triple formation of the hydrogen lines, but enormously developed; and the Dominion plates with a prolonged exposure might have provided additional evidence with further details.

1907 *June* 4.

Note on the Visual Spectrum of Mira Ceti in December 1906.

Rev. A. L. Cortie, S.J.

The spectrum of Mira Ceti was examined visually with a Maclean star-spectroscope attached to the Perry Memorial 15-inch equatorial on the evenings of Dec. 6 and Dec. 9. The visual magnitudes of the star on these two dates have been kindly furnished by Colonel E. E. Markwick, from means of observations by members of the variable star section of the British Astronomical Association, as 2·07 mag. on Dec. 6 and 2·00 on Dec. 9. The maximum intensity in the visual spectrum on these dates was unmistakably in the red and the orange part of the spectrum. The most brilliant part of the spectrum was the bright background or band which began sharply at the edge of Duner's second band at approximate λ 6164 and shaded off towards the violet. The red background to the Duner's bands beginning at approximate λ 6493 and 6164 was very brilliant. The bright space between these two absorption-bands had a central maximum of brightness. With regard to the brilliant space beginning at λ 6164, the edge was so sharp and bright and the intensity faded so perceptibly towards the violet that visually it appeared as a bright band. At least it is safe to affirm that the maximum [illegible]ightness of the star was in the region λ 6164–5958. [illegible] telescope on Dec. 6 the star was distinctly golden in [illegible] much redder than Aldebaran when compared with that [illegible] Dec. 9. These visual observations of the spectrum are co[illegible] all details by four plates taken with the Hilger compound [illegible] the nights [illegible] Dec. 6, 9, 13, and 14. The pl[illegible] were [illegible] pinachr[illegible] and although, as was proved [illegible] plat[illegible] a Cygni, [illegible] effect of the dye is to give an [illegible]ted [illegible] to the [illegible]

part of the spectrum, yet the general appearance of the photographic spectrum on a plate dyed with pinachrom approximates more to the visual spectrum, than photographs taken on isochromatic or ordinary plates. The hydrogen bright lines H_γ and H_δ were well seen on Dec. 6. Assuming that the increased brilliancy of the star at maximum is due to temperature, it is important to notice that the extension of the spectrum into the violet is also accompanied by a much greater radiative power in the red and orange part of the spectrum.

Stonyhurst College Observatory:
1907 *June* 6.

Note on the Colours of α and o (Mira) Ceti.
By W. S. Franks.

In the paper communicated by the Radcliffe observer on the magnitude of o (*Mira*) Ceti [*Monthly Notices*, April 1907, p. 412] there are some remarks on its colour, as compared with α Ceti and other stars. Perhaps some of my recorded observations of these objects may be of interest, as they cover a period from 1876 to 1893. Briefly they are as follows:—

o Ceti orange-red, 26 Nov. 1876, 5-inch o.g.
" splendid yellow, 28 Nov. 1877, " (near *maximum*).
" bright yellow, 22 Oct. 1878 "
" decidedly red, 14 Dec. 1884, $11\frac{1}{4}$ in spec. (about 8^m).
" orange-yellow, 21 Jan. 1885, " (" $3\frac{1}{2}^m$).
" OrR^3 (orange-red), 17 Sept. 1887, $8\frac{1}{2}$ in spec. (est. $7^m \pm$).
" OrR^2 (pale or. red), 24 Sept. 1887, " (invis. to n. eye).
" Or^3 (orange), 4 Oct. 1887, " (about 6^m).
" too small for colour, 1892, 3-inch o.g.
" Y^3 to OrR^3 (10 obs.), 1893, B.A.A. Coloured Star Section.

α Ceti bright orange, 28 Nov. 1877, 5-inch o.g.
" pale orange, 14 Dec. 1884, $11\frac{1}{4}$ in. spec.
" Or^2 (pale orange), 24 Sept. 1887, $8\frac{1}{2}$ in. spec.
" Or^3 (orange), 17 Sept. 1887, $8\frac{1}{2}$ in. spec.
" Or^3 (orange), 4 Oct. 1887, "
" Or^2 (pale orange), (5 obs.) 1892, 3-inch o.g.
" OrY^2 (pale or. yel.), (mean of 33 obs.), 1893, B.A.A. Coloured Star Section.

The colour symbols are those of the nomenclature proposed in vol. xlvii. p. 269 of *Monthly Notices*, which has been found convenient in practice, as by its aid over 36,000 observations have been made, by various persons, of the colours of all the brighter stars in both the Northern and Southern hemispheres, and mean results deduced.

There is no doubt but that *Mira* does exhibit fluctuations in its colour, as evidenced by many observers, but this is probably only an accompaniment of its variation in brightness.

Uxbridge: 1907 *May* 17.

The Relation between Star Colours and Spectra. By W. S. Franks.

During an enforced leisure, I have made some investigations into the connection between the colours of stars and their spectra (see *The System of the Stars*, by Miss Clerke, p. 149), which may possibly be of interest to the Fellows of this Society. The material was gathered from the following sources. For *colours* of the brighter stars, I used the mean values of the observations made by members of the B.A.A. in 1890–3, and published in vol. i. part 3 and vol. ii. part 4 of the *Memoirs* of that Association,—the 928 stars selected depending on about 30,000 observations; they extend from the N. Pole to 25° south declination. For *southern* stars, between $-25°$ and the S. Pole, I made use of the observations by the New South Wales star colour section, published in B.A.A. *Journal*, vol. vi. p. 416 and vol. viii. p. 17. The number of southern stars selected was 432, resting on nearly 5000 observations; the aggregate is therefore 1360 of the brighter stars in both hemispheres. The same colour symbols were used throughout—they are published in the *Monthly Notices*, vol. xlvii. p. 269. The spectra were derived from vol. xxviii. part 2 of the *Harvard Annals* for the southern objects; and for the remainder, from *H.A.*, vol. xlviii. part 4 (catalogue of 1520 bright stars).

At the commencement I made a careful analysis of the colours and spectra of the southern stars, entering all in tabulated form under the 24 headings into which the spectra were divided—a condensed summary of which is seen in Table I. The same was done for the northern stars, as shown in Table II. Table III. is a list of the most obvious *discordances*, which amount to about 2½ per cent. of the whole. Table IV. gives the results of Tables I. and II. combined, and is probably the most trustworthy for generalisation.

TABLE I.

Summary of Analysis of 432 Southern Stars from $-25°$ to S. Pole.

Division.	Spectra.	Type.	O	YG^1	Y^1	OrY^1	Y^2	OrY^2	Or^2	Y^3	OrY^3	Or^3	OrR^3	R^3	Stars
I*a*.	Oa to B9A	Helium stars	46	50	43	6	6	2	...	...	...	...	—	...	153
I*b*.	A to A5F	Hydrogen stars	7	9	28	9	13	...	...	...	...	...	—	...	66
I.–II.	F to F5G	α Carinæ type	1	5	6	6	11	8	...	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
II.	F8G to G5K	Solar stars	...	3	2	5	7	11	...	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
II.–III*a*.	K	Arcturus type	...	...	1	5	4	31	6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	67
II.–III*b*.	K2M and K5M	Aldebaran type	...	...	...	...	2	7	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	47
III.	Ma to Md	Betelgeuse type	...	...	...	1	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		Total	54	67	80	32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

s.—Three G^1 included with O; two YG^2 with Y … OrY^3; six Or^4 with Or^3; two OrR^4 with … Carinæ, in R^3.

Table II.

Summary of Analysis of 928 *Northern Stars, etc., from* – 25° *to N. Pole.*

Group.	Division.	Spectra.	Type.	0	YG^1	Y^1	OrY^1	Y^2	OrY^2	Or^2	Y^3	OrY^3	Or^3	Or
1	I*a*.	Oe to B9A	Helium stars	79	19	26	2	2	1	...	...	...	...	...
2	I*b*.	A to A5F	Hydrogen stars	161	71	76	2	1	...	...	...	...	...	...
3	I.–II.	F to F5G	α Carinæ type	2	6	72	2	12	...	...	1	...	...	...
4	II.	F8G to G5K	Solar stars	...	...	24	7	70	18	4	16	2	1	...
5	II.–III*a*.	K	Arcturus type	...	...	2	7	82	37	3	41	2	...	...
6	II.–III*b*.	K2M and K5M	Aldebaran type	...	...	...	...	2	11	3	6	6	...	...
7	III.	Ma to Md	Betelgeuse type	...	...	1	1	1	23	13	4	2	2	2
			Total	242	96	201	21	170	90	23	68	12	3	2

Notes.—One B^1 and one G^1 included with O; one Or^1 with OrY^1; and one Y^4 with Y^3 spectrum type H in the "Draper" Catalogue (*H.A.*, vol. xxvii.) has been in under G5K, as it does not find a place in the later classification of the se catalogue (*H.A.*, xxviii. 2).

Table III.

List of the Largest Discordances.

a. From Northern List (II.).

Star.	R.A. 1900	Decl.	Spect.	Col.	Remarks.
	h m	° ′			
κ Cassiop.	0 27·3	+62 23	B	OrY^1	Possibly *var.* in col.; previous obs. gi and Y^1.
P. III. 54 Cam.	3 21·0	+59 36	B9A	OrY^2	Certainly *yel.*; confirmed by previous
P. III. 51 ,,	3 21·9	+58 32	B9A	OrY^1	Possibly *var.* in col.; previous obs. ra from Y^1 to Y^3.
μ Persei	4 7·6	+48 9	G	Y^3	Strong *yel.*; confirmed by previous o
29 Can. Maj.	7 14·5	–24 23	Oe	Y^2	Col. confirmed by previous obs.
ξ Argûs	7 45·1	–24 37	G	Y^3	Strong *yel.*; confirmed by previous o
ρ ,,	8 3·3	–24 1	F5G	Y^3	My previous obs. make this YG^2?
ρ Ophiuchi	16 19·6	–23 14	B	Y^2	Col. confirmed by previous obs.
μ Aquilæ	19 29·2	+ 7 10	A?	Y^2	Decidedly *yel.*; previous obs. ra between Y^2 and Y^3.
β Capri.	20 15·4	–15 6	G comp.	Y^3	The brighter component of the known double; decidedly *yel.*; companion (A or A2F) appears *blu* contrast, but in reality it is prob *white.*

TABLE III.—*continued.*

b. From Southern List (I.).

Star.	R.A. 1900. h m	Decl. ° ′	Spect.	Col.	Remarks.
Eridani	2 36·0	−43 20	A	Y^2	Lines broad and ill-defined (*H.A.*, xxviii. 2).
Carinæ	6 21·8	52 39	F	O	Col. confirmed by A. S. Williams.
Can. Maj.	,, 46·1	32 24	B2A	Y^2	Lines broad and ill-defined (*H.A.* xxviii. 2).
Carinæ	7 54·2	52 43	B3A Pec.	OrY^2	Either spectrum or colour wrong?
Lac. 3105 Puppis	,, 55·3	48 58	B1A	Y^1	Spect. binary; fainter component B3A (*H.A.*, xxviii. 2).
Velorum	8 40·8	42 17	G5K	YG^1	Col. too pale for type of spectrum.
Centauri	11 30·0	53 42	B8A	OrY^2	Spectrum or colour wrong?
Hydræ	,, 47·9	33 21	B9A	Y^2	Do.?
Crucis	12 25·6	56 33	Mb	Y^2	But A. S. Williams makes it OrR^3, which agrees with spectrum.
Centauri	13 43·6	33 58	Mb	OrY^1	Spect. clearly type III.; col. wrong?
,,	,, 52·5	44 19	B3A	Y^2	Spectrum or colour wrong?
,,	14 0·8	35 52	K	Y^1	This type is strongly *yel.*; *maximum* falls in OrY^2.
,,	,, 13·3	55 56	B5A	YG^2	Possibly this ought to be YG^1?
Lupi	15 11·5	47 31	B8A	Y^2	Spectrum or colour wrong?
Circini	,, 15·4	58 58	B5A	Y^2	A. S. Williams makes this O. It is a composite spectrum; the other component F8G or G nearly = mags. (*H.A.*, xxviii. 2). Possibly spect. binary, with colour changes?
Lupi	,, 18·8	38 22	A	Y^2	Spectrum or colour wrong?
Normæ	,, 55·8	48 57	G5K	YG^1	Do.?
,,	16 19·8	47 20	B5A	Y^2	Composite spect., other component type I. (*H.A.*, xxviii. 2).
Scorpii	,, 29·7	28 0	B	Y^1	Tupman, Gage, and Baracchi make it O, which is probably correct.
Aræ	17 58·8	50 6	B1A	YG^2	Possibly should be YG^1?
Pisc. Aust.	22 47·0	33 24	A	Y^2	Spectrum or colour wrong?

TABLE IV.

Summary of Analysis of 1360 *bright Stars.*

(*Combined Results.*)

Division.	Spectra.	Type.	O	YG^1	Y^1	OrY^1	Y^2	OrY^2	Or^2	Y^3	OrY^3	Or^3	OrR^3	R^3	Stars.
I*a*.	Oa to B9A	Helium stars	125	69	69	8	8	3	...	...	...	...	...	...	282
I*b*.	A ,, A5F	Hydrogen stars	[illegible]	[illegible]	[illegible]04	11	14	...	...	...	...	...	...	...	377
I.–II.	F ,, F5G	α Carinæ type	[illegible]	[illegible]	78	8	23	8	...	6	...	...	...	...	137
II.	F8G ,, G5K	Solar stars	[illegible]	[illegible]	[illegible]6	12	77	29	4	21	5	3	...	...	180
II.–III*a*.	K	[illegible]	[illegible]	[illegible]	3	12	86	68	9	47	10	6	...	...	241
II.–III*b*.	K2M [illegible] K5[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	...	4	18	4	7	16	20	6	...	75
III.	Ma to [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	26	13	4	4	11	3	2	68
			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	152	30	85	35	40	9	2	1360

Grouping together O, YG^1, Y^1, and OrY^1—the last 3 being *white* with the faintest perceptible tinge of colour; Y^2, OrY^2, and Or^2 as *pale* tints; Y^3, OrY^3, and Or^3 as full or *normal* tints; OrR^3 and R^3 as *ruddy* tints; we have—

White, or nearly white, stars	793 :	p. cent.	58·3 :	55·2	(Fr's obs. 1884–6).
Pale tints, yel. to orange ,,	396 :	,,	29·1 :	35·2	,, ,,
Full ,, ,, ,,	160 :	,,	11·8 :	9·6	,, ,,
Ruddy tints, orange-red and red stars, ...	11 :	,,	·8 :	·0	,, ,,
	1360		100·0	100·0	

The last column gives the percentage results I found in 1884–6 from 1744 stars, and it is interesting to note the comparison.

It will be seen that the proportion of colourless stars is similar in groups 1 and 2, the *maximum* for both series being in O (white). In group 3 the typical tint is Y^1 (yellowish-white); in group 4 it is Y^2 (pale yellow); and in group 5 it lies between Y^2, OrY^2 (pale orange-yellow), and Y^3 (normal yellow). The reason for making a subdivision in II.–III. was that there were *two maxima*—the other falling in OrY^2, OrY^3 (normal orange-yellow), and Or^3 (normal orange) = group 6. The last group is very similar to 6 in colour distribution, and it would be difficult to tell merely by eye observation of colour to which of these groups a star belonged, although the spectra are essentially different. On the whole, there is undoubtedly a striking agreement between the colours and spectra of stars; but as the spectral types shade off almost imperceptibly into each other (though in a rational sequence), so likewise do the tints of the stars. It is this overlapping which is responsible for most of the apparent departures from the normal; and we must also remember that the tints of two stars may be apparently similar even with a different arrangement of lines (see *Problems in Astrophysics*, by Miss Clerke, p. 263), whilst two stars of similar spectra may differ perceptibly in tint.

In connection with this subject it may be mentioned that there is a curious affinity between Helium stars (B type) and *bright* line spectra (O type), with the *Galaxy*. Some of the latter type are associated, too, with *nebulæ*, as 15 (S) Monocerotis, ι and θ Orionis, and η Carinæ (which also has a bright line spectrum, though very peculiar). *All* the bright line spectra met with in this investigation were in or near the Milky Way; and this becomes still more remarkable when we remember that *all* the "Wolf-Rayet" stars (some 70 in number), *all* the temporary stars (novæ), and the majority of short-period variables are also found in this region. The Galaxy seems to be the plane of origin of some of the striking phenomena in the stellar universe; and such associ[illegible] cannot be merely accidental, but must be the result of some physical law, at present undiscovered.

Uxbridge: 1907 *May* 24.

On the "Owl" Nebula, Messier 97 = N.G.C. 3587. 1860.0. $\alpha = 11^h 6^m 40^s$. $\delta = +55° 46'.7$. By E. E. Barnard. (Plate 3.)

There has always been much interest attached to the great reflecting telescope of Lord Rosse. This giant instrument was used mostly in the study of the nebulæ, for which its great light-grasping power made it singularly useful. All the well-known nebulæ were studied with it, and drawings or sketches were made of many of them. There is little known as to the performance of the instrument on objects where definition was specially required. A comparison with the large modern refractors can only be had, therefore, through the agency of the nebulæ. Such a comparison must be crude at best from the difference in climate, observers, etc. Nevertheless, it would seem possible to form some idea of the power of the telescope in dealing with the nebulæ by a comparison of the drawings and descriptions with the present appearance of these objects as seen in the actual sky. To compare these drawings with photographs of the nebulæ would be very unfair, because of the extraordinary power of the sensitive plate compared with that of the eye when dealing with such objects.

A visual comparison of this kind would be well worth while, and might lead to important information concerning probable changes in the nebulæ, though there seems to be but little hope of anything in this direction except through the aid of photography at very long intervals of time.

The relative amount of light collected by the 40-inch refractor of the Yerkes Observatory and the 6-foot Rosse reflector would be 1 : 32 for a point source. If the relative effectiveness were the same, the 6-foot should have shown an object 32 times fainter than could be seen with the 40-inch telescope. Inasmuch as the great speculum was of metal, there must have been a serious loss of light to begin with. The power of the instrument must also have varied greatly according to the condition of the surface of the speculum, for Lord Rosse stated that it was subject to considerable tarnish because of the necessary amount of copper in the material.

I have examined, both with the 36-inch of the Lick Observatory and the 40-inch here, a large number of the nebulæ—especially of the planetary nebulæ—which were drawn by Lord Rosse and his assistants. From their observations, I believe, on the whole, that the 40-inch telescope will sl[illegible] v anything that could have been seen with the great reflector, [illegible] here light alone is concerned. True, there are some details [illegible] nebulæ in the Rosse drawings that are not visible in the [illegible] At the same time there are others that the gre[illegible] which are visible in the 40-inch. Ther[illegible] his point, therefore, without prejudice.

Very of[illegible] es around the planetary nebulæ, or [illegible] 40-inch. I do not know tha[illegible] One is almost

tempted to ask if this singular feature was not in some way a product of the great reflector itself—a want of exact definition. I offer this only as a possible explanation, which in itself seems to me to be very doubtful. Instances of this peculiarity are seen in the drawings of M 97, M 57, G.C. 450, G.C. 2098, G.C. 464, etc. But these appendages do not seem to be real.

A remarkable example of some of these peculiarities is the drawing of M 97—the celebrated "Owl" Nebula—made in 1848 with the great reflector. This picture for a half century was a classic among the drawings of nebulæ. And indeed it might well be, for it represented the nebula as a weird bewhiskered grinning face, with two dark spots where the eyes should be. In each of these dark spots or eyes a considerable star, which formed the pupil of the eye, was shown. The whole representation is strikingly like the face of an owl, or some uncanny cross-eyed goblin which only needs a pair of legs to execute some fiendish dance in space.

In recent years this fascinating picture seems to have dropped out of astronomical books, mainly, I believe, because when the nebula was examined in the sky with other powerful telescopes the two stars no longer formed the pupils of the eyes. In fact, there seemed after all to be only one star in the nebula; and this star, furthermore, was not in either of the eyes, but really occupied the bridge of the nose between the eyes!

Nevertheless, I believe that this drawing, shorn of some of its structural details and appendages, is important. There does not seem to be any question but that there were two stars in the nebula which were seen with the great telescope in 1848 and later. The main question is the true location of these two stars at that time with respect to the large dark spots in the nebula. If the drawing, which seems to be corroborated by the observer's notes, correctly represents the position of these two stars—both or either one—as being in the dark spots, then a change has occurred in the nebula, for the two stars are there and have not changed their places in the sky.

The brighter of these two stars occupies the centre of the nebula. In the observations of 1848 it is described as being in the centre. If this star is the nucleus of the nebula—and it probably is, though it is more star-like in appearance than are the nuclei of the nebulæ in general—then the nebula has not moved, for the star has not sensibly changed its place.

There is only one other means by which a displacement of the holes with respect to the two stars is possible, and that is a rotation of the nebula. Motion of translation or of rotation in the nebulæ is not improbable when time enough lapses to show it. We do not have, however, a single example where motion is proved, outside of spectroscopic results.

If this nebula were sensibly rotating from west to east, on an axis whose position angle is 50°, then the holes would have been further east on the disc in 1848; and if the velocity of rotation

were great enough, the two stars would have occupied the position in the holes shown in the drawing of Lord Rosse. While I do not believe this is the true explanation of the discrepancy in the early drawings of this nebula, it will at least serve as a working hypothesis.

Following is a collection of notes which relate to the stars in the nebula. They are taken from the papers of Lord Rosse in the *Philosophical Transactions of the Royal Society* and the *Scientific Transactions of the Royal Dublin Society*.

From the *Transactions of the Royal Society*, 1850.

1848 March 11. Two stars considerably apart in the central region. Dark penumbra around each, spiral arrangement, with stars as apparent centres of attraction; stars sparkling in it, resolvable; night excellent. Note by Mr Rambaut.

1848 March 11. Remarkably fine night; a brilliant star in the centre; also star to the right; round each a black space (see sketch). Note by Mr Rambaut.

1848 March 26. Second bright star visible; spiral arrangement hardly perceptible; not seen so well as on the 11th of March.

1848 March 27. Not seen so well as last night; second star seen at rare intervals, power 468.

1848 March 28. Night hazy, could not see second star.

1848 March 31. Caught one glimpse of second star, but saw the large star very plainly.

1848 April 3. Small star distinctly seen.

1848 April 6. First star seen easily, though hazy; the second only occasionally.

1848 March 9. Second star only seen for a moment. Several attempts were made to procure measures of position and distance of the two stars this spring, but in vain, the season was so unfavourable. . . . With the micrometer as at present mounted there would not have been the slightest difficulty in procuring measures.

From the *Transactions of the Royal Dublin Society*, vol. ii. (New Series), Aug. 1879.

[See p. 7, 1850. Plate xxxvii. fig. 11.]

1848 March 11. Brilliant star in the centre. After 5^m observation detected the star to t[illegible] [illegible]hich Dr Robinson immediately saw. Round each [illegible]

1848 March 26. Seen b[illegible] star seen at frequent intervals w[illegible] through the neb. part.

1848 March 27, 2[illegible] [illegible]ly seen at rare intervals.

1848 April 3. S[illegible]

1850 March 9. S star barely visible for a moment. Uncertain measure of its pos. from central star 331°·5 (Lord R.).

Between 1850 March 15 and 1858 March 8, viewed 18 times. S star not seen, nor minute details.

1858 March 11. *v*F star once suspected almost p the central star, but very uncertain. 2nd star not seen.

1858 April 3. Suspected star (see last obs.) not verified. A very minute star s f the middle one, just on the outer edge of dark hole, was more strongly suspected.

1863 Feb. 12. One star plainly seen, and at moments I fancied I saw a star n f it and one in s f edge, but only by glimpses. I have the merest suspicion of one in n p edge.

1868 Feb. 15. Star in centre, one on cross-bar near n extremity, susp. comp. to central star. S star on s f edge of f hole and sev. others susp. at moments.

1868 Feb. 28. Comp. susp. p central star.

1872 March 12. Star n f from central star. Pos. 22°·9 (3). Dist. 157″·6 (3). No other star distinctly seen. power 414, possibly one or two e F st f the central one, on the edge of the hole.

1874 March 9. Minor Axis Pos. 41°·4 (2) Diam. 147″·4 (2).
Major Axis „ 126·2 (2) „ 163·5 (2).

Central star 14 mag., no other st steadily seen.

As will be seen, the notes in these observations are frequently rather vague. Sometimes it is impossible to tell just what is meant.

It seems quite clear, however, that the "second star" frequently referred to was really the one which I have measured and called 2. The note of 1848 March 26, where it is referred to as "second bright star," is not reconcilable with their later observations of it, nor with its appearance to-day, for in the later notes it seems to have been a difficult object.

The direction of the 2nd star given by Lord Rosse on 1850 March 9 of 331°·5 must be 180° in error to agree with the previous descriptions. If this is so, and I think there is no doubt that it is, then the position angle 151°·5 would agree as closely as could be expected with my observation of 153°.

On 1907 April 5 I gave an exposure of $6^h\ 0^m$ on the place of this nebula with the Bruce 10-inch doublet. The sky was fair for such work, but not specially good. Clouds at one time covered the region.

The general appearance of the nebula on this photograph is that of a round mass, with a bright central star and two unequal dark places, one on each side of this star. The following is a description of the object under a considerable magnifying power as it appears on this plate. There seems to be a slightly elliptical disc, with its axis very roughly in P.A. 160°, superposed unsymmetrically upon a fainter circular disc. The greatest displacement of these two discs is on the following side, where the edge of the brighter one is $\frac{2}{10}$ of radius nearer the central star. There is some detail in the south preceding portion of the nebula in the form of a narrow curved mass of greate

brightness running toward the south side. There is also some detail of a similar nature near the north edge. The following hole is much smaller than the preceding one, its following edge is not quite $\frac{1}{2}$ radius from the nucleus. The preceding edge of the preceding hole extends over $\frac{7}{10}$ radius from the nucleus. The nucleus is so large that it almost obliterates the separation between the two holes, so that they appear nearly to merge into each other. There are suggestions of considerable detail on the nebula—especially in the north and south parts. The second star (2) is faintly visible just outside the south following edge of the following hole. There are no traces of fringes or other appendages to the nebula.

Among the many nebulæ photographed by the late Dr Roberts with his 20-inch reflector was M 97. A reproduction of his photograph is given in vol. ii. of Roberts' Celestial Photographs. Of this photograph which was made on 1895 April 20 with four hours' exposure, Dr Roberts says: "It (the nebula) measured about 203″ of arc in length, and the star referred to by Rosse is very conspicuously seen in the centre, its magnitude being about the 15th, but there is no other star anywhere in the nebula."

Dr Roberts' photograph is wrongly oriented in plate 19. South is above, and following is to the left. Below it, on the same plate, is a photograph of M 57 Lyrae, which is correctly oriented—south above, and preceding to the left. I call attention to this fact, otherwise it may cause confusion in comparing the picture of M 97 with observations. The photograph must be looked at in a mirror, which will correct the orientation. There is no faint star in the open sky outside the nebula on Roberts' plate that is not shown equally well or better on mine.

Following is a collection of my notes on the nebula as it appears with the 40-inch refractor.

1899 Feb. 6. The star (2) is just free of the dark opening. A sketch shows it outside the south following edge of the following hole.

Feb. 7. Small star difficult to measure.

1900 Dec. 18. The small star is not in the south following opening, but is just free of it and is on the nebulosity south following. This star is rather [illegible] to measure. Sketch.

Dec. 25. A sketch sho[illegible] mall star just free of the following part of the hole. [illegible] and difficult, and only seen by glimpses.

1902 Feb. 7. The cen[illegible] $16^{m}\cdot5$. It is very difficult. Can see the other star (2 [illegible] The star measured (1) is between the two eyes on th[illegible] of the nose, but nearer the following hole.

1907 March [illegible] nebula reaches just half way to the 10^{m} sta[illegible]

March 10 [illegible] less than the central star. The followin[illegible] and darker of the two. The central st[illegible] hole but not in it.

March 1[illegible] be 13^{m} if fr[illegible] on the

nebulosity. Can feebly see the two other stars. The following one (2) is on the south following edge of the following hole. It seems to be either on the edge or within the nebulosity outside the hole. The other small star (3) is very difficult. The preceding hole is further from the centre than the following hole, and there is not much contrast with the nebulosity. That side of the nebula seems somewhat less bright and less distinct. Cannot make out any spiral arrangement. Tried pulling out the eyepiece, but this did not seem to make any improvement. The edges of the nebula, though rather definite, are very indistinct—especially the preceding edge. With a magnifying power of 700 diameters, the south following star (2) is very close to the following edge of the following hole, but outside of the hole. The central star is north following the line between the two holes, and is nearer the following hole. The following hole is the more distinct of the two and is nearer the middle.

April 2. Sky fairly clear. The following estimates of brightness were made. Central star (1) = $13\frac{1}{2}^{m}$ or 14^{m}. Star (2) = 15^{m}. Star (3) is 1 magnitude less bright than (2). Tried drawing out eyepiece, but could not see any more details. There does not seem to be any structure, spiral or otherwise. The nebula is irregularly round—the edges very ill-defined.

I have the following observation of this object with the 36-inch, 1894 April 8, with a magnifying power of 350. There is a rough sketch of the central star and the two holes :—

"The nebula is round. The edges fairly well terminated. There is a 15^{m} star [the central star] just free of the preceding edge of the following dark hole ; there are one or two faint stars south following this hole, but no star in either hole. The following hole is the more noticeable. I do not see any spiral structure. The [central] star is on the following side of the bridge, but not in the hole."

I suppose the "one or two faint stars south following the hole" were stars (2) and (3). The description of the position of the central star on the bridge between the two dark holes would rather show there has been no decided change in the holes in the past ten or fifteen years.

I think the central star is variable. It has sometimes appeared much fainter than at others. Of course it is very difficult to decide on the variability of a faint star involved in nebulosity, for conditions of seeing, etc. affect it far more than when the star is in the open sky. On 1902 Feb. 7 it was estimated to be $16\frac{1}{2}$ magnitude, and was very difficult. I would not consider this alone as of any importance were it not that a note says that star (2) was visible. This would imply that there could be but little difference in their brightness at that time. It will be watched further for change.

Below are given my measures of the stars in this nebula. The diagram on Plate 3 shows the relative places of these stars with reference to the two dark spots.

Measures of Stars in Messier 97.

Nucleus (1) and (4).

	°	"	m	m	
1899 Feb. 6	24·85	157·52	14	10	
7	.....	158·58			Single distances.
	24·85	158·05			
1900 Dec. 18	24·86	158·28	13	10	
25	24·92	157·90		10·5	
28	24·70	157·99	14	10·5	
	24·83	158·06			
1902 Feb. 7	24·80	157·69	16·5	10	
8	24·97	158·26			
24	24·98	158·68			
	24·92	157·88*			
1907 Feb. 10	24·72	158·06			
Mar. 5	24·94	158·01			
	24·83	158·03			

Nucleus (1) and (5).

	°	"	m	
1900 Dec. 25	184·77	180·87	12	
28	184·88	181·20	12	
1901 Jan. 20	184·91	181·13	11·5	
	184·85	181·11*		

Nucleus (1) and (2).

	°	"	m	
1900 Dec. 25	152·89	40·76	16	
1907 Mar 10	153·99	40·78	16	
	153·44	40·77		

Nucleus (1) and (3).

	°	"	m	
1907 Mar. 5	205·1	36·8	16	[illegible] distances.

In volume ii., Publications of the Lick Observ[illegible] Professor Burnham gives three measures of the central [illegible] the 10 magnitude star north. The mean [illegible]

	°	"	m
1891·24	24·8	15[illegible]	14·2

* At the moment of going to [illegible] noticed th[illegible] means are not consistent with the figures abov[illegible] necessary [illegible] will be made in the next Number.

He does not mention any other stars in the nebula.

From the visual observations in March of the present year, the following facts were determined:—

Diameter of the following hole $= 34''\cdot0$ (1).

Diameter of the nebula in the direction of $25^\circ = 155''\cdot4$ (1).

Distance from following edge of following hole along the axis of the two holes to following edge of nebula $= 43''\cdot4$ (1). Position angle of the line between the centres of the two holes $= 134^\circ\cdot2$ (2). This line would pass 5″ or 10″ south preceding the central star.

To decide whether the "central star" was really central in the nebula, the distances from the star to the edges of the nebula in four directions were measured:—

P.A.	28°	81″·0	(2)
	118	81·6	(3)
	208	80·3	(2)
	298	80·0	(4)

It was difficult to set on the edges of the nebula, especially in the direction 28°. In that direction another night's measures gave $71''\cdot2$ (2), but the seeing was very bad, and the photograph shows that the fainter outline here was perhaps not seen, and that the measures doubtless refer to the edge of the brighter disc. I have rejected that set. The central star would therefore appear to be in the centre at least of the fainter disc. A mean of all the measures would make the diameter about 160″. The numbers in parenthesis indicate the number of settings of the wires.

A copy of my drawing of M 97 is given in Plate 3. It is a fair representation of the nebula as it appears in the 40-inch telescope. For comparison, I have mounted beside it a copy of the Rosse drawing, which I have assumed is correctly oriented in the *Philosophical Transactions* for 1850, south above, preceding to left. I have, however, tilted it so that the line between the eyes is parallel to that in my drawing. The Rosse picture is more interesting when turned upside down.

Yerkes Observatory:
1907 *May* 16.

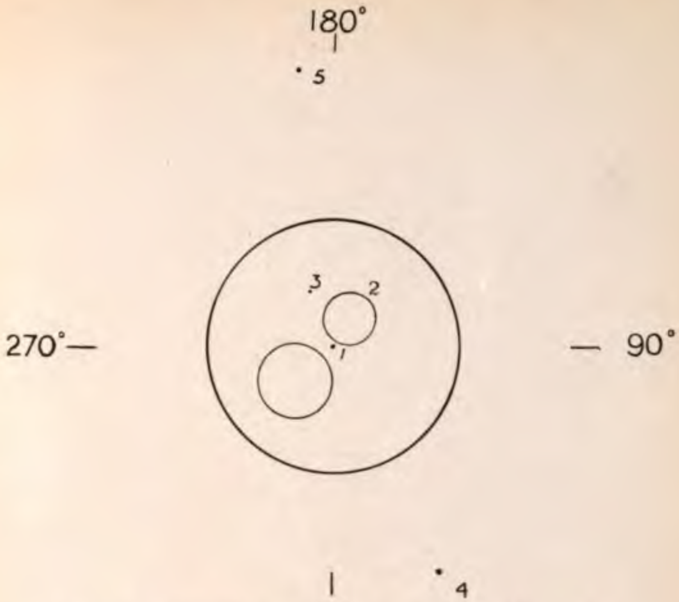


DIAGRAM OF THE "OWL" NEBULA FROM THE MEASURES.

SOUTH.

THE "OWL" NEBULA WITH LORD ROSSE'S GREAT REFLECTOR IN MARCH OF 1848.

...LA WITH THE 40-IN. ...E YERKES OBSERVA- ...907

MONTHLY NOTICES

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Determination of the Secular Perturbations of Minor Planet Ceres, arising from the actions of the eight Major Planets of the Solar System. By C. J. Merfield.

Introduction.—So far as the writer knows, there has been no complete determination of the secular perturbations of the minor planet Ceres.

Dr. Hill has published * a method of computing absolute perturbations, and as an example of the method he calculated † the secular perturbations of this asteroid in the cases of three of the major planets, Mars, Jupiter, and Saturn, the osculating elements by Schubert being adopted as a foundation of his work.

A system of Mean Elements of Ceres being now available, the writer was induced to undertake the complete determination of the secular perturbations of this body, as a contribution to the science of astronomy that seemed worthy of undertaking.

Elements.—The elements of the major planets, adopted in this work, are those given by Dr. Hill in his investigation ‡ of "A New Theory of Jupiter and Saturn." The Mean Elements of Ceres are by the same investigator, and are probably not very far from the true ones; no doubt, upon the whole, they will [illegible]ire very slight amendment, and the alterations necessary to [illegible] about a better representation of the normal places will not [illegible] the results to an extent requiring consideration.

* *Astronomische Nachrichten*, 1982.

† *Astronomical Journal*, 368.

‡ *Astronomical Papers, prepared f*[illegible] *Ephe*[illegible] *and Nautical Almanac*, vol. iv. Th[illegible] and Ur[illegible] adopted by Hill are 1/408134, 1/327[illegible]

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The values of the secular perturbations of Ceres, deduced from this inquiry, may be accepted as definitive if the preceding premise be found correct.

Methods of Calculation.—The method * of Gauss has been adopted, as explained by Dr. Hill. The values of the integrals, involved in the formulæ, have been interpolated from the table appended to his memoir. As the moduli in all cases came within the limits of the arguments given, this table was of much value, and considerably reduced the numerical work.

The values of the functions were deduced by the aid of seven figure logarithms; and, where it was advantageous, addition and subtraction logarithms, to a similar degree of accuracy, were used.

As the work proceeded, certain well-known checks were applied, and in all instances they were completely satisfied. The accuracy of the perturbations in the plane of the orbit has been verified by the usual formula; the residual is given in each case as an indication of the exactness of the calculation.

The equality of the summations of the functions for the odd and even divisions forms a useful control against large numerical errors, but it is not infallible, owing to certain circumstances arising in each special case.

Consulting the volumes, *locis citatis*, the significance of the notation will be found. The results of the determination are therefore given without further comment. Just so much of the numerical details are appended as will form a guide to a future investigation, and the detection of any errors that may have been made.

Elements of Ceres.

The following are the Mean Elements determined by Dr. Hill, and used as the basis of the investigation here presented.

Elements.

$$n = 281504''\!\cdot\!850309$$

$$\text{Log } e = 8\cdot89458493$$

$$\left.\begin{aligned} \pi &= 148^\circ\ 28'\ 32''\!\cdot\!5 \\ \Omega &= 80\ \ 48\ \ 5\cdot6 \\ i &= 10\ \ 37\ \ 6\cdot2 \end{aligned}\right\}\ 1850$$

$$\text{Log } a = 0\cdot4420738$$

Dividing the orbit into sixteen equal parts, with respect to the eccentric anomaly, the following values of the true anomalies and logarithms of the radii vectores are deduced.

* *Gauss's Method of Computing Secular Perturbations: Astronomical Papers, prepared for the use of the American Ephemeris,* vol., i. part v.

n	E	v	Log r	n	E	v	Log r
1	$0^\circ\!\cdot0$	0° $0'$ $0''\!\cdot00$	0·4065934	9	$180^\circ\!\cdot0$	180° $0'$ $0''\!\cdot00$	0·4748732
2	22·5	24 17 14·89	0·4093985	10	202·5	200 50 15·83	0·4724618
3	45·0	48 16 23·69	0·4172889	11	225·0	221 54 12·95	0·4655203
4	67·5	71 43 14·16	0·4288362	12	247·5	243 24 15·50	0·4549198
5	90·0	94 29 57·82	0·4420738	13	270·0	265 30 2·18	0·4420738
6	112·5	116 35 44·50	0·4549198	14	292·5	288 16 45·84	0·4288362
7	135·0	138 5 47·05	0·4655203	15	315·0	311 43 36·31	0·4172889
8	157·5	159 9 44·17	0·4724618	16	337·5	335 42 45·11	0·4093985

We also have the constant quantities used throughout the calculation—

$$\omega = 67^\circ\ 40'\ 26''\!\cdot90$$

$$\phi = 4\ 29\ 57\cdot82$$

Log sin $\phi = 8\cdot8945849$ Log sin $i = 9\cdot2654471$

,, cos $\phi = 9\cdot9986595$,, cos $i = 9\cdot9924988$

Log $\sin^2\frac{\phi}{2} = 7\cdot1877800$ Log $\sin^2\frac{i}{2} = 7\cdot9325686$

I. Mercury.

Elements.

$$n' = 5381016''\!\cdot260$$

Log $e' = 9\cdot31303314$

$\pi' = 75^\circ\ 7'\ 13''$ [illegible]

$\Omega' = 46$ [illegible]

$i' =$ [illegible]

Log $a' = 9$ [illegible]

$m' =$ [illegible]

$\mathrm{I} = 6^\circ\ 13$ [illegible] 39 $16''\!\cdot31$

$\Pi = 2$ [illegible] 9 33·89

$\Pi' =$ [illegible]

[illegible] 987395

Log C = [illegible] 987043

Secular Perturbations.

$$\left[\frac{de}{dt}\right]_{00} = -0''\!\cdot\!000020 \qquad \left[\frac{d\Omega}{dt}\right]_{00} = -0''\!\cdot\!000257$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +0''\!\cdot\!000521 \qquad \left[\frac{d\pi}{dt}\right]_{00} = +0''\!\cdot\!000517$$

$$\left[\frac{di}{dt}\right]_{00} = +0''\!\cdot\!000047 \qquad \left[\frac{dL}{dt}\right]_{00} = +0''\!\cdot\!076247$$

$$A_1^{(s)} \sin \phi + B_0^{(c)} \cos \phi = -0\cdot0000000013$$

II. Venus.

Elements.

$$n' = 2106641''\!\cdot\!357$$

$$\text{Log } e' = 7\cdot83525353$$

$$\left.\begin{aligned} \pi' &= 129^\circ\ 27'\ 42''\!\cdot\!83 \\ \Omega' &= 75\ 19\ 53\cdot08 \\ i' &= 3\ 23\ 35\cdot01 \end{aligned}\right\}\ 1850$$

$$\text{Log } a' = 9\cdot8593378$$

$$m' = 1/425000$$

$$I = 7^\circ\ 14'\ 52''\!\cdot\!52 \qquad K = 19^\circ\ 12'\ 48''\!\cdot\!63$$

$$\Pi = 245\ \ 6\ 39\cdot72 \qquad K' = 18\ 45\ 15\cdot08$$

$$\Pi' = 226\ \ 7\ 37\cdot99$$

$$\text{Log } k = 9\cdot9981963$$

$$\text{Log C} = 5\cdot3891827 \qquad \text{,, } k' = 9\cdot9983334$$

Secular Perturbations.

$$\left[\frac{de}{dt}\right]_{00} = -0''\!\cdot\!000025 \qquad \left[\frac{d\Omega}{dt}\right]_{00} = -0''\!\cdot\!026580$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +0''\!\cdot\!037022 \qquad \left[\frac{d\pi}{dt}\right]_{00} = +0''\!\cdot\!036567$$

$$\left[\frac{di}{dt}\right]_{00} = +0''\!\cdot\!000219 \qquad \left[\frac{dL}{dt}\right]_{00} = +1''\!\cdot\!395618$$

$$A_1^{(s)} \sin \phi + B_0^{(c)} \cos \phi = +0\cdot0000000004$$

Owing to certain circumstances, a verification of certain functions was undertaken. The orbit of Ceres was again divided into another sixteen parts, and the result of the investigation fully established the correctness of the previous calculation. The summations of the various functions, denoted by Σ_3, are given below the tabular statement, for comparison with $\Sigma_1 + \Sigma_2$. The check on the perturbations in the plane of the orbit in this instance gives a residual—

$$-0{\cdot}000000004$$

The above results are deduced from the several functions of the first investigation.

III. Earth.

Elements.

$$n' = 1295977''{\cdot}416$$

$$\text{Log } e' = 8{\cdot}22456263$$

$$\left.\begin{aligned} \pi' &= 100^\circ\ 21'\ 39''{\cdot}73 \\ \Omega' &= \text{—}\ \ \text{—}\ \ \text{—} \\ i' &= 0\ \ 0\ \ 0{\cdot}00 \end{aligned}\right\}\ 1850$$

$$\text{Log } a' = 0{\cdot}0000000$$

$$m' = 1/322800$$

$$\mathrm{I} = 10^\circ\ 37'\ 6''{\cdot}20 \qquad \mathrm{K} = 48^\circ\ 25'\ 29''{\cdot}13$$

$$\Pi = 247\ 40\ 26{\cdot}90 \qquad \mathrm{K}' = 47\ 48\ \ 1{\cdot}33$$

$$\Pi' = 199\ 33\ 34{\cdot}13$$

$$\text{Log } k = 9{\cdot}9991720$$

$$\text{Log } \mathrm{C} = 6{\cdot}4491253 \qquad \text{,, } k' = 9{\cdot}9933526$$

Secul[illegible]bations.

$$\left[\frac{de}{dt}\right]_{00} = -0''{\cdot}000547 \qquad \left[\frac{d\Omega}{dt}\right]_{00} = -0''{\cdot}108987$$

$$\left[\frac{d\chi}{dt}\right]_{00} = -\ [illegible] \qquad \left[\frac{d\pi}{dt}\right]_{00} = +0''{\cdot}094236$$

$$\left[\frac{di}{dt}\right]\ [illegible] \qquad \left[\frac{d\mathrm{L}}{dt}\right]_{00} = +1''{\cdot}0\ [illegible]$$

00000[illegible]

IV. Mars.

Elements.

$$n' = 689050''\!\cdot\!784$$

$$\text{Log } e' = 8\cdot96973284$$

$$\left.\begin{aligned}\pi' &= 333^\circ\ 17'\ 51''\!\cdot\!74\\ \Omega' &= 48\ 23\ 54\cdot59\\ i' &= 1\ 51\ 2\cdot24\end{aligned}\right\}1850$$

$$\text{Log } a' = 0\cdot1828971$$

$$m' = 1/3093500$$

$\mathrm{I} = 9^\circ\ 6'\ 33''\!\cdot\!95$	$\mathrm{K} = 175^\circ\ 21'\ 16''\!\cdot\!67$
$\Pi = 241\ 23\ 55\cdot82$	$\mathrm{K}' = 174\ 49\ 10\cdot35$
$\Pi' = 66\ 18\ 46\cdot43$	
	$\text{Log } k = 9\cdot9953869$
$\text{Log } \mathrm{C} = 8\cdot3052599$	,, $k' = 9\cdot9991198$

Secular Perturbations.

$$\left[\frac{de}{dt}\right]_{00} = +0''\!\cdot\!000068 \qquad \left[\frac{d\Omega}{dt}\right]_{00} = -0''\!\cdot\!039884$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +0''\!\cdot\!064691 \qquad \left[\frac{d\pi}{dt}\right]_{00} = +0''\!\cdot\!064008$$

$$\left[\frac{di}{dt}\right]_{00} = +0''\!\cdot\!000357 \qquad \left[\frac{d\mathrm{L}}{dt}\right]_{00} = +0''\!\cdot\!238781$$

$$\mathrm{A}_1^{(s)} \sin \phi + \mathrm{B}_0^{(c)} \cos \phi = +0\cdot000000007$$

Owing to the inequality of the quantities Σ_1 and Σ_2, for several of the functions, it was decided to divide the orbit of Ceres into another sixteen equal parts and make a separate calculation.

The results of the summations of the several functions in each case are given below the tabular statement; they are denoted by Σ_3. A comparison of these, with the quantity $\Sigma_1 + \Sigma_2$, completely verifies the accuracy of the first investigation.

The secular perturbations given above are determined from the first computed coefficients, as no alteration would take place by combining the results for the thirty-two divisions. The residual of the check equation for testing the accuracy of the perturbations in the plane of the orbit for the second division is $+0\cdot000000002$.

V. Jupiter.

Elements.

$$n' = 109256''{\cdot}62552$$

$$\text{Log } e' = 8{\cdot}68354330$$

$$\left.\begin{aligned} \pi' &= 11^\circ\ 54'\ 31''{\cdot}67 \\ \Omega' &= 98\ 56\ 19{\cdot}79 \\ i' &= 1\ 18\ 42{\cdot}10 \end{aligned}\right\} 1850$$

$$\text{Log } a' = 0{\cdot}7162374$$

$$m' = 1/1047{\cdot}879$$

$$\mathrm{I} = 9^\circ\ 22'\ 50''{\cdot}37 \qquad \mathrm{K} = 136^\circ\ 19'\ 14''{\cdot}71$$

$$\Pi = 250\ 10\ 47{\cdot}57 \qquad \mathrm{K}' = 136\ 53\ 10{\cdot}58$$

$$\Pi' = 113\ 34\ 30{\cdot}27$$

$$\text{Log } k = 9{\cdot}9950989$$

$$\text{Log C} = 8{\cdot}7995614 \qquad ,,\ k' = 9{\cdot}9990752$$

Secular Perturbations.

$$\left[\frac{de}{dt}\right]_{00} = -\ 0''{\cdot}675238 \qquad \left[\frac{d\Omega}{dt}\right]_{00} = -\ 52''{\cdot}157759$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +\ 56''{\cdot}772540 \qquad \left[\frac{d\pi}{dt}\right]_{00} = +\ 55''{\cdot}879400$$

$$\left[\frac{di}{dt}\right]_{00} = -\ 0''{\cdot}577398 \qquad \left[\frac{d\mathrm{L}}{dt}\right]_{00} = -\ 56''{\cdot}026115$$

$$\mathrm{A}_1^{(s)} \sin\phi + \mathrm{B}_0^{(c)} \cos\phi = -\ 0{\cdot}000000001$$

VI. Saturn.

Elements.

$$n' = 43996''{\cdot}21506$$

$$\text{Log } e' = 8{\cdot}74865\ldots$$

$$\left.\begin{aligned} \pi' &= 90^\circ \ldots \\ \Omega' &= 112 \ldots \\ i' &= 2 \ldots \end{aligned}\right\} 1850$$

$$\text{Log } a' = 0{\cdot}979\ldots$$

$$m' = 1/350\ldots$$

$$\mathrm{I} = 8^{\circ}\,35'\,27''\!\cdot\!85 \qquad \mathrm{K} = 58^{\circ}\,40'\,26''\!\cdot\!61$$

$$\Pi = 256\;26\;31\cdot11 \qquad \mathrm{K}' = 58\;17\;41\cdot41$$

$$\Pi' = 197\;57\;23\cdot97$$

$$\mathrm{Log}\;k = 9\cdot9995389$$

$$\mathrm{Log\;C} = 9\cdot4563013 \qquad ,,\; k' = 9\cdot9955700$$

Secular Perturbations.

$$\left[\frac{de}{dt}\right]_{00} = -0''\!\cdot\!030004 \qquad \left[\frac{d\Omega}{dt}\right]_{00} = -1''\!\cdot\!410009$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +1''\!\cdot\!339545 \qquad \left[\frac{d\pi}{dt}\right]_{00} = +1''\!\cdot\!315400$$

$$\left[\frac{di}{dt}\right]_{00} = -0''\!\cdot\!040972 \qquad \left[\frac{d\mathrm{L}}{dt}\right]_{00} = -2''\!\cdot\!122041$$

$$\mathrm{A}_1^{(s)} \sin\phi + \mathrm{B}_0^{(c)} \cos\phi = -0\cdot000000007$$

VII. Uranus.

Elements.

$$n' = 15425''\!\cdot\!752$$

$$\mathrm{Log}\;e' = 8\cdot67139132$$

$$\left.\begin{array}{l} \pi' = 168\;15\;\;6\cdot70 \\ \Omega' = \;\;73\;14\;\;8\cdot00 \\ i' = \;\;\;0\;46\;20\cdot54 \end{array}\right\} 1850$$

$$\mathrm{Log}\;a' = 1\cdot2831044$$

$$m' = 1/22800$$

$$\mathrm{I} = 9^{\circ}\,51'\,11''\!\cdot\!74 \qquad \mathrm{K} = 340^{\circ}\,15'\,40''\!\cdot\!28$$

$$\Pi = 247\;\;4\;47\cdot40 \qquad \mathrm{K}' = 340\;10\;\;5\cdot55$$

$$\Pi' = 266\;51\;55\cdot71$$

$$\mathrm{Log}\;k = 9\cdot9935658$$

$$\mathrm{Log\;C} = 9\cdot9089914 \qquad ,,\; k' = 9\cdot9999810$$

Secular Perturbations.

$$\left[\frac{de}{dt}\right]_{00} = +0''\!\cdot\!000195 \qquad \left[\frac{d\Omega}{dt}\right]_{00} = -0''\!\cdot\!026973$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +0''\!\cdot\!023392 \qquad \left[\frac{d\pi}{dt}\right]_{00} = +0''\!\cdot\!022930$$

$$\left[\frac{di}{dt}\right]_{00} = +0''\!\cdot\!000000{,}2 \qquad \left[\frac{dL}{dt}\right]_{00} = -0''\!\cdot\!036902$$

$$A_1^{(s)} \sin\phi + B_0^{(c)} \cos\phi = +0\cdot00000000002$$

VIII. Neptune.

Elements.

$$n' = 7864''\!\cdot\!935$$

$$\text{Log } e' = 7\cdot9292247$$

$$\left.\begin{aligned} \pi' &= 43^\circ\ 17'\ 30''\!\cdot\!30 \\ \Omega' &= 130\ \ 7\ 31\cdot83 \\ i' &= 1\ 47\ \ 1\cdot68 \end{aligned}\right\}\ 1850$$

$$\text{Log } a' = 1\cdot47781414$$

$$m' = 1/19700$$

$$I = 9^\circ\ 33'\ 3''\!\cdot\!35 \qquad K = 104^\circ\ 38'\ 4''\!\cdot\!92$$

$$\Pi = 255\ 51\ 14\cdot32 \qquad K' = 105\ 18\ 52\cdot87$$

$$\Pi' = 150\ 52\ 49\cdot92$$

$$\text{Log } k = 9\cdot9985797$$

$$\text{Log } C = 8\cdot8147322 \qquad \text{,, } k' = 9\cdot9953889$$

Secular Perturbations.

$$\left[\frac{de}{dt}\right]_{00} = +0''\!\cdot\!000014 \qquad \left[\frac{d\Omega}{dt}\right]_{00} = -0''\!\cdot\!007647$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +0''\!\cdot\!007602 \qquad \left[\frac{d\pi}{d[illegible]}\right] - [illegible]07471$$

$$\left[\frac{di}{dt}\right]_{00} = -0''\!\cdot\!000213 \qquad [illegible]10975$$

$$A_1^{(s)} \sin\phi + B_0^{(c)} \cos [illegible]$$

Secular Perturbations of Minor Planet Ceres, arising from the actions of the Eight Major Planets of the Solar System.

Planet	$\left[\frac{de}{dt}\right]_{00}$	$\left[\frac{d\chi}{dt}\right]_{00}$	$\left[\frac{di}{dt}\right]_{00}$	$\left[\frac{d\Omega}{dt}\right]_{00}$	$\left[\frac{d\pi}{dt}\right]_{00}$	$\left[\frac{dL}{dt}\right]_{00}$
Mercury	−0″·000020	+0″·000521	+0″·000047	−0″·000257	+0″·000517	+0″·07624
Venus	−0·000025	+0·037022	+0·000219	−0·026580	+0·036567	+1·39561
Earth	−0·000547	+0·096102	+0·000011	−0·108987	+0·094236	+1·92603
Mars	+0·000068	+0·064691	+0·000357	−0·039884	+0·064008	+0·23878
Jupiter	−0·675238	+56·772540	−0·577398	−52·157759	+55·879400	−56·02611
Saturn	−0·030004	+1·339545	−0·040972	−1·410009	+1·315400	−2·12204
Uranus	+0·000195	+0·023392	+0·000000	−0·026973	+0·022930	−0·03690
Neptune	+0·000014	+0·007602	−0·000213	−0·007647	+0·007471	−0·01097
Summation	−0·705557	+58·341415	−0·617949	−53·778096	+57·420529	−54·55935

Total Secular Perturbations of Ceres.

$$\left[\frac{de}{dt}\right]_{00} = -0''\!\cdot\!70556 \qquad \left[\frac{d\Omega}{dt}\right]_{00} = -53''\!\cdot\!77810$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +58''\!\cdot\!34142 \qquad \left[\frac{d\pi}{dt}\right]_{00} = +57''\!\cdot\!42053$$

$$\left[\frac{di}{dt}\right]_{00} = -0''\!\cdot\!61795 \qquad \left[\frac{dL}{dt}\right]_{00} = -54''\!\cdot\!55936$$

Sydney: 1907 *March* 1.

[*The Tables to accompany Mr. Merfield's paper are bound and placed in the Library.*]

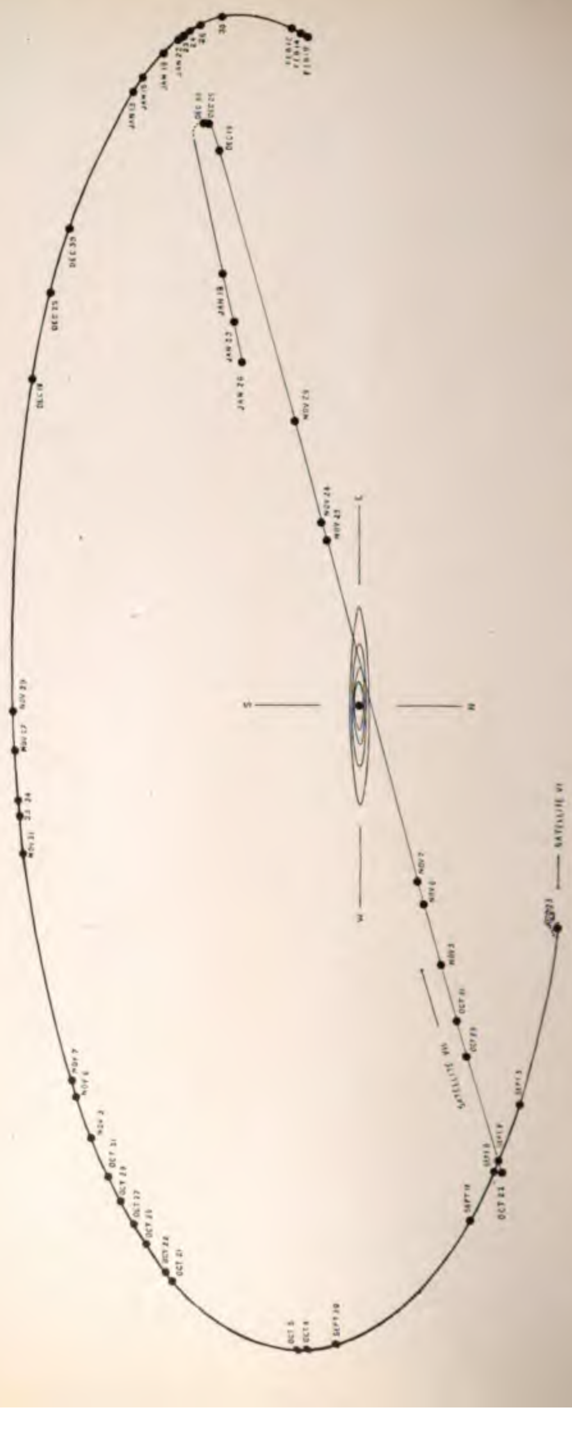


Diagram showing the positions of Jupiter's Satellites VI. and VII., from photographs taken at the Royal Observatory, Greenwich, during the Opposition 1905-6.

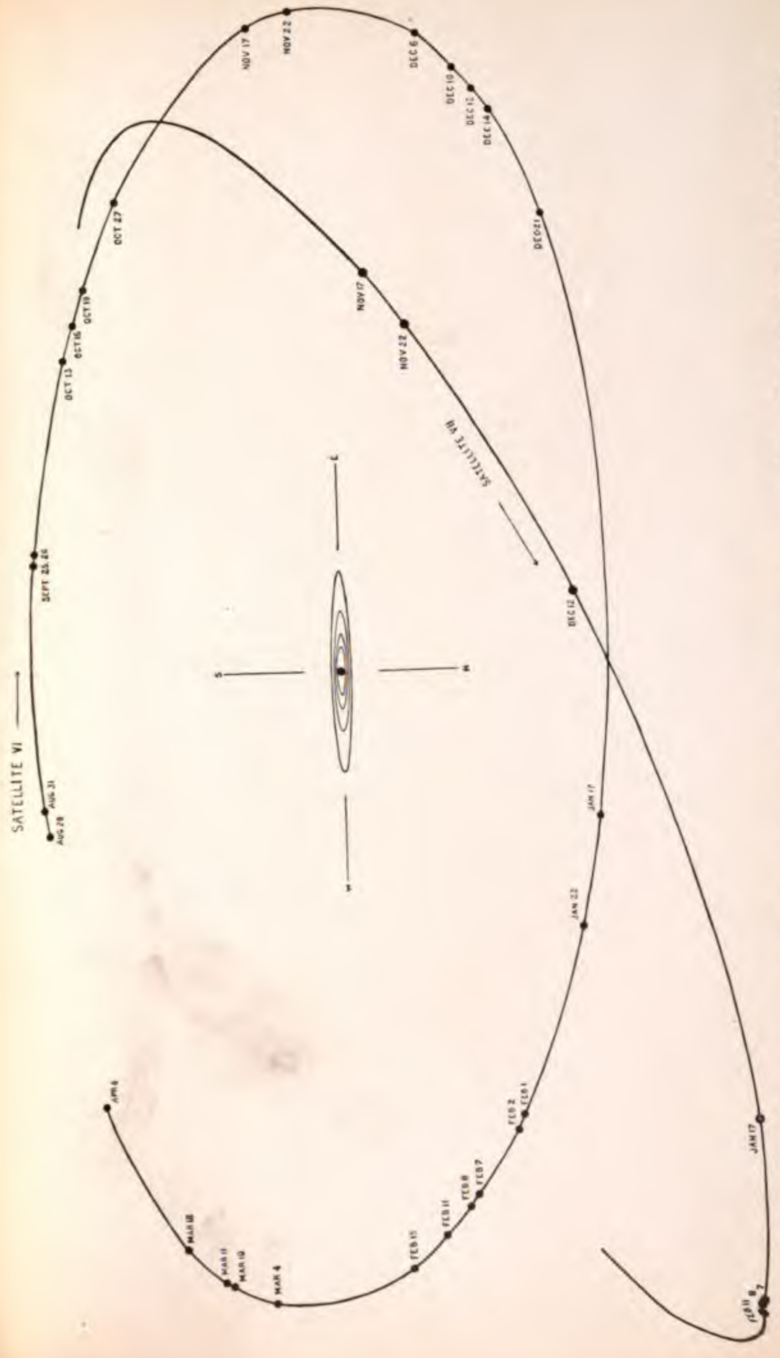


Diagram showing the positions of Jupiter's Satellites VI. and VII., from photographs taken at the Royal Observatory, Greenwich, during the Opposition 1906-7.

Diagrams showing the positions of Jupiter's Satellites VI and VII from Photographs taken at the Royal Observatory, Greenwich, during the Oppositions of 1905–6 *and* 1906–7. (Plates 4, 5.)

(*Communicated by the Astronomer Royal.*)

These diagrams are a graphical representation of the observations of Jupiter's Satellites VI and VII from photographs taken at the Royal Observatory with the 30-inch Reflector during the Oppositions of 1905–6 and 1906–7, printed in the *Monthly Notices*, vol. lxvi. p. 438 and vol. lxvii. p. 479.

During the 1905–6 opposition eighty-six photographs of J. VI were secured on thirty-six nights from 1905 August 23 to 1906 February 15—a period of 177 days. Of J. VII nineteen photographs were secured on fifteen nights from 1905 October 22 to 1906 January 26—a period of 97 days.

The opposition 1906–7 was not so favourable as regards weather conditions, but fifty-six photographs of J. VI were obtained on twenty-eight nights over a period of 222 days, and twelve photographs of J. VII on seven nights over a period of 87 days.

The positions plotted in the diagrams are the mean when more than one photograph was taken on any night. The curves were drawn through these points, but in the case of 1906–7 observations of J. VII the curve is continued beyond the observations to indicate the apparent form.

For comparison, the orbits of the four larger satellites are shown, the major axes being plotted to scale, but the minor axes intentionally exaggerated.

Royal Observatory, Greenwich:
1907 *September* 30.

On the Measurement of a Meteor Trail on a Photographic Plate.
By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. On August 22, 1907, Mr. F. W. Longbottom, of Haslemere, Queen's Park, Chester, sent me the print of a nebula photograph across which a meteor had flashed, asking for advice about the measurement. I have not seen any note on the easiest way of utilising such trails, which have already appeared in a good many photographs, and will doubtless occur in a good many more; and the following method may be found useful in other cases.

2. The rule is simply—measure on any convenient uniform scale, and by any convenient method, the distances of any known stars from the trail. (Three stars are theoretically sufficient, but more will give better results.) These measures will give the desired information.

3. The information obtainable is the position of the pole of the trail. The trail is assumed to be part of a straight line on the plate or of a great circle on the sphere. We cannot determine the radiant from a single trail, only the great circle on which the radiant lies; and this is conveniently specified by its pole, in R.A. and Decl. A_m and D_m say. If we can find other trails belonging to the same radiant, their poles will lie on a great circle of which the radiant is the pole.

4. Let (A, D) be the co-ordinates of the plate centre; (ξ, η) the standard co-ordinates of any point on the plate, *expressed in parts of the radius*, the R.A. and Decl. being (α, δ). If (α, δ) lies on the meteor trail, it is 90° distant from (A_m, D_m), and hence

$$\sin\delta \sin D_m + \cos\delta \cos D_m \cos(\alpha - A_m) = 0 \quad . \qquad (1)$$

Now (α, δ) may be expressed in terms of (ξ, η) by the following formulæ (*Monthly Notices*, lxiv. p. 648):—

$$\frac{\cos\delta \sin(\alpha - A)}{\xi} = \frac{\sin\delta}{\sin D + \eta \cos D} = \frac{\cos\delta \cos(\alpha - A)}{\cos D - \eta \sin D} \qquad (2)$$

and since we may re-write (1) in the form

$$d \cos\delta \sin(\alpha - A) + q \sin\delta + r \cos\delta \cos(\alpha - A) = 0 \qquad (3)$$

where

$$d = \cos D_m \sin(A_m - A),\ q = \sin D_m,\ r = \cos D_m \cos(A_m - A) \qquad (4)$$

It may also be written

$$d.\xi + q(\sin D + \eta \cos D) + r(\cos D - \eta \sin D) = 0 \quad . \qquad (5)$$

or

$$d.\xi + e.\eta + f = 0 \quad . \quad . \quad . \quad . \qquad (6)$$

where q and r are replaced by e and f such that

$$\left.\begin{aligned} e &= q \cos D - r \sin D = \sin D_m \cos D - \cos D_m \sin D \cos(A_m - A) \\ f &= q \sin D + r \cos D = \sin D_m \sin D + \cos D_m \cos D \cos(A_m - A) \end{aligned}\right\} \quad (7)$$

The equation (6) is the equation to the meteor trail on the plate in standard co-ordinates.

5. Now let (ξ_s, η_s) be the standard co-ordinates of any star, and let us measure, in terms of any convenient unit, its distance p_s from the meteor trail. Then

$$k'.p_s\ (d^2+e^2)^{\frac{1}{2}} = d\xi_s + e\eta_s + f$$

where k' is a constant representing the arbitrary scale value; and we can replace $k'\ (d^2+e^2)^{\frac{1}{2}}$ by a new arbitrary constant k^{-1}. Thus

$$p_s = kd.\xi_s + ke.\eta_s + kf$$

From the measures of three different stars for which (ξ_s, η_s) are known we get kd, ke, kf. We can utilise the measures of other stars by any of the well-known processes.

6. Then from (7) we get kq and kr; and from (4) we have

$$\tan\ (\mathrm{A}_m - \mathrm{A}) = \frac{d}{r} = \frac{kd}{kr} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\tan\ \mathrm{D}_m = \frac{q}{\pm(d^2+r^2)^{\frac{1}{2}}} = \frac{kq}{\pm(k^2d^2+k^2r^2)^{\frac{1}{2}}} \quad . \quad . \quad (9)$$

which give A_m and D_m, since A is known.

7. For the trail caught by Mr. Longbottom on August 4, 1907, while he was photographing N.G.C. 6960, with a $12\frac{1}{4}$-in. reflector of 24-in. focus, exposure 45 minutes; time of meteor $10^h\ 52^m$: it will be a sufficiently good approximation (as the scale is small and the star images large owing to coma and a small driving error) to take as the plate-centre that used for the plate of this region in the Oxford Astrographic Catalogue, viz.—

$$\mathrm{A} = 20^h\ 42^m \qquad \mathrm{D} = +30^\circ \qquad (1900{\cdot}0)$$

There was no difficulty in identifying several stars from the Cambridge Catalogue, and a rough reduction from these made it easy to identify on the reflector photograph the other stars shown on the Oxford plate in the neighbourhood. The following measures were then made by Mr. Bellamy:—

		Oxford.			
No. in +30°.	Diam.	x	y	p	O-[illegible]
61506	14	7·785	16·391	+1·12	
61507	12	8·018	·418	+1	
61508*	22	8·616	·508		
61509	14	10·580	·420		
61510	20	11·184	·439		
61511	22	11·622	·228		
61512*	40	11·715	·407		
61513	20	12·689	·544		
61514	14	12·725	16·3[illegible]		

No. in +30°.	Diam.	x	y	p	O−C.
61542	11	8·444	17·446	+0·44	+·02
61543	12	10·563	410	+0·32	·00
61545	12	10·993	·952	−0·04	·00
61546*	90	11·292	·138	+0·45	−·02
61547	13	15·090	17·401	−0·03	−·02
61580*	35	8·695	18·422	−0·14	−·01
61581	40	9·390	·385	−0·16	−·02
61582	15	9·975	·363	−0·21	·00
61583	20	11·096	·165	−0·17	−·01
61584	12	11·256	·156	−0·18	·00
61585	17	12·179	·631	−0·52	−·02
61586	14	13·643	·877	−0·78	−·01
61587	30	14·341	·929	−0·88	·00
61588	24	15·450	18·888	−0·92	−·02
61606*	30	9·744	19·337	−0·77	·00
61607	12	12·222	·708	−1·17	−·01
61608	12	13·189	·027	−0·84	−·01
61609*	38	14·796	·524	−1·27	+·01
61610	14	15·290	19·659	−1·46	+·08

Measures of 61610 marked as very faint, ? a star.

The values of x and y are not standard co-ordinates but the measures on the Oxford plate; but they can be readily converted into standard co-ordinates by means of the formulæ given at the plate-heading in the Oxford Catalogue. No doubt it is scarcely necessary to measure so many stars, but it seemed possible that there might be systematic error depending on the shape of the star images with a reflector of so short a focus; and thus stars of different magnitudes might give different results.

8. The constants in the linear relation

$$d'x+e'y+f'=p$$

were then found to be as follows:—

$$d'=-\cdot 0762,\quad e'=-\cdot 5915,\quad f'=+11\cdot 416$$

We may omit the last figure as probably not justified by the measures and write

$$d'=-\cdot 076,\quad e'=-\cdot 592,\quad f'=+11\cdot 42$$

With these values the column O − C was formed, and it will be seen that the effect of the brightness of the star is not large.

9. The formulæ for getting standard co-ordinates from x and y are (*see* heading of plate)

$$\xi=x-13+\cdot 00054x-\cdot 00081y+\cdot 5124$$
$$\eta=y-13+\cdot 00061x+\cdot 00043y+\cdot 1011$$

Let us write

$$\xi_0 = \xi + 13 - \cdot 51 = \xi + 12\cdot 49$$
$$\eta_0 = \eta + 13 - \cdot 10 = \eta + 12\cdot 90$$

adopting the standard of accuracy found sufficient above.

Then $(x - \xi_0)$, $(y - \eta_0)$ differ by less than $\cdot 001$ of x and y, and hence we may neglect squares of these differences. Thus

$$x = \xi_0 - \cdot 0005\xi_0 + \cdot 0008\eta_0 = \xi - \cdot 0005\xi + \cdot 0008\eta + 12\cdot 49$$
$$y = \eta_0 - \cdot 0006\xi_0 - \cdot 0004\eta_0 = \eta + \cdot 0006\xi + 0004\eta + 12\cdot 91$$

Hence $p = -\cdot 076\xi - \cdot 592\eta + 11\cdot 42 - \cdot 076 \times 12\cdot 49 - \cdot 592 \times 12\cdot 91$
$\qquad = -\cdot 076\xi - \cdot 592\eta + 2\cdot 84$

But $\xi\ \eta$ are expressed in reseau intervals, and it was assumed in § 4 that they were in parts of the radius. We must multiply their coefficients by $687\cdot 6$.

10. Hence as in Eq^n. (7)

$$kq \cos D - kr \sin D = ke = -\cdot 592 \times 687\cdot 6 = -407\cdot 1$$
$$kq \sin D + kr \cos D = kf = +2\cdot 84$$

$$kq = -407 \times \cdot 866 + 2\cdot 84 \times 0\cdot 500 = -351$$
$$kr = +407 \times \cdot 500 + 2\cdot 84 \times 0\cdot 866 = +206$$

and $$\tan(A_m - A) = \frac{kd}{kr} = \frac{-\cdot 076 \times 688}{+206} = -\cdot 254$$

$$\therefore\ A_m - A = -14^\circ 15' \text{ or } +165^\circ 45'$$

We shall always get an ambiguity, as there are two poles to the trail. But when we choose one or other of these values, we settle the ambiguity of Eq^n. (9), which may be otherwise written

$$\tan D_m = \frac{kq}{kr} \sec(A_m - A)$$

$$= \frac{-351}{206} \times 1\cdot 032 = -1\cdot 758 \text{ taking } (A_m - A) = -14^\circ 15'$$

$$D_m = -60^\circ 22'$$

Hence the two poles are for 1900·0

$A_m = 20^h\ 42^m - 57^m\ 0^s$	and	$A_m = 7^h\ 45^m\ 0^s$
$= 19^h\ 45^m\ 0^s$		
$D_m = -60^\circ\ 22'$		$D_m = +60^\circ\ 22'$

They may be brought up to date by well known rules.

Note on a Meteoric Shower. (The August Draconids.)
By W. F. Denning.

On 1907 August 15 I saw five meteors during a watch of forty minutes from a radiant at $288° + 61°$. One of them was stationary at that point. On August 26 at 9^h 18^m I observed a brilliant member of the same shower falling from $231° + 57°$ to $213° + 50°$, and showing some curious fluctuations in its light. On August 28, during a short period of clear sky, I recorded two other meteors from the same radiant. On August 15, Mr. H. Corder saw a Draconid equal to Mars flashing out with an orange colour at $288° - 10°$, and falling 8° further in a vertical path directed from a point one degree west of β Cygni.

The shower is evidently active between August 15 and 28, and I recognised it between these dates in the years 1887, 1899, and 1900. Mr. Corder had also seen it pretty richly manifested on 1879 August 16 from $286° + 61°$ (10 meteors).

This display of Draconids, coming, as it does, concurrently with the rapidly declining stages of the great Perseid shower, deserves special mention, and particular efforts should be made in future years to re-observe it. In 1879 August 21 to 25, of 225 meteors which I recorded, no less than 56 were Draconids, and nearly all of them were bright, slow-moving objects with yellow trains. During the twenty-eight years which have passed since 1879 I have never seen the shower richly exhibited until last month, when, however, my observations were very imperfect on account of ill-health. Particulars of this evidently prominent and periodically visible system of meteors, as it appeared in 1879, will be found in the *Monthly Notices*, vol. xl. pp. 127–8, and in *The Observatory*, vol. iii. pp. 172–3.

Bishopston, Bristol:
1907 *September* 1.

On Trigonometrical Differences of Altitude.

By G. Tyrrell McCaw, M.A.

In the present methods in use in practical geodesy for determining the difference of altitude of two stations on the Earth's surface, it is common to assume that the vertical refraction at both ends of the line joining two stations is the same. Sometimes even the error seems to be made of implicitly assuming that refraction is eliminated by simultaneous and reciprocal observations at the two ends.

The refraction at each station is commonly regarded as a linear factor of the "contained angle" subtended by the stations at the Earth's centre; or, in other words, is considered as being directly proportional to the distance between the two stations projected on the geodetic surface. The refraction r is therefore expressed thus—

$$r = k \, . \, \psi,$$

ψ being the contained angle and k the so-called "coefficient of refraction."*

Numerous experiments in various parts of the world have shown that this coefficient is by no means constant, not even for the same station. Thus in the United Kingdom, though in 72 cases the value lay between 0·0733 and 0·0804, among 144 cases the values varied between the extremes of 0·0320 and 0·1058. Even in the clear air of mountain heights, where one usually obtains consistent results, there may be wide differences. Thus in the line Ben Lomond to Ben Nevis, the value on Ben Lomond was 0·0719, while on Ben Nevis it was 0·0942.

In Western America, when the refraction at a single station was investigated, the coefficient was found to have a diurnal range as great as 0·04 in some cases. Periods of rapid change occurred about an hour before sunrise and an hour after sunset. The time of *maximum* was about 3 a.m., and the time of *minimum* varied at different stations between 10 a.m. and 4 p.m., the hour from 2 to 3 p.m. marking apparently the mean period. On the line Mt. Diablo—Martinez East, the value at 3 p.m. at the former station (height 1173 metres) was 0·0653, and at the latter (57 m.) 0·1007, while at the latter the value rose to 0·1415 at 3 a.m. The value for the interior of the continent was about 0·065, over parts of the sea near the Atlantic coast about 0·078, and over parts of the sea near the Pacific coast about 0·085. Generally spe
elevations gave higher values and greater range;
were also obtained in the excessive moisture of th
range of temperature seems chiefly to determi
curve which represents the diurnal variation
height or moisture, its position.

* In Continental practice it is the custom to
ratio of the total refraction to the contained angl

Taking the mean value of the two stations on a line, the Coast Survey obtained the following results :—

	Jackson Butte (714 m.)—Round Top (3174 m.) (Inland).	Sea Horizon—Ragged Mt. (397 m. (Atlantic coast).	Martinez East (57 m.)—Mt. Diablo (1173 m.) (Pacific ocean).
Daily Mean,	0·0646	0·0849	0·1000
Diurnal Range,	0·0168	0·0293	0·0333

In the computations of the great American Arc of Parallel 39°, the mean coefficients for the various lines in the western portion of the arc were tabulated, the mean heights of the stations and mean temperatures also being recorded. These results seemed to suggest an empirical formula of the form—

$$k = k_0 + (t - t_0)x + (h - h_0)y ,$$

t_0 and h_0 being the mean temperature (Centigrade) and height (in hectometres) of all the stations in a given area of the triangulation.

For the more eastern area $k_0 = 0{\cdot}0534$, $t_0 = 9^{\circ}{\cdot}9$, $h_0 = 37{\cdot}7$
" " western " $= 0{\cdot}0578$, $= 11^{\circ}{\cdot}2$, $= 32{\cdot}5$

Forming the equations for all the observations in each area, the following values of x and y were obtained by mean squares :—

For the more eastern area $x = 0{\cdot}00036$, $y = 0{\cdot}00087$
" " western " $= 0{\cdot}00081$, $= 0{\cdot}00091$

The agreement between the observed and computed values was, in general, fairly good.

Over the hot plains of India much greater discrepancies were observed. In one case a depression of 4′ 52″·6 at 4.30 p.m. changed to an elevation of 2′ 24″·0 at 10.50 p.m., the stations being only 10·5 miles apart. The refraction had sometimes a negative value, and signals which were invisible during the day were seen above the horizon at sunset. The coefficient in India actually ranged from − 0·09 to + 1·21.

In Cape Colony and Natal the refraction was more constant, the extreme values of the coefficient being found to be 0·045 and 0·106, but by no means all the lines were computed. The value was seen to increase by about 0·00045 per mile for distances between 20 and 60 miles—a result which shows that the refraction is not a linear factor of the distance.

Careful observations in France indicated that refraction there is greatest about daybreak, and then diminishes very rapidly till 8 a.m., and from that very slowly till 10 a.m. Thence the refraction remains nearly constant till 4 p.m., after which it begins again to increase.

Very few experiments have been made with a view to determine seasonal variation, but there is evidence enough to show that refraction changes very considerably with the season.

Enough has been said to prove how very variable the coefficient may be. It is otherwise obvious, *à priori*, that the refraction at a

point in a hot, low, moist plain must be very different from that on a cool, dry, mountain height. Indeed there is a very special reason for this refraction remaining even so constant as it is: in primary triangulation work, practically all measurement of vertical angle is made within the range of 87° and 93° zenith distance, and the great majority of the observations are confined within 89° 45′ and 90° 45′—a range of only one degree.

One of the practical methods devised to overcome the difficulty of the variability of refraction is that which assumes that the coefficient is constant for a given station rather than for the extremities of a line. This is also a fairly extravagant hypothesis. It ignores the variation of zenith distance with which the refraction is doubtless in some way connected; it ignores the varying length of the lines by which the refraction is affected in some measure, as shown in South Africa; it ignores, further, those fortuitous variations which take place from moment to moment, due to rain, clouds, wind, heat waves, etc., the effect of which in an individual measure may have a preponderating effect on the mean of a series of observations.

The actual method of determining the coefficients on this assumption is as follows:—If $k_1, k_2, k_3, \ldots\ldots$ be the mean values of the coefficients at the stations $P_1, P_2, P_3 \ldots\ldots$ respectively, then it is easily seen that these coefficients are connected by equations of the form

$$k_1 + k_2 = \text{a constant},$$

the constant being determined from the observations; forming the equations for all the stations in a circuit, the values of $k_1, k_2, k_3 \ldots\ldots$ are determined.

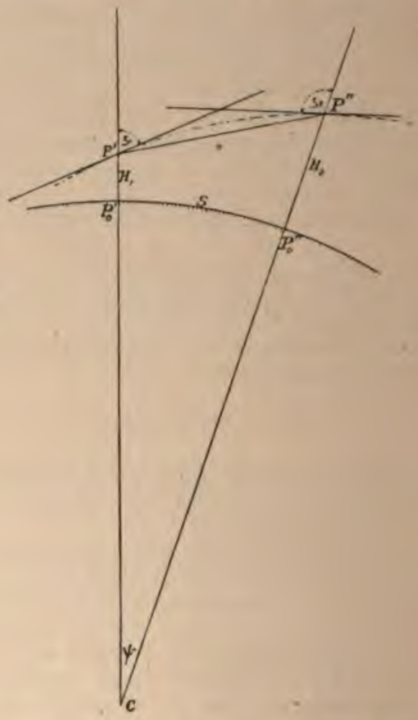
It is, of course, well recognised that it is an entirely gratuitous assumption to express the refraction at either end of a line, or the total refraction, as simply proportional to the arc between the two stations. It is, therefore, in more refined methods, usual to endeavour to find an expression for k which will introduce the necessary meteorological conditions. These investigations lead to equations almost unmanageable in practice, and depend on atmospheric hypotheses not generally recognised.

In all present methods, so far as the writer is aware, there are assumptions which can with difficulty be defended on theoretical grounds, or on the ground that the coefficients adopted are found by experience to give concordant results. Recognising this fact, the Seventh Congress of the International Geodetic Association, in 1883, after hearing a report from Dr. Bauernfeind, expressed the desire that the various States which contribute to the Associ tion should undertake exact investigations to determine influence of the varying conditions of ground and climate or terrestrial refraction.

As the problem remains unsolved, the writer off following theoretical solution, based on generally accepted

laws and recognised hypotheses. In this solution it will be seen that the use of the coefficient k has been entirely abandoned.

It is necessary to premise at the outset that the observations at the ends of the line are reciprocal and simultaneous. The temperature and pressure must be observed at both stations with standardised instruments. For the present no consideration will be taken of the effect of aqueous vapour; the influence of this vapour may be subsequently investigated.



In the figure, P', P'' are two stations on the Earth's surface; P_0', P_0'' their projections on the ellipsoid of reference; the mean radius of curvature of the ellipsoid along the line between P_0' and P_0'' is R; Z_1, Z_2 are the true, and ζ_1, ζ_2 the observed, zenith distances of the mutually observed points; the contained angle $P'OP''$ is ψ; the length of the arc P_0' P_0'' is S; H_1, H_2 are the heights of the two stations above the spheroid, and $H_2 - H_1 = h$, the height-difference.

We adopt the recognised hypothesis that the air in its normal condition consists of spherical strata, the density of each layer being constant, and varying continuously from layer to layer. Then, if a beam of light pass through media of continuously

varying density, the density and refractive index of a layer being any function whatever of the distance from a fixed point, we have the well-known relation

$$\mu . r . \sin \phi = a \ constant,$$

where

μ is the absolute index of refraction of any stratum,
r the distance of that stratum from the fixed origin,
ϕ the angle between the vector r and the tangent to the path of the beam at the point where it cuts the stratum whose index is μ.

Applying this law to the present case

$$\mu_1(R+H_1) \sin \zeta_1 = \mu_2(R+H_2) \sin \zeta_2$$

whence

$$\frac{R+H_2}{R+H_1} = \frac{\mu_1 \sin \zeta_1}{\mu_2 \sin \zeta_2} \quad . \quad . \quad . \quad (1)$$

Put the latter $= \omega$.

Then $$H_2 - H_1 = h = (R+H_1)(\omega - 1) \quad . \quad . \quad (2)$$

Thus the height-difference is known if we know ω, that is, if the ratio $\mu_1 : \mu_2$ is known.

Before proceeding to investigate this ratio we shall consider two particular cases.

From the geometrical conditions of the figure we have obviously

$$\frac{R+H_2}{R+H_1} = \frac{\sin Z_1}{\sin Z_2},$$

whence

$$H_2 - H_1 = h = 2(R+H_1) \frac{\sin \frac{1}{2}(Z_1 - Z_2) \cos \frac{1}{2}(Z_1 + Z_2)}{\sin Z_2} \quad (3)$$

which reduces to

$$h = 2(R+H_1) \sin \tfrac{1}{2}\psi . \frac{\sin \frac{1}{2}(Z_2 - Z_1)}{\cos \frac{1}{2}(Z_2 - Z_1 + \psi)} \quad . \quad . \quad (4)$$

which is the rigorous expression for the height-difference if the difference of the true zenith distances be known.

Consider first the particular case where the refraction at both ends of the line is the same.

Then, since $Z_1 - \zeta_1 = Z_2 - \zeta_2$, equation (4) becomes

$$h = 2(R+H_1) \sin \tfrac{1}{2}\psi . \frac{\sin \frac{1}{2}(\zeta_2 - \zeta_1)}{\cos \frac{1}{2}(\zeta_2 - \zeta_1 + \psi)} \quad . \quad (5)$$

which is the rigorous expression for h when the above assumptior is supposed to hold good, the plumbline being uninfluenced local attraction at either station, and the mean surface of curv of the geoid between the two stations being regarded as pa sphere.

Again, suppose that $\mu_1 = \mu_2$, *i.e.* that the index of refraction at both stations is the same; then equation (1) becomes

$$\frac{R+H_2}{R+H_1} = \frac{\sin \zeta_1}{\sin \zeta_2},$$

wherefore the height-difference is expressed by the equation (3), in which the observed zenith distance ζ is substituted for the true zenith distance Z.

To proceed with the general investigation, it is necessary to determine the ratio $\mu_1 : \mu_2$.

Suppose that a ray of light passes from a vacuum into a gas whose absolute index of refraction is μ and density ρ. Then if the density be varied, the index of refraction varies according to the formula

$$\mu - 1 = c.\rho, \text{ where } c \text{ is a constant} \quad . \quad . \quad (6)$$

This, the well-known law of Gladstone and Dale, is that which is generally accepted for solids, liquids, and gases. In the case of gases, however, where μ is nearly equal to unity, some have supposed that

$$\mu^2 - 1 = c.\rho$$

more nearly represents the true law.* We shall adopt Gladstone's law; but even should a somewhat different formula be discovered in the future, it is obvious that the present investigation is readily modifiable accordingly.

It only remains to find the density in terms of the temperature and pressure according to the usual relations dependent on the Boyle-Marriotte-Charles law.

Let p_1, p_2 be the atmospheric pressures at the stations P', P''.
t_1, t_2 ,, ,, temperatures ,, ,,
b_1, b_2 ,, heights of the barometers ,, ,,
g_1, g_2 ,, accelerations of gravity ,, ,,
τ_1, τ_2 ,, temperatures indicated by the thermometers attached to the mercurial columns at P', P'' when the corresponding atmospheric temperatures are t_1, t_2.
σ_0 be the density of mercury at 0° C.
β ,, coefficient of expansion of mercury.

Then we have the following relations:

$$\begin{aligned} p_1 &= \kappa.\rho_1.(1 + a.t_1) \\ p_2 &= \kappa.\rho_2.(1 + a.t_2) \end{aligned} \quad . \quad . \quad . \quad (7)$$

But

$$\begin{aligned} p_1 &= g_1.\sigma_0.(1 - \beta.\tau_1).b_1 \\ p_2 &= g_2.\sigma_0.(1 - \beta\,\tau_2).b_2 \end{aligned} \quad . \quad . \quad . \quad (8)$$

Accordingly

$$\begin{aligned} \rho_1 &= \frac{g_1.\sigma_0.(1 - \beta.\tau_1).b_1}{\kappa(1 + a\,t_1)} \\ \rho_2 &= \frac{g_2\,\sigma_0.(1 - \beta.\tau_2).b_2}{\kappa(1 + a.t_2)} \end{aligned} \quad . \quad . \quad . \quad (9)$$

* This latter relation is attributed to Biot and Arago.

To put these into more workable forms, let T_1, T_2 be the absolute temperatures, so that

$$T_1 = \frac{1}{\alpha} + t_1, \quad T_2 = \frac{1}{\alpha} + t_2;$$

and let

$$\mathfrak{T}_1 = \frac{1}{\beta} - \tau_1, \quad \mathfrak{T}_2 = \frac{1}{\beta} - \tau_2;$$

then

$$\omega = \frac{\mu_1 \sin \zeta_1}{\mu_2 \sin \zeta_2} = \frac{(1 + c.\rho_1). \sin \zeta_1}{(1 + c.\rho_2). \sin \zeta_2}.$$

Substituting ρ_1 and ρ_2 from (9).

$$\omega = \frac{\kappa\alpha + c\sigma_0\beta g_1 b_1 \dfrac{\mathfrak{T}_1}{T_1}}{\kappa\alpha + c\sigma_0\beta g_2 b_2 \dfrac{\mathfrak{T}_2}{T_2}} \cdot \frac{\sin \zeta_1}{\sin \zeta_2} \quad . \quad . \quad (10)$$

But g_1, g_2 may generally be put equal to g, where g is computed in the first instance by the formula

$$g = g_0 . \frac{R^2}{(R + H)^2},$$

g_0 being the acceleration of gravity at sea-level at the mean latitude of the survey, and H the average height of the stations. This will usually be sufficiently exact if for no other reason than that g_1, g_2 will often be considerably affected by the distribution of Earth-mass in the vicinity of the stations. Hence putting

$$\frac{\kappa.\alpha}{c.\sigma_0.\beta.g} = A,$$

we have finally

$$\omega = \frac{A + b_1 . \dfrac{\mathfrak{T}_1}{T_1}}{A + b_2 . \dfrac{\mathfrak{T}_2}{T_2}} . \frac{\sin \zeta_1}{\sin \zeta_2} \quad . \quad . \quad . \quad (11)$$

If the units throughout be centimetres and degrees Centigrade,

A is about 5,262,250 cms.

Thus ω is completely determined from the [illegible] h distances and the temperatures and pressures a[illegible] s. This value of ω substituted in equation (2)

$$h = (R + H_1)(\omega - 1)$$

gives the difference of height of the two [illegible] usual correction for "eye and object."

. . . .

It is obvious that, if the air is not in its normal condition, if it is locally disturbed by storms, rain, cloud, excessive moisture, and such-like, the distribution of density in spherical strata may be considerably upset, and the theory will no longer hold good. Should, however, such abnormal conditions occur, they will generally reveal themselves at one or both of the stations by considerable perturbations of the thermometer or barometer. Moreover, it must be remembered that the theory of celestial refraction is vitiated by the same disturbances, though it must be granted that these disturbances are more likely to occur in the strata near the surface, and in which geodetic observations take place, than in the upper regions of the atmosphere. Where long lines pass very close to the Earth's surface, or to intervening plateaus or mountain ranges, considerable discordances may be expected. It must be remembered, however, that conditions such as these, which frequently occur in practical geodesy, must upset any theory. It cannot, therefore, be expected that any theory of terrestrial refraction will yield results so accurate as the Besselian theory of celestial refraction. Still, much better results must follow from expressions based on acknowledged principles rather than blind hypotheses.

For a full account of the previous treatment of the subject, the reader may consult the following references. For drawing his attention to the last publication on this list, the writer would desire to acknowledge his indebtedness to Major E. H. Hills, C.M.G.

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Comparisons of the places of Mars calculated from Newcomb's Tables, with the places calculated from Le Verrier's Tables, near the times of Opposition in 1907 *and* 1909. By A. M. W. Downing, D.Sc., F.R.S.

These comparisons have been made by reducing the quantities given in the *Nautical Almanac* from Greenwich noon to Paris noon at convenient intervals, and then finding the differences between the reduced quantities and the corresponding quantities in the *Connaissance des Temps.*

It will be noticed that at these oppositions, when the planet is nearly at its least possible distance from the Earth, the differences in the heliocentric places derived from the two sets of Tables are much magnified when the transformation to geocentric places is effected. Also it must be remembered that different Tables of the Sun (Newcomb's and Le Verrier's respectively) are employed in making the transformation in the two cases. It will be remarked, however, that the difference in heliocentric longitude, which vanishes in 1907 September, is as much as 4″ in 1909 September. It appears desirable, at the present time, to draw attention to this discordance, as well as to the corresponding discordances in the geocentric places, which amount to comparatively large quantities at the epochs considered.

Mars 1907—*Corrections to Le Verrier's Tables.*

Day 1907.		R.A. Time.	R.A. Arc.	Decl.
		s	″	″
May	26	+·16	+2·2	−0·8
June	3	·17	2·3	0·8
	11	·23	3·1	0·6
	19	·28	3·8	0·8
	27	·28	3·8	1·2
July	5	·29	3·9	1·4
	13	·28	3·7	1·2
	21	·25	3·4	1·8
	29	·19	2·5	1·7
Aug.	6	·14	1·9	1·6
	14	+·13	+1·7	−1·1

On July 5 (near the time of Opposition):—

the correction to Le Verrier's heliocentric longitude of Mars is + 0″·7;
the correction to Le Verrier's longitude of the Sun is − 0″·5;
the distance of Mars from the Earth is 0·41.

Mars 1909—*Corrections to Le Verrier's Tables.*

Day 1909.		R.A. Time.	R.A. Arc.	Decl.
		s	"	"
Aug.	14	−·46	−6·9	−4·5
	22	·50	7·5	4·8
	30	·58	8·7	5·1
Sept.	7	·66	9·9	5·8
	15	·70	10·5	5·8
	23	·70	10·5	5·5
Oct.	1	·72	10·8	5·6
	9	·70	10·5	4·9
	17	·60	9·0	4·2
	25	·58	8·7	4·3
Nov.	2	−·52	−7·8	−4·0

On September 23 (near the time of Opposition):—

the correction to Le Verrier's heliocentric longitude of Mars is − 4″·1;
the correction to Le Verrier's longitude of the Sun is − 0″·9;
the distance of Mars from the Earth is 0·39.

1907 *September* 23.

Errata in Mr. Innes's Paper on the Computation of Secular Perturbations,—M. N., vol. lxvii., No. 7.

To distinguish k_1 on pp. 431, 438, 439, lines 19 and 20, and 443, from the k_1 of the cubic on p. 432, etc., write k'.

p. 431, table, and p. 432, line 2,

$$\textit{for}\quad \frac{a}{r_0} \qquad\qquad \textit{read}\quad \frac{a}{r}.$$

p. 440. In

$$\left(3\chi - \frac{1}{4}g_2 r_0\right)\frac{F_B}{\lambda^{\frac{3}{2}}} + \frac{F_A}{\lambda^{\frac{3}{2}}}$$

interchange suffices $_A$ and $_B$, and then

$$\textit{for}\quad \frac{F_B}{\lambda^{\frac{3}{2}}} \qquad\qquad \textit{read}\quad \phi\frac{F_B}{\lambda^{\frac{3}{2}}}$$

p. 442 and p. 443. **Interchange the numerical values of the** F functions.

For a numerical application of the formulæ see Mr. C. J. Merfield's paper to be published shortly in the *Astr. Nachr.*

Corrections to Prof. E. E. Barnard's Paper on the "Owl" Nebula,—M. N., vol. lxvii., No. 8.

On page 549, for nucleus (1) and (4) the distance measure on 1902 Feb. 24 should be $157''\cdot68$ instead of $158''\cdot68$. The mean for the observation for these stars for 1902 is correctly printed.

For nucleus (1) and (5) the error is in the mean; it should be $181''\cdot07$ instead of $181''\cdot11$.

On page 543, line 25, *for* 1 : 32, *read* 1 : 3·2; and on line 26, *for* 32 times, *read* 3·2 times.

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LIST OF ADDITIONS

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An asterisk (*) indicates that the work is an excerpt.

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